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Boundary value problem for a class of nonlinear second order ordinary differential equations with variable coefficients

In this article the existence of continuous solutions of two-point boundary value problem in an interval of positive real line for a class of nonlinear second order ordinary differential equations with variable coefficients is proved. To prove of the existence theorem about two-point boundary value problem the authors constructed the general solution of the corresponding linear second order differential equations with variable coefficients and used Schauder fixed point principle. The method of building of the solution of linear second order differential equations with variable coefficients and the general solution can be useful for various applications of science. For simplicity the coefficient and the nonlinear part of the ordinary second order differential equation are taken from the class of continuous functions. They can be taken from the class of measurable and essentially bounded functions. One can easily verify that the results remain in force and in this case.

Key words: two-point boundary value problem, ordinary differential equation, the second order, nonlinear equation, general solution, Schauder principle.

1 Introduction

Let $0 < x_1 < \infty$. We consider the equation

$$\frac{d^2 y}{dx^2} + a(x)y = f(x, y) \quad (1)$$

in the interval $[0,\ x_1]$, where $a(x) \in C[0,\ x_1]$ and the function $f(x, y)$ is continuous in the set of variables in the domain $D = \{(x,\ y) : 0 \le x \le x_1,\ | y - \alpha | \le \sigma\}$. Here $y(0) = \alpha,\ \sigma > 0.$ The connection between the numbers x_1 and σ will be defined later. In this article we will solve the following boundary value problem.

Problem *D*. Find the solution of the equation (1) from the class $C^2[0, x_1]$ satisfying the conditions

$$y(0) = \alpha,\ \ \frac{du}{dx}(x_0) = \beta, \quad (2)$$

where α, β — are given real numbers.

2 Construction of the general solution of the linear equation

Let $S[0,\ x_1]$ is the class of measurable, essentially bounded functions in $[0,\ x_1]$, with the norm

$$| f |_0 = \sup_{x\in[0, x_1]} vrai\,| f(x) | = \lim_{p\to\infty} \| f \|_{L_p[0, x_1]} .$$

By $W_\infty^2[0, x_1]$ we denote the class of functions $f(x)$, such that $\frac{d^2 f}{dx^2} \in S[0,\ x_1]$.

We consider the equation

$$\frac{d^2 y}{dx^2} + a(x)u = f(x) \quad (3)$$

in $[0,\ x_1]$, where $a(x),\ f(x) \in S[0,\ x_1]$.

Two-point boundary value problems for the ordinary second order differential equations is a classic of research domain of the theory of ordinary differential equations and due to their wide application in mechanics, mathematical physics and geometry (see, for example, [1–9]) have been actively investigated so far. However, in the mathematical literature mainly it is studied equations of the form (3) with continuous coefficients and sufficient conditions for the solvability of boundary value problems for them. In [10, 11] the general solution of the equation (1) is constructed and the Cauchy problem for him with initial point $x = 0$ is solved. In this section an explicit solution to the boundary-value problem for the equation (3) in the class

$$W_\infty^2[0,\ x_1] \cap C^1[0,\ x_1], \quad (4)$$

where

$$x_1 < \sqrt{\frac{2}{|a|_0}} \qquad (5)$$

is found and the following problem is solved.

Problem D_1. Find the solution of the equation (3) from the class (4) satisfying the conditions (2).

Integrating twice the equation (3), we have

$$y(x) = (By)(x) + g(x) + c_1 x + c_2, \qquad (6)$$

where c_1, c_2 — are arbitrary real numbers,

$$(By)(x) = \int_0^x \int_\tau^{x_1} a(t) y(t) dt d\tau, \quad g(x) = -\int_0^x \int_\tau^{x_1} f(t) dt d\tau.$$

Applying the operator B to both sides of equation (6), we get

$$(By)(x) = (B^2 y)(x) + (Bg)(x) + c_1 a_1(x) + c_2 b_1(x), \qquad (7)$$

where

$$(B^2 y)(x) = (B(By)(x))(x), \ a_1(x) = \int_0^x \int_\tau^{x_1} t a(t) dt d\tau; \quad b_1(x) = \int_0^x \int_\tau^{x_1} a(t) dt d\tau.$$

