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ON THE PERIODIC CAMASSA – HOLM EQUATION WITH A SOURCE

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We consider the Camassa – Holm (CH) equation with a self-consistent source

$$u_t - u_{xxt} = uu_{xxx} + 2u_x u_{xx} - 3uu_x + \sum_{k=0}^{\infty} \alpha_k(t) s(\pi, \lambda_k, t) q_x(x,t) \psi^2(x, \lambda_k, t) + 2q(x,t)\left(\psi^2(x, \lambda_k, t)\right) \quad (1)$$

in the class of real-valued π- periodic on the spatial variable x function $u = u(x,t)$ which satisfy the regularity of assumption $u \in C_x^3(t>0) \cap C_t^1(t>0) \cap C(t \geq 0)$ with the initial condition

$$u(x,0) = u_0(x), x \in R, \quad (2)$$

where $q(x,t) = u(x,t) - u_{xx}(x,t)$ and $u_0(x) \in C^3(R)$ is the given real-valued π- periodic function and $\psi(x, \lambda_k, t)$ are the Floquet solution (normalized by the condition $\psi(0, \lambda_k, t) = 1$ of the weighted Sturm-Liouville equation.

$$y'' = \frac{1}{4} y + \lambda q(x,t) y, \ \ x \in R. \quad (3)$$

Here λ_k is zeros of the function $\Delta^2(\lambda) - 4$, where $\Delta(\lambda) = c(\pi, \lambda, t) + s'(\pi, \lambda, t)$. We denote by $c(x, \lambda, t)$ and $s(x, \lambda, t)$ the solutions of equation (3) satisfying the initial conditions $c(0, \lambda, t) = 1, c'(0, \lambda, t) = 0$ and $s(0, \lambda, t) = 0$, $s'(0, \lambda, t) = 1$ respectively. In system (1), the functions $\alpha_k(t)$, $k \in Z$, can be chosen freely within the class of real-valued continuousfunctions having uniform asymptotic decay $\alpha_k = O\left(\frac{1}{k^2}\right), k \to +\infty$, thus providing uniform convergence of the series in equation (1).

The aim of this work is to provide a procedure for constructing the solution $u(x,t)$, $\psi(x, \lambda_k, t)$ of problem (1)-(3) using the inverse spectral theory for the weighted Sturm-Liouville equation (3).

For a discussion of integration of the CH equation we refer to works [1-4]. With regard to their applications we refer to works [5-6].

In [7], the CH equation with a self-consistent source was constructed and investigated using the Darboux transformation. In [8], the CH equation with a self-consistent source was integrated by the method of inverse scattering theory.

The main result of the paper is stated in the theorem below.

Theorem 1. Let $u(x,t)$ and $\psi(x,\lambda_k,t)$ be solution of the problem (1)-(3). Then the spectrum of the problem (3) does not depend on t, and the spectral parameters $\xi_n = \xi_n(t)$, $\sigma_n = \sigma_n(t)$, $n \ge 1$ satisfy the analogue of the system of Dubrovin equations

$$\dot{\xi}_n = \left\{ \frac{1}{2\xi_n} - \frac{1}{2}\sum_{j=1}^{\infty}\frac{1}{\xi_j} + \frac{1}{4}\sum_{k=0}^{\infty}\frac{1}{\lambda_k} + \sum_{k=0}^{\infty}\frac{\xi_n \alpha_k(t) s(\pi,\lambda_k,\mathrm{t})}{\xi_n - \lambda_k} \right\} h_n(\xi),$$

where

$$h_n(\xi) = -\frac{\sigma_n \xi_n \sqrt{\left(1-\frac{\xi_n}{\lambda_0}\right)\prod_{i=1}^{\infty}\left(1-\frac{\xi_n}{\lambda_{2i-1}}\right)\left(1-\frac{\xi_n}{\lambda_{2i}}\right)}}{\prod_{j\neq n,\, j=1}^{\infty}\left(1-\frac{\xi_n}{\xi_j}\right)}.$$

The sign $\sigma_n(t) = \pm 1$ changes at each collision of the point $\xi_n(t)$ with the boundaries of its gap $[\lambda_{2n-1}, \lambda_{2n}]$. Moreover, the following initial conditions are fulfilled:

$$\xi_n(t)\big|_{t=0} = \xi_n^0, \quad \sigma_n(t)\big|_{t=0} = \sigma_n^0,\ n \ge 1,$$

where ξ_n^0, σ_n^0, $n \ge 1$ are the spectral parameters of the weighted Sturm-Liouville equations (3) corresponding to the coefficients $q_0(x) = u(x,0) - u_{xx}(x,0) < 0$.

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INTEGRATION OF THE FINITE COMPLEX TODA CHAIN WITH A SELF-CONSISTENT SOURCE

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The finite Toda lattice is a nonlinear Hamiltonian system which describes the motion of N particles moving in a straight line, with “exponential interactions”[1]. A huge number of papers has been devoted to the investigation of the Toda lattices and their various generalizations, from which we indicate here only [2, 3]. With regard to their applications we refer to works [4, 5].

We consider the following system of equations