

of differentially perfect fields of characteristic p are strongly convex theories, which means that they are AP-theories.

Therefore, the study of the connection between AP and JEP requires a detailed semantic and syntactic approach.

In accordance with the notions and assertions considered, we can obtain the following results:

Theorem 3. T is an AP-theory if and only if $(*) \rightarrow (**)$ is true.

Theorem 4. T is a JEP-theory if and only if $(**) \rightarrow (*)$ is true.

Theorem 5. T is an AJ-theory if and only if $((*) \rightarrow (**)) \wedge ((**) \rightarrow (*))$ is true.

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THE PERFECT JONSSON S-ACTS

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One can find in [1] that any complete theory of first order can be compared in exactly way with some complete theory of S-acts. We can bring sufficient full list of references to this topic and there is new consideration of main settlements of such problem in the incomplete way. As we know the class of incomplete theories is sufficiently huge and the technical apparatus which is operating in this area is not developed like in the complete case. It is not a surprise because one can get that in classical model theory there are two main branches, they are: “western” and “eastern” directions by historical remarks. As we can see in [2] in the introducing chapter written by J. Keisler, he wrote that in the model theory two great founders of model theory A.Tarski and A.Robinson have formed the main features of details of main streams in developing of model theory. Actually, A.Tarski and A.Robinson lived correspondingly on western and eastern coasts of USA and so-called names of their researching topics conditionally became now such denoting as “western” and “eastern”. Our notes in this abstract paper are dedicated to researchers of Jonsson theories regarding the notion of S-acts. Let us recall the main definitions from the matter of model theory for Jonsson theories and S-acts features. We give the definitions about jonssonness and their immediate consequences from the matter of this definitions and related results necessary for further work in this paper. All of these ones i.e. classical concepts on this topic one can extract from [3]. More new and fresh results of other authors in this area one can find in [4-7].

Recall the main definition on Jonsson theories.

Definition 1. [3] A theory T is said to be Jonsson if this theory satisfies for 4 natural conditions:

- 1) theory T has infinite models;
- 2) theory T is inductive;
- 3) theory T has the joint embedding property (JEP);
- 4) theory T has the property of amalgam (AP).

All of that above statements natural with the following meaning. Because there are many natural samples in the algebra and correspondingly in the linked areas which are very actively used of ones. The following notions are examples of Jonsson theories: group theory; theory of Abelian groups; theory of fields of fixed characteristics; theory of Boolean algebras; theory of S-acts over a fixed monoid; theory of modules over a fixed ring; theory of linear order. All of these examples are very important not only in an algebra but also in other parts of mathematics and their applications.

We have to give the main definitions of other features which playing important roles in Jonsson model theory as a notion-units or constructions of ones.

Definition 2. Let $\kappa \geq \omega$. Model M of theory T is called κ -universal for T , if each model T with the power strictly less κ isomorphically imbedded in M ; κ -homogeneous for T , if for any two models A and A_1 of theory T , which are submodels of M with the power strictly less than κ and for isomorphism $f: A \rightarrow A_1$ for each extension B of model A , which is a submodel of M and is model of T with the power strictly less than κ there is exist the extension B_1 of model A_1 , which is a submodel of M and an isomorphism $g: B \rightarrow B_1$ which extends f .

Definition 3. Model C of Jonsson theory T is called semantic model, if it is ω^+ -homogeneous-universal.

Definition 4. The center of Jonsson theory T is called an elementary theory of the its semantic model. And denoted through T^* , i.e. $T^* = Th(C)$.

The following two facts speak about the "good" exclusivity of the semantic model.

Fact 1 [3;160]. Each Jonsson theory T has k^+ -homogeneous-universal model of power $2k$. Conversely, if a theory T is inductive and has infinite model and ω^+ -homogeneous-universal model then the theory T is a Jonsson theory.

Fact 2 [3;160]. Let T is a Jonsson theory. Two k -homogeneous-universal models M and M_1 of T are elementary equivalent.

Definition 5. Jonsson theory T is called a perfect theory, if each a semantic model of theory T is saturated model of T^* .

The following theorem is a criterion of perfectness of Jonsson theory.

Theorem 1 [3;158]. Let T is a Jonsson theory. Then the following conditions are equivalent:

- 1) Theory T is perfect;
- 2) Theory T^* is a model companion of theory T .

Theorem 2 [3;162]. If T is a perfect Jonsson theory then $E_T = Mod T^*$.