From (6) and (7) it follows

$$y(x) = (B^2 y)(x) + c_1(x + a_1(x)) + c_2(1 + b_1(x)) + g(x) + (Bg)(x). \qquad (8)$$

In the following we use functions and operators

$$a_k(x) = \int_0^x \int_\tau^{x_1} a(t) a_{k-1}(t) dt d\tau, \quad b_k(x) = \int_0^x \int_\tau^{x_1} a(t) b_{k-1}(t) dt d\tau, \quad (k = \overline{2,\infty}),$$

$$(B^k y)(x) = (B(B^{k-1} y)(x))(x), \quad (k = \overline{2,\infty}).$$

Applying the operator B to both sides of the equation (8), we have

$$(By)(x) = (B^3 y)(x) + c_1(a_1(x) + a_2(x)) + c_2(b_1(x) + b_2(x)) + (Bg)(x) + (B^2 g)(x).$$

From (6) and (9) it follows

$$y(x) = (B^3 y)(x) + c_1(x + a_1(x) + a_2(x)) + c_2(1 + b_1(x) + b_2(x)) + g(x) + (Bg)(x) + (B^2 g)(x). \qquad (9)$$

Continuing this procedure n times, we obtain an integral representation of the solution of equation (1):

$$y(x) = (B^n y)(x) + c_1\left(x + \sum_{k=1}^{n-1} a_k(x)\right) + c_2\left(1 + \sum_{k=1}^{n-1} b_k(x)\right) + \sum_{k=0}^{n-1} (B^k g)(x), \qquad (10)$$

where $(B^0 g)(x) = g(x)$.

Let $y(x) \in C[0, x_1]$. Taking into consideration the definition of the operators $(B^n y)(x)$ and the functions $a_k(x)$, $b_k(x)$, the following estimates are obtained

$$|(B^n y)(x)| \le 2|y|_1 \cdot \frac{|a|_0^n \cdot x_1^{2n}}{2^n}, \quad (n = \overline{1,\infty}), \qquad (11)$$

$$|a_k(x)| < \frac{|a|_0^k \cdot x_1^{2k}}{2^k} \cdot x, \quad |b_k(x)| < \frac{2 \cdot |a|_0^k \cdot x_1^{2k}}{2^k}, \quad (k = \overline{1,\infty}), \qquad (12)$$

where $|f|_1 = \max\limits_{x \in [0, x_1]} |f(x)|$.

Passing to the limit with $n \to \infty$ in the representation (10) and taking estimates (11), (12) into account, we receive

$$y(x) = c_1 I_1(x) + c_2 I_2(x) + F(x), \qquad (13)$$

where

$$I_1(x) = x + \sum_{k=1}^{\infty} a_k(x), \ I_2(x) = 1 + \sum_{k=1}^{\infty} b_k(x), \quad F(x) = \sum_{k=0}^{\infty} (B^k g)(x).$$

Using inequalities (11), (12), we have

$$|I_1(x)| \le \frac{2}{2 - |a|_0 \cdot x_1^2}, \ |I_2(x)| < \frac{2 + |a|_0 \cdot x_1^2}{2 - |a|_0 \cdot x_1^2}, \ |F(x)| \le |g|_1 \cdot \frac{2 + |a|_0 \cdot x_1^2}{2 - |a|_0 \cdot x_1^2}. \qquad (14)$$

