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On the boundedness of solution of the second order ordinary differential equation with damping term and involution

In the present paper the initial value problem for the second order ordinary differential equation with damping term and involution is investigated. We obtain equivalent initial value problem for the fourth order ordinary differential equations to the initial value problem for second order linear differential equations with damping term and involution. Theorem on stability estimates for the solution of the initial value problem for the second order ordinary linear differential equation with damping term and involution is proved. Theorem on existence and uniqueness of bounded solution of initial value problem for second order ordinary nonlinear differential equation with damping term and involution is established.

Keywords: differential equation with damping term and involution, stability, boundedness, existence and uniqueness.

Introduction

Differential equations with involution appear in mathematical models of ecology, biology, and population dynamics (see, e.g, [1–6] and the reference given therein).

Our goal in this paper is to investigate the boundedness of the solution of the initial value problem for the second order ordinary differential equation with damping term and involution

$$y''(t) = f(t, y(t), y'(t), y(u(t))),\ t \in I = (-\infty, \infty),\ y(t_0) = y_0,\ y'(t_0) = y'_0. \quad (1)$$

Here and in future $u(t)$ is involution function, that is $u(u(t)) = t$, and t_0 is a fixed point of u. Problem (1) does not seem to yield directly to any techniques that can be used for ordinary differential equations without involution term [1, 2]. Therefore, we consider the second order linear differential equations with damping term and involution. We obtain equivalent initial value problem for the fourth order ordinary differential equations to the initial value problem for second order linear differential equations with damping term and involution. Theorem on stability estimates for the solution of the initial value problem for the second order ordinary linear differential equation with damping term and involution is proved. Finally, theorem on existence and uniqueness of bounded solution of initial value problem for the second order nonlinear ordinary differential equation with damping term and involution is established. Note that some of the results of this work was presented, without proof, in [7].

Linear ordinary differential equation with damping term and involution

Let $C^\infty[I]$ be the set of all differentiable functions for all degrees.

Theorem 1. Let $a(t)$, $b(t), \alpha(t)$ be functions of class C^∞ on I, such that $b(t)$ does not vanish on the interval I, then the problem

$$y^{''}(t) + \alpha(t)y'(t) = a(t)y(t) + b(t)y(-t) + f(t),\ t \in I,\ y(0) = \varphi,\ y^{'}(0) = \psi$$

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is equivalent to the following problem for the fourth order ordinary differential equation

$$\begin{cases} y^{(4)}(t) = p(t)y(t) + q(t)y^{'}(t) + r(t)y^{''}(t) + s(t)y^{'''}(t) + F(t),\ t \in I, \\ y(0) = \varphi, y^{'}(0) = \psi, \\ y^{''}(0) = a(0)\varphi + b(0)\varphi - \alpha(0)\psi + f(0), \\ y^{'''}(0) = \left[-\alpha(0)\left[a(0) + b(0)\right] + a^{'}(0) + b^{'}(0)\right]\varphi \\ + \left[-\alpha^{'}(0) + \alpha^2(0) + a(0) - b(0)\right]\psi + f^{'}(0) - \alpha(0)f(0), \end{cases}$$

where

$$p(t) = a^{''}(t) + b(-t)b(t) - \left[2b^{'}(t) + b(t)\alpha(-t)\right]\frac{1}{b(t)}a^{'}(t)$$

$$-\left[b^{''}(t) + b(t)a(-t) - \left[2b^{'}(t) + b(t)\alpha(-t)\right]\frac{1}{b(t)}b^{'}(t)\right]\frac{1}{b(t)}a(t),$$

$$q(t) = -\alpha^{''}(t) + 2a'(t) + \left[2b^{'}(t) + b(t)\alpha(-t)\right]\frac{1}{b(t)}\left[\alpha^{'}(t) - a(t)\right]$$

$$-\left[b^{''}(t) + b(t)a(-t) - \left[2b^{'}(t) + b(t)\alpha(-t)\right]\frac{1}{b(t)}b^{'}(t)\right]\frac{1}{b(t)}\alpha(t),$$

$$r(t) = -2\alpha^{'}(t) + a(t) + \left[2b^{'}(t) + b(t)\alpha(-t)\right]\frac{1}{b(t)}\alpha(t)$$