The next notion which we would like to define is a well-known algebraic object as a S -acts.

Definition 6. [7] Let A be non-empty set, $\langle S; \cdot, e \rangle$ - monoid. Algebraic system $\langle A; \langle f_\alpha: \alpha \in S \rangle \rangle$ with unary operations $f_\alpha, \alpha \in S$, is called a S -act S , if the following conditions hold: $f_e(a) = a$ for all $a \in A, f_{\alpha\beta}(a) = f_\alpha(f_\beta(a))$ for all $a \in A$ and all $\alpha, \beta \in S$.

Let $a \in A$, then $S_a = \{f_\alpha(a): \alpha \in S\}$; if \bar{a} -tuple of elements from A , then $S_{\bar{a}} = \bigcup_{a_i \in \bar{a}} S_{a_i}$. The set $C_a = \{b \in A: b \in S_a \text{ or } a \in S_b\}$ is called a component.

Proposition 1. [7] If T is a S -act theory and for any $f: S_{\bar{a}} \simeq S_{\bar{b}}$ there exists a $g \supset f$ such that $g: C_{\bar{a}} \simeq C_{\bar{b}}$, then T admits the elimination of the quantifiers.

Hereafter, we consider S -acts over the group G and correspondingly the theory of S -acts over the group. If A is a S -act over the group G , $a \in A$, then $id(a) = \{g \in G: f_g(a) = a\}$; $p(G) = \{H: H \leq G\}$. If $H \leq G$, then $\mathfrak{F}(H) = |\{gH: g \in G, \{\varphi \in G: \{\varphi gH\} = gH\} = H\}|$.

Definition 7. [4] A hybrid $H(T_1, T_2)$ of Jonsson theories T_1, T_2 is called the theory $Th_{\forall\exists}(C1 \sqcap C2)$, if it is Jonsson. Herewith, the algebraic construction $(C1 \sqcap C2)$ is called a semantic hybrid of the theories T_1, T_2 .

Note the following fact:

Fact 3. [4] In order for the theory $H(T_1, T_2)$ to be Jonsson enough to $(C1 \sqcap C2) \in E_T$.

Definition 8. Let X be a set, then the fragment of set X will be the following theory $Th_{\forall\exists}(X)$.

Now we are ready to give the main results of this abstract paper.

Theorem 1. Let A_1, A_2 be existentially closed models of some perfect Jonsson existentially complete theory T of given S -act which is closed under Cartesian products of their models. Let T_1, T_2 be existentially complete Jonsson fragments of A_1 and A_2 . With the condition that T_1 is perfect. Then T_2 will be perfect iff T_1 is model consistent with T_2 .

Theorem 2. Let A_1, A_2 be existentially closed models of some perfect Jonsson existentially complete theory T of given S -act which is closed under Cartesian products of their models. Let T_1, T_2 be existentially complete Jonsson fragments of A_1 and A_2 . With the condition that T_1 is perfect. The hybrid of T_1 and T_2 will be perfect iff it is model consistent with T .

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THE PROPERTY OF NON-MULTIDIMENSIONALITY FOR J -BEAUTIFUL PAIRS IN ADMISSIBLE ENRICHMENTS

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We need the following definitions in order to formulate the main result.

Definition 1. [1] Jonsson theory T is called a perfect theory, if each a semantic model of theory T is saturated model of T^* .

Definition 2. [2] An enrichment of the Jonsson theory T is said to be permissible if any \exists -type in this enrichment is definable in the framework of considered stability.

Definition 3. [2] The Jonsson theory is said to be hereditary, if in any of its permissible enrichment any extension of it in this enrichment will be Jonsson theory.

Definition 4. [3] A set X is called a Jonsson set in the theory T , if it satisfies the following properties:

- 1) X is a definable subset of C_T , where C_T is a semantic model of the theory T ;
- 2) $dcl(X)$ is a universe of existentially closed submodel C_T , where $dcl(X)$ is definable closure of X .

Definition 5. Let T be an \exists -complete Jonsson theory of a countable language L , $N, M \in E_T$ and $M \preceq_{\exists_1} N$. We will call a pair (N, M) is called a J -beautiful pair if it satisfies the following conditions:

1. M is $|T|^+ \exists_1$ -saturated;
2. for each $\bar{b} \in N$ each \exists -type over $M \cup \{\bar{b}\}$ is realized in N .