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CONSTRUCTION OF AN ELECTRON-OPTICAL SCHEME FOR ELECTROSTATIC QUASI-SPHERICAL DEFLECTOR-TYPE ENERGY ANALYZER

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This study investigates electrostatic spherical deflector-type energy analyzers designed to analyze energy of charged particle beams. The research aims to eliminate quadratic angular aberrations characteristic of classical spherical deflector-type energy analyzers, which limit the quality of angular focusing and the energy resolution of the instruments. An electron-optical scheme of an electrostatic quasi-spherical deflector-type energy analyzer has been proposed, whose field is synthesized using a multipole approach.

The electrostatic field of the energy analyzer is a superposition of an axisymmetric hexapole and a spherical field. Analytical calculations have shown that by choosing the coefficient that determines the weighting contribution of the axisymmetric hexapole, it is possible to completely compensate for second-order angular aberrations and significantly improve the focusing properties of the quasi-spherical energy analyzer.

The profile of the deflecting electrodes of the energy analyzer has been determined, providing the necessary spatial distribution of the deflecting field potential to achieve the specified electron-optical parameters. Numerical simulation of the electron-optical scheme of the quasi-spherical energy analyzer and calculation of the trajectories of charged particles were carried out using the "Focus" numerical program. The electron-optical scheme of the energy analyzer implements a second-order angular focusing mode of the "axis-ring" type.

The instrumental function of the scheme has been constructed. The relative energy resolution and luminosity of the energy analyzer were estimated. The numerical results show that the relative energy resolution of the energy analyzer is 1.6% at a luminosity of 17.5% of 2π, which confirms effectiveness of the proposed scheme. The proposed electron-optical scheme could be used to design real structures of new high-resolution spectrometers intended for the analysis of charged particle beams

Keywords: multipole approach, electrostatic deflector-type energy analyzer, quasi-spherical field, axisymmetric multipole, angular aberrations

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1. Introduction

Corpuscular-optical systems, designed to form and analyze charged particle beams, are a crucial tool in modern experimental physics, space research, and physical electronics. They are used for diagnosing space plasma and solar wind, and for studying the energy and angular distributions of secondary electrons emitted from the surface of solid bodies.

The current stage of physical scientific instrumentation evolution is characterized by a striving for miniaturization, increased energy and angular resolution, and expansion of the range of measurable particle energies. These tasks require the improvement of corpuscular-optical systems capable of ensuring high accuracy in focusing particle beams and minimizing aberrations. Classical spherical and cylindrical electrostatic analyzers, despite their long history of development, have limited capabilities in controlling the aberration properties of the fields.

Therefore, methods for synthesizing new types of electrostatic fields that allow for the targeted formation of the spatial structure of the potential and control of electron-optical parameters are attracting increasing attention. Advancing this area opens up prospects for designing a new generation of instruments with improved characteristics and expanded functionality.

Thus, research on designing new electrostatic systems intended to form and analyze charged particle beams is relevant for modern scientific instrumentation and space technologies.

2. Literature review and problem statement

Work [1] proposes an electrostatic analyzer design that provides high temporal, angular, and energy resolution for detecting solar wind protons. The instrument structure is based on a spherical electrostatic analyzer with sector geometry, which provides a wide range of energies and angles. Although the instrument demonstrates high nominal resolution parameters, questions remain regarding the aberration optimization of the electrode system, the stability of the angular focus, and the preservation of energy selectivity under real operating conditions.

The authors of [2] implemented a triple-dome analyzer with a 360° field of view, allowing simultaneous detection of ions and electrons, which significantly expands the diagnostic capabilities of the instrument. Its design consists of several nested spheroids whose typical dimensions ensure optimal field distribution and compactness, making the instrument suitable for use in space missions. Despite the wide field of view and versatility of detection, the structure of the triple-dome analyzer has limitations related to maintaining resolving power during combined measurements.

In [3], a parametric study on the performance of an analyzer with a hemispherical field of view was carried out, which made it possible to optimize the geometry of the electrodes to increase sensitivity and focusing stability. However, despite the improved operating parameters, this configuration retains the dependence of energy resolution on the particle entry angle.