From the forms of the functions $I_1(x),\ I_2(x)$ and $F(x)$ for $x \in [0,\ x_1],$ where x_1 satisfies (5), it follows

$$\frac{dI_1}{dx} = 1 + \int_x^{x_1} a(t) I_1(t) dt, \quad \frac{dI_2}{dx} = \int_x^{x_1} a(t) I_2(t) dt;$$
$$\frac{dF}{dx} = -\int_x^{x_1} f(t) dt + \int_x^{x_1} a(t) F(t) dt; \tag{15}$$

$$\frac{d^2 I_1}{dx^2} = -a(x) I_1(x), \quad \frac{d^2 I_2}{dx^2} = -a(x) I_2(x); \tag{16}$$

$$\frac{d^2 F}{dx^2} = f(x) - a(x) F(x). \tag{17}$$

By virtue of (15) and the form of the function $I_1(x),\ I_2(x),\ F(x)$ we obtain

$$I_1(0) = F(0) = \frac{dF}{dx}(x_1) = \frac{dI_2}{dx}(x_1) = 0,\ I_2(0) = \frac{dI_1}{dx}(x_1) = 1. \tag{18}$$

From (16) and (17) it follows that the functions $I_1(x),\ I_2(x)$ are particular solutions from class (4) of the homogeneous equation

$$\frac{d^2 y}{dx^2} + a(x) y = 0 \tag{19}$$

and $F(x)$ is a particular solution of non-homogeneous equation (3).

We will compute the Wronskian of functions $I_1(x)$ and $I_2(x)$. By definition $W(x) = I_1(x) \cdot I_2^{'}(x) - I_2(x) \cdot I_1^{'}(x).$ Hence by virtue of (16) we have $W^{'}(x) = I_1(x) \cdot I_2^{''}(x) - -I_2(x) \cdot I_1^{''}(x) = a(x)(-I_1(x) \cdot I_2(x) + I_2(x) \cdot I_1(x)) = 0.$ Hence, $W(x) = const.$ On the other hand from (18) it follows that $W(0) = -I_1^{'}(0)$ and $W(x_1) = -I_2(x_1),$ so $W(x) = -I_2(x_1) = -I_1^{'}(0).$ Thus we have proved the following theorem.

Theorem 1. Let $I_2(x_1) \neq 0.$ Then the function $y(x),$ defined by the formula (13), is the general solution of equation (3) from the class (4).

For the equation (3) we consider the problem D_1. To solve the problem D_1 we use the general solution of equation (3) given by the formula (13). Substituting the function defined by the formula (13) into the boundary conditions (2) taking into (18), we have $c_2 = \alpha,\ c_1 = \beta.$

Hence the solution of the problem D_1 has the form

$$y(x) = \beta I_1(x) + \alpha I_2(x) + F(x). \tag{20}$$

Thus we have proved the following theorem.

Theorem 2. Problem D_1 has the solution, which is given by (20).

Remark 1. The obtained results remain in force and in the case $a(x),\ f(x) \in C[0, x_1];\ x < \sqrt{\frac{2}{|a|_1}}.$ In this case the solutions given by (13) and (20) belong to the class $C^2[0,\ x_1].$

3 Reduction of the equation (1) to integral equation

In section 2 we construct the general solution of the equation (19) in the form $y(x) = c_1 I_1(x) + c_2 I_2(x),$ where $c_1,\ c_2$ are arbitrary real numbers,

$$I_1(x) = x + \sum_{k=1}^{\infty} a_k(x), \quad I_2(x) = 1 + \sum_{k=1}^{\infty} b_k(x);$$

$$a_1(x) = \int_0^x \int_\tau^{x_1} t a(t) dt d\tau, \quad b_1(x) = \int_0^x \int_\tau^{x_1} a(t) dt d\tau;$$

$$a_k(x) = \int_0^x \int_\tau^{x_1} a(t) a_{k-1}(t) dt d\tau, \qquad b_k(x) = \int_0^x \int_\tau^{x_1} a(t) b_{k-1}(t) dt d\tau, \qquad (k = \overline{2, \infty}).$$