$$+\left[b^{''}(t) + b(t)a(-t) - \left[2b^{'}(t) + b(t)\alpha(-t)\right]\frac{1}{b(t)}b^{'}(t)\right]\frac{1}{b(t)},$$

$$s(t) = -\alpha(t) + \left[2b^{'}(t) + b(t)\alpha(-t)\right]\frac{1}{b(t)},$$

and

$$F(t) = -\left[b^{''}(t) + b(t)a(-t) - \left[2b^{'}(t) + b(t)\alpha(-t)\right]\frac{1}{b(t)}b^{'}(t)\right]\frac{1}{b(t)}f(t)$$

$$-\left[2b^{'}(t) + b(t)\alpha(-t)\right]\frac{1}{b(t)}f^{'}(t) + b(t)f(-t) + f^{''}(t).$$

The proof of Theorem 1 is based on approaches of proof of Theorem 1 of paper [1] on the first order linear differential equation with involution.

Now, we consider the initial value problem

$$y^{''}(t) + \alpha y^{'}(t) = by(-t) + ay(t) + f(t),\ t \in\ I,\ y(0) = \varphi,\ y'(0) = \psi \tag{2}$$

for the second order involutory ordinary differential equation with damping term. We are interested in studying the stability of problem (2) on I. In general cases of α, a and b the solution of (2) is not bounded on I. Applying Theorem 1, we get the equivalent initial value problem

$$\begin{cases} y^{(4)}(t) + (a^2 - b^2)y(t) - \left(2a + \alpha^2\right)y^{''}(t) = F(t), \\ F(t) = -af(t) + bf(-t) - \alpha f'(t) + f^{''}(t),\ t \in I, \\ y(0) = \varphi, y^{'}(0) = \psi, y^{''}(0) = (b + a)\varphi - \alpha\psi + f(0), \\ y^{'''}(0) = -\alpha\left(b + a\right)\varphi + \left(-b + a + \alpha^2\right)\psi + f'(0) - \alpha f(0) \end{cases} \tag{3}$$

for the fourth order ordinary differential equation. We will obtain the solution of problem (3). Assume that $|b| < |a|$, $a \in \left(-\left(\frac{\alpha^2}{4} + \frac{b^2}{\alpha^2}\right), -\frac{\alpha^2}{2}\right)$. Then, it is easy to see that

$$\frac{d^4y(t)}{dt^4} - \left(2a + \alpha^2\right)\frac{d^2y(t)}{dt^2} + \left(a^2 - b^2\right)y(t)$$

$$= \left(\frac{d^2}{dt^2} - \left(a + \frac{\alpha^2}{2} + \sqrt{a\alpha^2 + \frac{\alpha^4}{4} + b^2}\right)\right)\left(\frac{d^2}{dt^2} - \left(a + \frac{\alpha^2}{2} - \sqrt{a\alpha^2 + \frac{\alpha^4}{4} + b^2}\right)\right)y(t).$$

Therefore problem (3) can be written as initial value problem

$$\begin{cases} \left(\frac{d^2}{dt^2} - \left(a + \frac{\alpha^2}{2} + \sqrt{a\alpha^2 + \frac{\alpha^4}{4} + b^2}\right)\right) y\,(t) = v(t), \\ y(0) = \varphi,\ y'(0) = \psi, \\ \left(\frac{d^2}{dt^2} - \left(a + \frac{\alpha^2}{2} - \sqrt{a\alpha^2 + \frac{\alpha^4}{4} + b^2}\right)\right) v(t) = F(t), \\ F(t) = -af(t) + bf(-t) - \alpha f'(t) + f^{''}(t),\ t \in I, \\ v(0) = \left(b - \frac{\alpha^2}{2} - \sqrt{a\alpha^2 + \frac{\alpha^4}{4} + b^2}\right) \varphi - \alpha\psi + f(0), \\ v'(0) = -\alpha\,(b+a)\,\varphi + \left(-b + \frac{\alpha^2}{2} - \sqrt{a\alpha^2 + \frac{\alpha^4}{4} + b^2}\right)\psi \\ +f'(0) - \alpha f(0) \end{cases}$$

for the system of second order differential equations. Applying the d'Alembert's formula, we get

$$y\,(t) = \cos\,(mt)\,\varphi + \frac{\sin\,(mt)}{m}\psi + \int\limits_0^t \frac{\sin\,(m\,(t-s))}{m} v(s)ds, \tag{4}$$