All the solutions reported in above papers demonstrate that spherical geometry provides high efficiency in particle collection, but the accuracy of energy analysis remains limited due to aberrations in a wide angular range of particle detection.

In work [4], a numerical simulation of an electrostatic spherical analyzer was performed, showing that the input energies of particles can be determined as a function of the exit position. The authors did not analyze the effects of electron-optical aberrations that arise when trajectories deviate from the central orbit. The simulation was carried out assuming ideal spherical symmetry, which does not take into account real field distortions.

Study [5] describes a 90° spherical deflector for spin-polarized photoelectron spectroscopy. Electron spin detection using Mott scattering can be advantageously combined with such a spectrometer, providing energy analysis in spin-polarized photoemission. Although the 90° deflector demonstrates successful implementation of spin-resolved spectroscopy, questions remain regarding the optimization of potential distribution and compensation of aberrations affecting energy and angular resolution.

Work [6] reports a method for three-dimensional calculations of the trajectory of charged particles in a spherical sector electrostatic analyzer, allowing the determination of focusing properties, energy and angular responses. It was revealed that even minor deviations in the shape of the electrodes or an asymmetric potential distribution lead to the appearance of aberrations, which worsen focusing and distort the energy response.

Paper [7] shows an improvement of the prototype of a spherical mirror analyzer previously designed by the author, achieving uniformity of resolution with respect to the angle of radiation. However, even in the modernized version, limitations associated with aberrational field distortions remain. In particular, the non-uniformity of the potential between the mirror surfaces and the imperfect spherical symmetry lead to the appearance of aberrations.

Study [8] numerically modeled the response function of a hemispherical electrostatic analyzer with a curved aperture plate, providing collimation of the scattered field at the input apertures, and investigated the effect of deformation of the inner hemisphere on the transfer characteristics of the device. The results showed that even small geometric deviations lead to a noticeable change in the structure of the electrostatic field and a shift in the focal regions, which causes a deterioration in energy resolution. Despite the detailed numerical analysis, the work does not propose methods for compensating for aberrations arising from imperfect field symmetry.

In all above studies, attention was mainly paid to improving the design parameters of spherical analyzers and their focusing properties. Despite the successes achieved in increasing the energy resolution and expanding the functionality of the devices, the issue of improving focusing properties by controlling the structure of the electrostatic field has not been considered.

Modern technological advancements in energy-analyzing systems for analyzing charged particle flows are aimed at increasing energy and spatial resolution while simultaneously reducing aberrations. A number of papers have proposed various engineering solutions that provide partial compensation for distortions. For example, the authors of [9] describe a compact ion-electron analyzer for space and laboratory measurements, based on a 260° spherical electrostatic sector. The geometry of the spherical plates provides only an approximate fulfillment of isochromatic conditions, which reduces the resolution when operating over a wide energy range.

Two papers [10, 11] investigate double deflection analyzer schemes. Study [10] analyzes the imaging properties of hemispherical deflection analyzers, showing that in an aberration-compensated double scheme (double HDA), mirror symmetry of the trajectories ensures complete compensation of α^2-aberration, while in a double-pass configuration, the summation of dispersions allows for improved energy resolution while maintaining high focusing quality. Work [11] considers a modified Jost double analyzer, which implements a "double-pass" scheme with a four-element decelerating lens. Numerical modeling and experiments showed that double beam passage provides a doubling of energy dispersion and improved resolution. Despite their effectiveness, such systems retain residual distortions.

Several studies [12–15] consider improving spherical analyzers. Work [12] describes an improved display spherical mirror analyzer (DIANA), designed for in situ mapping of the electronic and atomic structure of the surface. Despite all the achievements, DIANA retains a dependence of characteristics on the stability of the spherical field and the accuracy of focusing. The authors of [13] implements a new type of mirror spherical analyzer that provides simultaneous resolution in energy and momentum of photoelectrons. Significant improvement in parameters was achieved due to the geometric optimization of the electrodes and symmetrical beam focusing. It is noted that ensuring high resolution requires strict symmetry of potentials and adjustment of the mirror electrodes, however, aberration compensation was not structurally implemented. Work [14] shows that taking into account the real structure of the edge fields significantly affects the focusing properties of spherical deflectors. The authors showed that optimizing the position of the limiting electrodes relative to the input plane allows for compensation of trajectory distortions and reduction of the influence of angular aberrations caused by edge effects. Nevertheless, the proposed measures are geometric in nature and do not take into account the possibility of actively controlling the field structure.