The following inequality can be easily verified

$$|I_1(x)| < \frac{2x}{2-|a|_1\cdot x_1^2}, \qquad |I_2(x)-1| < \frac{2|a|_1\cdot x_1\cdot x}{2-|a|_1\cdot x_1^2}; \tag{21}$$

$$|I_1(x_3)-I_1(x_2)| < (x_3-x_2)\cdot\frac{2}{2-|a|_1\cdot x_1^2},\ |I_2(x_3)-I_2(x_2)| \le (x_3-x_2)\cdot\frac{2|a|_1\cdot x_1}{2-|a|_1\cdot x_1^2}, \tag{22}$$

where $|a|_1 = \max\limits_{0\le x\le x_1}|a(x)|$, $\quad 0\le x_2 < x_3 \le x_1$. Following the method of variation of arbitrary constants, we choose the solution of equation (1) in the form

$$y(x) = c_1(x)I_1(x) + c_2(x)I_2(x), \tag{23}$$

where $c_1(x)$, $c_2(x)$ — unknown functions from the class $C^1[0,\ x_1]$. Counting of the right-hand side of equation (1) is known by the method of variation of arbitrary constants, we obtain a system for determining the functions $c_1(x)$ and $c_2(x)$:

$$c_1'(x)\cdot I_1(x) + c_2'(x)\cdot I_2(x) = 0;$$
$$c_1'(x)\cdot I_1'(x) + c_2'(x)\cdot I_2'(x) = f(x,y).$$

Hence, by virtue of the equation $W(x) = -I_2(x_1)$ we have

$$c_1(x) = \frac{1}{I_2(x_1)}\int_0^x I_2(t)f(t,y)dt + c_1; \quad c_2(x) = \frac{-1}{I_2(x_1)}\int_0^x I_1(t)f(t,y)dt + c_2, \tag{24}$$

where c_1 and c_2 — are arbitrary real numbers. We have assumed here that $I_2(x_1)\ne 0$.

From (23) and (24) it follows

$$y(x) = \int_0^x K(x,t)f(t,y)dt + c_1I_1(x) + c_2I_2(x), \tag{25}$$

where $K(x,t) = \dfrac{1}{I_2(x_1)}(I_1(x)\cdot I_2(t) - I_2(x)\cdot I_1(t))$.

Now we show that any solution of the equation (25) of the class $C[0,x_1]$ satisfies the equation (1).

Theorem 3. Let $I_2(x_1)\ne 0$. Then any solution of the equation (25) from the class $C[0,x_1]$ belongs to the class $C^2[0,x_1]$ and satisfies the equation (1).

Proof. Obviously, the right-hand side of the equation (25) belongs to the class $C^1[0,x_1]$. Therefore, from (25) by virtue of $K(x,\ x)=0$ we get

$$y'(x) = \frac{I_1'(x)}{I_2(x_1)}\int_0^x I_2(t)f(t,y)dt - \frac{I_2'(x)}{I_2(x_1)}\int_0^x I_1(t)f(t,y)dt + c_1I_1'(x) + c_2I_2'(x). \tag{26}$$

Right side of (26) belongs to the class $C^1[0,\ x_1]$. Therefore, $y(x)\in C^2[0,x_1]$. Hence, from (26) by virtue of (16) and equality $W(x) = -I_2(x_1)$ it follows

$$y''(x) = -a(x)\cdot\int_0^x K(x,t)f(t,y)dt + f(x,y) - a(x)(c_1I_1(x)+c_2I_2(x)) = -a(x)y(x) + f(x,y).$$

The theorem is proved.

4 Proof of the existence of continuous solutions of the boundary value problem for the equation (1)

In section 3 it is proved that any solution of the equation (25) from the class $C[0,x_1]$ belongs to the class $C^2[0,x_1]$ and satisfies the equation (1). Now we consider the solution of the equation (25) satisfying the boundary conditions (2). To do this, from (25) and (26) taking into account (18), we obtain

$$c_2 = \alpha,\ c_1 = \beta - \frac{1}{I_2(x_1)}\int_0^{x_1} I_2(t)f(t,y(x_1))dt.$$