$$v\,(t) = \cos\,(nt)\left[\left(b - \frac{\alpha^2}{2} - \sqrt{a\alpha^2 + \frac{\alpha^4}{4} + b^2}\right)\varphi - \alpha\psi + f(0)\right]$$
$$+\frac{\sin\,(nt)}{n}\left[-\alpha\,(b+a)\,\varphi + \left(-b + \frac{\alpha^2}{2} - \sqrt{a\alpha^2 + \frac{\alpha^4}{4} + b^2}\right)\psi + f'(0) - \alpha f(0)\right]$$
$$+\int\limits_0^t \frac{\sin\,(n\,(t-s))}{n} F(s)ds,$$

where

$$m = \sqrt{-\left(a + \frac{\alpha^2}{2} + \sqrt{a\alpha^2 + \frac{\alpha^4}{4} + b^2}\right)},\ n = \sqrt{-\left(a + \frac{\alpha^2}{2} - \sqrt{a\alpha^2 + \frac{\alpha^4}{4} + b^2}\right)}.$$

Since $F(t) = -af(t) + bf(-t) - \alpha f'(t) + f^{''}(t)$ and

$$\int\limits_0^t \frac{\sin\,(n\,(t-s))}{n} f^{'}(s)ds = -\frac{\sin nt}{n} f(0) + \int\limits_0^t \cos\,(n\,(t-s))\,f(s)ds,$$

$$\int\limits_0^t \frac{\sin\,(n\,(t-s))}{n} f^{''}(s)ds = -\frac{\sin nt}{n} f'(0) - \cos nt f(0) + f(t) - \int\limits_0^t n\sin\,(n\,(t-s))\,f(s)ds,$$

we can write

$$v\,(t) = \cos\,(nt)\left[\left(b - \frac{\alpha^2}{2} - \sqrt{a\alpha^2 + \frac{\alpha^4}{4} + b^2}\right)\varphi - \alpha\psi\right] \tag{5}$$
$$+\frac{\sin\,(nt)}{n}\left[-\alpha\,(b+a)\,\varphi + \left(-b + \frac{\alpha^2}{2} - \sqrt{a\alpha^2 + \frac{\alpha^4}{4} + b^2}\right)\psi\right]$$

$$-a\int\limits_0^t \frac{\sin\left(n\left(t-s\right)\right)}{n}f(s)ds$$

$$+b\int\limits_{-t}^0 \frac{\sin\left(n\left(t+s\right)\right)}{n}f(s)ds-\alpha\int\limits_0^t \cos\left(n\left(t-s\right)\right)f(s)ds$$

$$+f(t)-\int\limits_0^t n\sin\left(n\left(t-s\right)\right)f(s)ds.$$

Applying formulas (4) and (5), we get

$$y\left(t\right)=\cos\left(mt\right)\varphi+\frac{\sin\left(mt\right)}{m}\psi \tag{6}$$

$$+\frac{\cos\left(nt\right)-\cos\left(mt\right)}{m^2-n^2}\left[\left(b-\frac{\alpha^2}{2}-\sqrt{a\alpha^2+\frac{\alpha^4}{4}+b^2}\right)\varphi-\alpha\psi\right]$$

$$+\frac{\frac{1}{n}\sin\left(nt\right)-\frac{1}{m}\sin\left(mt\right)}{m^2-n^2}\left[-\alpha\left(b+a\right)\varphi+\left(-b+\frac{\alpha^2}{2}-\sqrt{a\alpha^2+\frac{\alpha^4}{4}+b^2}\right)\psi\right]$$

$$+\frac{1}{m^2-n^2}\int\limits_0^t\left[-n\sin\left(n\left(t-s\right)\right)+m\sin\left(m\left(t-s\right)\right)\right]f(s)ds$$

$$+\frac{\alpha}{m^2-n^2}\int\limits_0^t\left[\cos\left(n\left(t-s\right)\right)-\cos\left(m\left(t-s\right)\right)\right]f(s)ds$$

$$+\frac{a}{m^2-n^2}\int\limits_0^t\left[-n\sin\left(n\left(t-s\right)\right)+m\sin\left(m\left(t-s\right)\right)\right]f(s)ds$$

$$-\frac{b}{m^2-n^2}\int\limits_{-t}^0\left[-\frac{1}{n}\sin\left(n\left(t+s\right)\right)+\frac{1}{m}\sin\left(m\left(t+s\right)\right)\right]f(s)ds.$$

