

References

- [1] Барвайс, Дж. (1982). Теория моделей: справочная книга по математической логике. Часть 1. Москва: Изд. "Наука".
- [2] Yeshkeyev, A. R., Issaeva, A. K., Mussina, N. M. (2019). The atomic definable subsets of semantic model. Bulletin of the Karaganda University-Mathematics, 94(2), 84-91.
- [3] Yeshkeyev, A. R., Mussina, N. M. (2021). An algebra of the central types of the mutually model-consistent fragments. Bulletin of the Karaganda University-Mathematics, 101(1), p. 111-118.

THE CONNECTION OF AP AND JEP-THEORIES IN THE LANGUAGE OF \exists -FORMULAS

Tungushbayeva I.O., Rzabayev A.A.

Karaganda Buketov University, Karaganda, Kazakhstan

E-mail: intng@mail.ru; assylbekrr@gmail.com

Definition 1 [1, p. 80] A theory T has the joint embedding property (JEP) if for any models U, B of the theory T there exists a model M of the theory T and isomorphic embeddings $f: U \rightarrow M, g: B \rightarrow M$.

Definition 2 [1, p. 68] A theory T has the amalgam property (AP) if for any models U, B_1, B_2 of the theory T and isomorphic embeddings $f_1: U \rightarrow B_1, f_2: U \rightarrow B_2$ there are $M \models T$ and isomorphic embeddings $g_1: B_1 \rightarrow M, g_2: B_2 \rightarrow M$ such that $g_1 \circ f_1 = g_2 \circ f_2$.

Initially, the AP and JEP are algebraic properties. In [1], one can find their definitions, commonly used in Model Theory in semantic key. In 1988, Mustafin T.G. proved the statements (Theorem 1 and Theorem 2), where he formulated amalgam and joint embedding properties in a syntactic way.

Let L be a first-order language and T be a theory of L . $(*)$ is a statement that states the following:

Let $\bar{x} \cap \bar{y}$ be empty and $p(\bar{x}), q(\bar{y})$ be such arbitrary sets of Σ -formulas that $T \cup p(\bar{x})$ and $q(\bar{y})$ are consistent, $\bar{x} = \langle x_1, x_2, \dots, x_n \rangle$ and $\bar{y} = \langle y_1, y_2, \dots, y_n \rangle$. Then $T \cup p(\bar{x}) \cup q(\bar{y})$ is also consistent.

Theorem 1 [3]. The following conditions are equivalent: T has JEP if and only if $(*)$ is true.

Let $(**)$ be a statement that says the following:

Let $\bar{x} = \langle x_1, x_2, \dots, x_n \rangle$ and $\bar{y} = \langle y_1, y_2, \dots, y_n \rangle$. If $p(\bar{x})$ and $q(\bar{x})$ are the sets of Σ -formulas such that the sets $T \cup p(\bar{x}), T \cup q(\bar{x}), T \cup \{\neg\phi(\bar{x}) \in \Sigma, \phi(\bar{x}) \notin p(\bar{x}) \cap q(\bar{x})$ are consistent, then the set $T \cup p(\bar{x}) \cup q(\bar{x})$ is consistent.

Theorem 2 [3]. T has AP if and only if $(**)$ is true.

The study of AP, JEP and their connection is of great importance in the development of Model Theory and related fields. It is well known that amalgam property and joint embedding property are independent of each other. This fact can be supported by the theories of various classes of unars [4]. However, when a theory has both AP and JEP, one can reveal some theory's syntactic and semantic specificity that causes the possession of the properties on consideration.

The following definition was introduced by Yeshkeyev A.R. with regard to the connection between AP and JEP in some theories:

Definition 3 [2, p. 130]. Let T be an inductive theory. A theory T is called to be

1. an AP-theory if in the theory T amalgam property implies joint embedding property;
2. a JEP-theory if in the theory T joint embedding property implies amalgam property;
3. an AJ-theory if in the theory T both properties are equivalent.
4. Otherwise, we say that for the theory T , the properties of AP and JEP are independent of each other.

The described classes of theories in Definition 3 form the corresponding subclasses within Jonsson theories. In [2], the authors considered some classical algebras as examples of AP-theories. In this manner, it was shown that the theory of differential fields of characteristic 0, and the theory

of differentially perfect fields of characteristic p are strongly convex theories, which means that they are AP-theories.

Therefore, the study of the connection between AP and JEP requires a detailed semantic and syntactic approach.

In accordance with the notions and assertions considered, we can obtain the following results:

Theorem 3. T is an AP-theory if and only if $(*) \rightarrow (**)$ is true.

Theorem 4. T is a JEP-theory if and only if $(**) \rightarrow (*)$ is true.

Theorem 5. T is an AJ-theory if and only if $((*) \rightarrow (**)) \wedge ((**) \rightarrow (*))$ is true.

This work was supported by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan (grant AP09260237).

References

1. Барвайс, Дж. (1982). Теория моделей: справочная книга по математической логике. Часть 1. Москва: Издательство "Наука".
2. Yeshkeyev, A.R., Tungushbayeva I.O., Kassymetova M.T. (2022). Connection between the amalgam and joint embedding properties. Bulletin of the Karaganda University-Mathematics, 105(1), P. 127–135.
3. Мустафин, Т.Г. (1998) Обобщенные условия Йонсона и описание обобщенно-йонсоновских теорий булевых алгебр, Матем. тр., 1998, том 1, № 2, 135–197.
4. Forrest, W.K. (1977). Model theory for universal classes with the Amalgamation Property: A Study in the foundations of model theory and Algebra. Annals of Mathematical Logic, 11(3), P. 263–366.

THE PERFECT JONSSON S-ACTS

Yarullina A.R., Kaliolla D.Ye.

Karaganda Buketov University, Karaganda, Kazakhstan

E-mail: linka14221@mail.ru, kaliolladana@mail.ru

One can find in [1] that any complete theory of first order can be compared in exactly way with some complete theory of S-acts. We can bring sufficient full list of references to this topic and there is new consideration of main settlements of such problem in the incomplete way. As we know the class of incomplete theories is sufficiently huge and the technical apparatus which is operating in this area is not developed like in the complete case. It is not a surprise because one can get that in classical model theory there are two main branches, they are: “western” and “eastern” directions by historical remarks. As we can see in [2] in the introducing chapter written by J. Keisler, he wrote that in the model theory two great founders of model theory A.Tarski and A.Robinson have formed the main features of details of main streams in developing of model theory. Actually, A.Tarski and A.Robinson lived correspondingly on western and eastern coasts of USA and so-called names of their researching topics conditionally became now such denoting as “western” and “eastern”. Our notes in this abstract paper are dedicated to researchers of Jonsson theories regarding the notion of S-acts. Let us recall the main definitions from the matter of model theory for Jonsson theories and S-acts features. We give the definitions about jonssonness and their immediate consequences from the matter of this definitions and related results necessary for further work in this paper. All of these ones i.e. classical concepts on this topic one can extract from [3]. More new and fresh results of other authors in this area one can find in [4-7].

Recall the main definition on Jonsson theories.

Definition 1. [3] A theory T is said to be Jonsson if this theory satisfies for 4 natural conditions:

- 1) theory T has infinite models;
- 2) theory T is inductive;
- 3) theory T has the joint embedding property (JEP);
- 4) theory T has the property of amalgam (AP).