Therefore, any solution from the class $C[0,x_1]$ of the equation

$$y(x) = (Ay)(x), \tag{27}$$

where

$$(Ay)(x)=\int_0^x K(x,t)f(t,y)dt-\frac{I_1(x)}{I_2(x_1)}\int_0^{x_1} I_2(t)f(t,y(x_1))dt+\beta I_1(x)+\alpha I_2(x), \tag{28}$$

will be a solution of problem D to the equation (1). Let

$$\left(\gamma K_1+\frac{2(2+|a|_1\cdot x_1^2)\cdot x_1}{(2-|a|_1\cdot x_1^2)^2\,|I_2(x_1)|}+\frac{2|\beta|+2|\alpha|\cdot|a|_1\cdot x_1}{2-|a|_1\cdot x_1^2}\right)\cdot x_1<\sigma, \tag{29}$$

where $K_1=\max\limits_{0<x,t<x_1}|K(x,t)|$; γ — maximum of the function $|f(x,y)|$ in D.

Inequality (29) always might be obtained for the small value of the number x_1. Let us prove the existence of continuous solution of the equation in the interval $[0,\,x_1]$.

Theorem 4. Let $I_2(x)\neq 0,\ a(x)\in C[0,x_1]$ and $f(x,\,y)$ is continuous in the set of variables in the domain D. Then in the interval $[0,x_1]$, where the number x_1 satisfies (29) and (5), there is at least one solution of the equation (1) of class $C^2[0,x_1]$, satisfying (2).

Proof. By virtue of theorem 3 it is sufficient to prove the existence of solutions from the class $C[0,x_1]$ of the equation (27). Let $\|y\|=\max\limits_{0\le x\le x_1}|y(x)|$. We consider the operator A, defined by (28), on the sphere $\|y-\alpha\|\le\sigma$ of the space $C[0,x_1]$. We show that the operator A is continuous on the sphere $\|y-\alpha\|\le\sigma$. If the sequence $\{y_n(x)\}$, belonging to the sphere $\|y-\alpha\|\le\sigma$, converges uniformly to $y(x)$, obviously belonging to the same sphere, then by virtue of the continuity of the function $f(x,y)$ the sequence $\{f(x,y_n(x))\}$ converges uniformly to $f(x,y(x))$ in $[0,x_1]$. Hence, by the passage to the limit under the integral sign the uniform convergence implies that the sequence $\{(Ay_n)(x)\}$ converges uniformly to $(Ay)(x)$, i.e. we have a continuous operator on the sphere $\|y-\alpha\|\le\sigma$. For any element $y(x)$ of sphere $\|y-\alpha\|\le\sigma$ by virtue of (14) we obtain

$$|(Ay)(x)|\le K_1\cdot\gamma\cdot x_1+\frac{2\gamma x_1(2+|a|_1\,x_1^2)}{(2-|a|_1\,x_1^2)^2\,|I_2(x_1)|}+\frac{2|\beta|+2|\alpha|\cdot x_1^2\cdot|a|_1}{2-|a|_1\cdot x_1^2}. \tag{30}$$

If x_2 and x_3 are two arbitrary points of the interval $[0,x_1]$, then by virtue of (22) we will have

$$|(Ay)(x_3)-(Ay)(x_2)|\le\left(K_1\gamma+\frac{2\gamma x_1(2+|a|_1\,x_1^2)}{(2-|a|_1\,x_1^2\,|I_2(x_1)|}+\frac{2|\beta|+2|\alpha|\cdot x_1\cdot|a|_1}{2-|a|_1\cdot x_1^2}\right)\cdot|x_3-x_2|. \tag{31}$$

Inequalities (30) and (31) by virtue of the Arzela show that the operator A transforms the sphere $\|u-\alpha\|\le\sigma$ into a compact set. We now show that the operator A transforms this sphere into itself. In fact, by virtue of (21)

$$|(Ay)(x)-\alpha|<\left(K_1\gamma+\frac{2(2+|a|_1\cdot x_1^2)\cdot x_1}{(2-|a|_1\cdot x_1^2)^2\,|I_2(x_1)|}+\frac{2|\beta|+2x_1\,|a|_1\cdot|\alpha|}{2-|a|_1\cdot x_1^2}\right)\cdot x_1$$

and by virtue of the inequality (29), we obtain here $|(Ay)(x)-\alpha|<\sigma$.