Theorem 2. Assume that $|b|<|a|$, $a\in\left(-\left(\frac{\alpha^2}{4}+\frac{b^2}{\alpha^2}\right),-\frac{\alpha^2}{2}\right)$. Then problem (2) is stable and the following stability estimate holds

$$\sup_{t\in I}|y(t)|\leq M(a,b,\alpha)\left[|\varphi|+|\psi|+\int\limits_{-\infty}^{\infty}|f(s)|\,ds\right].$$

The proof is based on formula (6) and the triangle inequality.

Nonlinear ordinary differential equation with involution

We consider the initial value problem

$$y''(t)+\alpha y'(t)=by(-t)+ay(t)+f(t,y(t),\ y'(t)),\ t\in\ I,\ y(0)=\varphi,y'(0)=\psi \tag{7}$$

for the second order nonlinear involutory ordinary differential equation. We are interested in studying the existence and uniqueness of bounded solution of problem (7) on I. In general cases of α, a and b the solution of (7) is not bounded on I. We will apply a fixed point theorem.

Let $C^{(1)}(I)$ be the metric space of all continuously differentiable functions defined on the interval I with the metric d defined by

$$d(x,y)=\sup_{t\in I}|x(t)-y(t)|+\sup_{t\in I}\left|\frac{dx\left(t\right)}{dt}-\frac{dy\left(t\right)}{dt}\right|.$$

Note that $C^{(1)}(I)$ is the complete space. This is first condition of a fixed point theorem in metric space (see [9]).

Theorem 3. Assume that $|b| < |a|$, $a \in \left(-\left(\frac{\alpha^2}{4}+\frac{b^2}{\alpha^2}\right), -\frac{\alpha^2}{2}\right)$, and f is continuous and bounded function on the region

$$P = \{(t,x,y) : -\infty < t < \infty,\ \ |x-\varphi| < M,\ \ |y-\psi| < M\}.$$

Suppose that f satisfies a Lipschitz condition on P with respect to its second and third arguments, that is, there is a constant l such that for (t,x,u), $(t,y,v) \in P$

$$|f(t,x,u) - f(t,y,v)| \le l\,(|x-y| + |u-v|). \tag{8}$$

Then, initial value problem (7) has a unique solution $y \in C^{(1)}(I)$.

Proof. The procedure of proving theorem on the existence and uniqueness of a bounded solution of problem (7) is based on reducing this problem to an integral equation

$$y(t) = Ty(t), \tag{9}$$

where

$$Ty(t) = \cos(mt)\,\varphi + \frac{\sin(mt)}{m}\psi$$
$$+\frac{\cos(nt)-\cos(mt)}{m^2-n^2}\left[\left(b-\frac{\alpha^2}{2}-\sqrt{a\alpha^2+\frac{\alpha^4}{4}+b^2}\right)\varphi-\alpha\psi\right]$$
$$+\frac{\frac{1}{n}\sin(nt)-\frac{1}{m}\sin(mt)}{m^2-n^2}\left[-\alpha(b+a)\,\varphi-\left(b-\frac{\alpha^2}{2}+\sqrt{a\alpha^2+\frac{\alpha^4}{4}+b^2}\right)\psi\right]$$
$$+\frac{1}{m^2-n^2}\int_0^t\left[-n\sin(n(t-s))+m\sin(m(t-s))\right]f(s,y(s),y'(s))ds$$
$$+\frac{\alpha}{m^2-n^2}\int_0^t\left[\cos(n(t-s))-\cos(m(t-s))\right]f(s,y(s),y'(s))ds$$
$$+\frac{a}{m^2-n^2}\int_0^t\left[-n\sin(n(t-s))+m\sin(m(t-s))\right]f(s,y(s),y'(s))ds$$
$$-\frac{b}{m^2-n^2}\int_{-t}^0\left[-\frac{1}{n}\sin(n(t+s))+\frac{1}{m}\sin(m(t+s))\right]f(s,y(s),y'(s))ds.$$