Thus, the operator A satisfies all the conditions of Schauders theorem. Therefore, there is the fixed point of this operator, i.e such a function $y(x)$, so that

$$y(x)=\int_0^x K(x,t)f(t,y)dt-\frac{I_1(x)}{I_2(x_1)}\int_0^{x_1} I_2(t)f(t,y(x_1))dt+\beta I_1(x)+\alpha I_2(x).$$

Hence, by theorem 3 there is the solution of the boundary value problem D for the equation (1). The theorem is proved.

Remark 2. This theorem will be suitable and in more general case, when $a(x)\in S[0,\,x_1]$ and $y(x)\in W_\infty^2[0,\,x_1]\cap C^1[0,\,x_1]$.

References

1 *Kamke E.* Differentialgleichungen. Losungsmethoden and Losungen, Teil 1. Akad. Verlag. — Leipzid, 1959.

2 *Федорук М.Ф.* Асимптотические методы для линейных обыкновенных дифференциальных уравнений. — М.: Наука, 1983.

3 *Наймарк М.А.* Линейные дифференциальные операторы. — 1969.

4 *Сансоне Дж.* Обыкновенные дифференциальные уравнения. — Т. 1. — 1953.

5 *Кодингтон Э.А., Левинсон Н.* Теория обыкновенных дифференциальных уравнений. — 1958.

6 *Boucherif A.* Boundary value problems for second order differential inclusions / A. Boucherif, B. Chanane // Int. I. Differ. Equ. Appl. 7. 2003. — № 2. — 10. 147–151.

7 *Boucherif A.* Second order multivalued boundary value problem / A. Boucherif, B. Chanane // Comm. Appl. Nonlinear Anal. — 2004. — № 1. — P. 85–91.

8 *Donchev T.* A two point boundary value problem for a class of differential inclusions / T. Donchev, M. Quincampoix // I. Nonlinear Complex Anal. 5 (2004). — № 1. — P. 59–69.

9 *Erbe L.* On two point boundary value problems for second order differential inclusions / L. Erbe, R. Ma, CC Tisdell / Dynam. Systems Appl. 15. — 2006. — № 1. — P. 79–88.

10 *Tungatarov A., Akhmed-Zaki.* General solution of second order linear ordinary differential equations with variable coefficients // J. of Inequalities and Special Functions ISSN: 2217-4303. URL: HTTP: / / www. ILIRIAS.com-2012. — Vol. 3, Issue 4. — P. 42–49.

11 *Tungatarov A., Akhmed-Zaki.* Cauchy problem for one class of ordinary differential equations // Int. J. of Mathematical Analyses. — 2012. — Vol. 6. — № 14. — P. 695–699.

Ә.Түнғатаров, М.Байжанова

Айнымалы коэффициенттері бар екінші ретті сызықты емес жай дифференциалдық теңдеулердің бір класы үшін шеттік есеп

Мақалада сандар осінің бір кесіндісінде сингулярлы коэффициенттері бар екі ретті сызықты емес жай дифференциалдық теңдеудің бір класы үшін екі нүктелі шеттік есептің шешімінің бар болуы дәлелденген. Екі нүктелі шеттік есептің шешімі бар болу теоремасын авторлардың құрған сызықтық теңдеулердің жалпы шешімі мен Шаудер әдісі арқылы негізделген. Жалпы шешімдерді табу әдісі және құрылған жалпы шешімдер әр түрлі қолданбалы есептерді шығаруда қолданылуы мүмкін. Алынған нәтижелерді оңай түсіндіру үшін теңдеудің коэффициенттері және сызықты емес бөлік үзіліссіз функциялар класынан алынған. Оларды тек өлшемді және ақырлы дерлік кеңістіктерде алуға болады. Бұл жағдайда да нәтижелер дұрыс деп есептеледі.

А.Тунгатаров, М.Байжанова

Краевая задача для одного класса нелинейных обыкновенных дифференциальных уравнений второго порядка с переменными коэффициентами

В статье доказано существование непрерывных решений двухточечной краевой задачи в некотором отрезке положительной числовой прямой для одного класса нелинейных обыкновенных дифференциальных уравнений второго порядка с переменными коэффициентами. Для доказательства теоремы существования двухточечной краевой задачи использованы построенное авторами общее решение соответствующих линейных дифференциальных уравнений второго порядка с переменными коэффициентами и принцип неподвижной точки Шаудера. Метод построения общего решения линейных дифференциальных уравнений второго порядка с переменными коэффициентами и само общее решение могут быть полезны для решения различных прикладных задач естествознания. Для простоты изложения коэффициент и нелинейная часть обыкновенного дифференциального уравнения второго порядка взяты из класса непрерывных функций, т.е. из класса измеримых и существенно ограниченных функций. Легко можно проверить, что результаты работы остаются в силе и в этом случае.

References

1 Kamke E. *Differentialgleichungen. Losungsmethoden and Losungen*, Teil 1. Akad. Verlag, Leipzid, 1959.

2 Fedoruk M.F. *Asymptotic methods for linear ordinary differential equations*, Moscow: Nauka, 1983.

3 Naimark M.A. *Linear differential operators*, 1969.

4 *Sansone Dzh. Ordinary Differential Equations*, 1, 1953.

5 Kodington E.A., Levinson N. *Theory of ordinary differential equations*, 1958.

6 Boucherif A. *Int. I. Differ. Equ. Appl. 7*, 2003, 2, p. 147–151.

7 Boucherif A. *Comm. Appl. Nonlinear Anal*. 2004, 1, p. 85–91.

8 Donchev T.A. *Two point boundary value problem for a class of differential inclusions* // T. Donchev, M. Quincampoix // I. Nonlinear Complex Anal. 5, 2004, 1, p. 59–69.

9 Erbe L. *On two point boundary value problems for second order differential inclusions* // L. Erbe, R. Ma, CC Tisdell / Dynam. Systems Appl. 15, 2006, 1, p. 79–88.

10 Tungatarov A., Akhmed-Zaki. *J. of Inequalities and Special Functions ISSN: 2217–4303*. URL: HTTP: // www. ILIRIAS.com-2012, 3, 4, p. 42–49.

11 Tungatarov A., Akhmed-Zaki. *Int. J. of Mathematical Analyses*. 2012, 6, 14, p. 695–699.

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Method of constructing general solution of the second order linear ordinary differential equations with variable coefficients

In this article new method of constructing general solution of the second order linear ordinary differential equations with variable coefficients is presented. The general solutions of Airy equation, of the second order linear ordinary differential equations with variable coefficients and coefficient e^x are constructed by this method. Constructed in explicit form general solutions can be used for solving of the Cauchy problem and of the two point's boundary value problems for ordinary differential equations with variable coefficients arising in solving various applied problems of science.

Key words: the second order, linear ordinary differential equation, variable and singular coefficients, general solution.

1 Introduction

Let $-\infty < x_1 < x_2 < \infty$. By $S[x_1, x_2]$ we denote the class of measurable, essentially bounded functions in $[x_1, x_2]$. The norm of an element from S$[x_1, x_2]$ is defined by the formula

$$|f|_0 = \sup_{x\in[x_1,x_2]} vrai|f(x)| = \lim_{p\to\infty}\|f\|_{L_p[x_1,x_2]}.$$

We consider the equation

$$\frac{d^2y}{dx^2} - a(x)y = f(x) \tag{1}$$

in interval $[x_1, x_2]$, where $a(x), f(x) \in S[x_1, x_2]$.

The solution of the equation (1) from class

$$W_\infty^2[x_1,x_2] \cap C^1[x_1,x_2] \tag{2}$$

is sought.

Here $W_\infty^2[x_1,x_2]$ is a class of functions $y(x)$, such that $\frac{d^2y}{dx^2} \in S[x_1,x_2]$.

If $a(x), f(x) \in C[x_1,x_2]$, then general solutions that are found in this article belong to the class $C^2[x_1,x_2]$.