The proof of equation (9) is based on the formula (6). Note that integral form is a Volterra type integro-differential equation of the second kind. Therefore, the recursive formula for the solution of problem (7) is

$$y_0(t) = \cos(mt)\,\varphi + \frac{\sin(mt)}{m}\psi$$
$$+\frac{\cos(nt)-\cos(mt)}{m^2-n^2}\left[\left(b-\frac{\alpha^2}{2}-\sqrt{a\alpha^2+\frac{\alpha^4}{4}+b^2}\right)\varphi-\alpha\psi\right]$$
$$+\frac{\frac{1}{n}\sin(nt)-\frac{1}{m}\sin(mt)}{m^2-n^2}\left[-\alpha(b+a)\,\varphi-\left(b-\frac{\alpha^2}{2}+\sqrt{a\alpha^2+\frac{\alpha^4}{4}+b^2}\right)\psi\right],$$
$$y_j(t) = y_0(t) + \frac{1}{m^2-n^2} \tag{10}$$

$$\times \int_0^t \left[-n\sin\left(n\left(t-s\right)\right)+m\sin\left(m\left(t-s\right)\right)\right] f(s,y_{j-1}(s)),y'_{j-1}(s))ds$$

$$+\frac{\alpha}{m^2-n^2}\int_0^t \left[\cos\left(n\left(t-s\right)\right)-\cos\left(m\left(t-s\right)\right)\right] f(s,y_{j-1}(s)),y'_{j-1}(s))ds$$

$$+\frac{a}{m^2-n^2}\int_0^t \left[-n\sin\left(n\left(t-s\right)\right)+m\sin\left(m\left(t-s\right)\right)\right] f(s,y_{j-1}(s)),y'_{j-1}(s))ds$$

$$-\frac{b}{m^2-n^2}\int_{-t}^0 \left[-\frac{1}{n}\sin\left(n\left(t+s\right)\right)+\frac{1}{m}\sin\left(m\left(t+s\right)\right)\right]$$

$$\times f(s,y_{j-1}(s)),y'_{j-1}(s))ds,\ j\geq 1.$$

According to the method of recursive approximation (10), we get

$$y(t)=y_0(t)+\sum_{j=0}^{\infty}\left[y_{j+1}(t)-y_j(t)\right]. \tag{11}$$

We have that

$$y_{j+1}(t)-y_j(t)=\frac{1}{m^2-n^2}\int_0^t \left[-n\sin\left(n\left(t-s\right)\right)+m\sin\left(m\left(t-s\right)\right)\right] \tag{12}$$

$$\times\left[f(s,y_j(s)),y'_j(s))-f(s,y_{j-1}(s)),y'_{j-1}(s))\right]ds$$

$$+\frac{\alpha}{m^2-n^2}\int_0^t \left[\cos\left(n\left(t-s\right)\right)-\cos\left(m\left(t-s\right)\right)\right]$$

$$\times\left[f(s,y_j(s)),y'_j(s))-f(s,y_{j-1}(s)),y'_{j-1}(s))\right]ds$$

$$+\frac{a}{m^2-n^2}\int_0^t \left[-n\sin\left(n\left(t-s\right)\right)+m\sin\left(m\left(t-s\right)\right)\right]$$

$$\times\left[f(s,y_j(s)),y'_j(s))-f(s,y_{j-1}(s)),y'_{j-1}(s))\right]ds$$

$$-\frac{b}{m^2-n^2}\int_{-t}^0 \left[-\frac{1}{n}\sin\left(n\left(t+s\right)\right)+\frac{1}{m}\sin\left(m\left(t+s\right)\right)\right]$$

$$\times\left[f(s,y_j(s)),y'_j(s))-f(s,y_{j-1}(s)),y'_{j-1}(s))\right]ds,\ j\geq 1,$$

therefore, applying the triangle inequality, formula (12) and Lipschitz condition (8), we get

$$\left|y_{j+1}(t)-y_j(t)\right|,\left|y'_{j+1}(t)-y'_j(t)\right|$$

$$\leq M(a,b,\alpha)l\int_{-|t|}^{|t|}\left[\left|y_j(s)-y_{j-1}(s)\right|+\left|y'_j(s)-y'_{j-1}(s)\right|\right]ds \tag{13}$$

for any $t\in I$ and $j\geq 1$. Moreover, applying the triangle inequality, we get

$$\left|y_0(t)\right|,\left|y'_0(t)\right|\leq M_1(a,b,\alpha,\varphi,\psi),$$

$$|y_1(t) - y_0(t)|\,, |y_1'(t) - y_0'(t)| \le M_2(a,b,\alpha)\,|t|\,, \tag{14}$$

for any $t \in I$. Applying estimates (13) and (14), we can prove that

$$|y_{j+1}(t) - y_j(t)|\,, \left|y_{j+1}'(t) - y_j'(t)\right| \le [4M(a,b,\alpha) l M_2(a,b,\alpha)]^j \,\frac{|t|^{j+1}}{(j+1)!} \tag{15}$$

for any $t \in I$ and $j \ge 1$. Therefore, applying the triangle inequality, formula (11) and estimates (13) and (15), we get

$$|y(t) - y_n(t)|\,, |y'(t) - y_n'(t)|$$

$$\le \sum_{j=n+1}^{\infty} [4M(a,b,\alpha) l M_2(a,b,\alpha)]^j \,\frac{|t|^{j+1}}{(j+1)!} \to 0,\ n \to \infty,$$

$$|y(t)|\,, |y'(t)| \le M_1(a,b,\alpha,\varphi,\psi) + M_2(a,b,\alpha)\,|t|$$

$$+ \sum_{j=1}^{\infty} [4M(a,b,\alpha) l M_2(a,b,\alpha)]^j \,\frac{|t|^{j+1}}{(j+1)!}$$

for any $t \in I$. Theorem 3 is proved.

Conclusion

In the present paper the initial value problem for the second order differential equation with damping term and involution is investigated. We obtained equivalent initial value problem for the fourth order ordinary differential equations to the initial value problem for second order differential equations with damping term and involution. Theorem on stability estimates for the solution of the initial value problem for the second order ordinary linear differential equation with damping term and involution is proved. Theorem on existence and uniqueness of bounded solution of initial value problem for the second order ordinary nonlinear differential equation with damping term and involution is established. Moreover, applying this result, the two-step stable difference schemes for the numerical solution of the initial value linear and nonlinear problems (2) and (7) for the second order linear and nonlinear differential equations with damping term and involution can be presented and studied.

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Оғызхан атындағы Түрікменстан инженерлік-технологиялық университеті, Ашхабад, Түрікменстан

Инволюциясы мен жойылып бара жатқан мүшесі бар екінші ретті қарапайым дифференциалдық теңдеудің шектелген шешімі туралы

Мақалада демпингтік мүше пен инволюциясы бар қарапайым екінші ретті дифференциалдық теңдеудің бастапқы есебі зерттелді. Екінші ретті сызықтық дифференциалдық теңдеулер үшін қарапайым, төртінші ретті дифференциалдық теңдеулер үшін бастапқы есептерге эквивалентті есептер алынды. Демпингтік мүше мен инволюциясы бар қарапайым екінші ретті сызықтық дифференциалдық теңдеу үшін бастапқы есепті шешудің тұрақтылығын бағалау теоремасы дәлелденді. Инволюциясы мен жойылып бара жатқан мүшесі бар екінші ретті қарапайым сызықты емес дифференциалдық теңдеу үшін бастапқы есепті шектелген шешімнің бар болуы мен жалғыздығы туралы теорема анықталды.

Кілт сөздер: жойылатын мүшесі және инволюциясы бар дифференциалдық теңдеу, тұрақтылық, шектелген, бар болуы мен жалғыздығы.

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Об ограниченности решения обыкновенного дифференциального уравнения второго порядка с затухающим членом и инволюцией

В статье исследована начальная задача для обыкновенного дифференциального уравнения второго порядка с демпинговым членом и инволюцией. Получены задачи, эквивалентные начальной задаче для обыкновенных дифференциальных уравнений четвертого порядка, начальной задаче для линейных дифференциальных уравнений второго порядка с затухающим членом и инволюцией. Доказана теорема об оценках устойчивости решения начальной задачи для обыкновенного линейного дифференциального уравнения второго порядка с демпинговым членом и инволюцией. Установлена теорема о существовании и единственности ограниченного решения начальной задачи для обыкновенного нелинейного дифференциального уравнения второго порядка с затухающим членом и инволюцией.

Ключевые слова: дифференциальное уравнение с затухающим членом и инволюцией, устойчивость, ограниченность, существование и единственность.

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