Paper [15] reports a numerical simulation of electron trajectories in real boundary regions (Herzog zones) of hemispherical energy analyzers. It is shown that aberrations arising from edge fields can be effectively compensated by simply changing the angle of the input beam, which restores focusing and improves the energy resolution of the instrument. Correction by selecting the angle of incidence is effective only for a limited range of conditions and does not eliminate the root cause – the non-ideal potential distribution in the Herzog zones. Thus, aberration compensation in the work was achieved primarily by geometric means – by shifting the electrodes or correcting the shape of the input beam – without structural synthesis of the field, which limits the possibilities for further improvement of electron-optical properties.

The authors of [16] describe a new numerical approach to analyzing the electron-optical properties of an imaging spherical deflector analyzer, based on the simulation of electron trajectories. Aberration compensation is achieved by selecting the shape of the electrodes and potentials, but not through structural control of the field components.

The next stage of evolution is represented by works [17–19], which propose new generation imaging energy analyzers. For example, paper [17] reports an experimental implementation of an electron emission spectrum microscope based on the α-SDA (Spherical Deflector Analyzer) imaging energy analyzer developed by the author. The authors of [18] proposed and implemented a new type of α-SDA imaging energy filter based on a double spherical deflecting field with a total angle of 2π. The symmetrical design provides partial compensation of chromatic and geometric aberrations. The authors of [19] designed a high-energy-resolution 2D photoelectron analyzer for the UV range, based on the implementation of an ideal spherical electrostatic field. This structure made it possible to significantly increase the energy resolution and reduce angular distortions. However, as in previous solutions, distortion compensation is achieved through symmetry and optimization of the electrode shape, and not through targeted field synthesis.

Previously, electron-optical schemes for energy analyzers were proposed and studied, one of the elements of which is a spherical electrostatic mirror. It was noted that the calculated electron-optical systems can be implemented in practice and possess high electron-optical characteristics and wide functionality [20, 21].

Modern devices for analyzing charged particle flows place increasingly high demands on the accuracy of angular focusing and resolving power. However, classic spherical electrostatic deflection-type energy analyzers have a significant drawback – the presence of quadratic angular aberrations, which distort the trajectories of charged particles and limit the efficiency of the instruments. Existing methods for correcting aberrations, as a rule, either complicate the structure of the analyzers or require the introduction of additional electron-optical elements. In this regard, it is necessary to search for and theoretically justify new methods for synthesizing electrostatic fields with controllable focusing properties.

Of particular interest is the multipole approach for synthesizing new deflecting fields. Previously, schemes for energy analyzers based on the superposition of multipole and cylindrical fields were built, possessing high energy resolution and significant luminosity, which makes them promising for use in electron spectroscopy and analysis of charged particle flows [22, 23]. The demonstrated advantages of this approach create prerequisites for its extension to spherical systems, where a similar effect of aberration compensation is expected.

Our review of related literature demonstrates that, despite the variety of structures designed for spherical electrostatic analyzers, their resolving power and focusing stability are still limited by the influence of aberrations. All studies note that even with precise electrode geometry, aberrations persist due to imperfections in the potential distribution, edge effects, and electrode surface deformations. These distortions lead to a decrease in energy resolution and a deterioration of the focusing properties of the instruments. Existing methods for eliminating aberrations, such as optimizing the shape of the electrodes, introducing limiting plates, and selecting the angle of the input beam, only partially compensate for the distortions, but do not provide complete compensation of aberrations. Moreover, most solutions are focused on geometric alignment, rather than on physical control of the electrostatic field structure.

Consequently, the task that must be addressed is to design electrostatic systems that enable targeted control over focusing properties by regulating the type of potential field distribution and the ratio of field components. Solving this problem requires the application of a multipole approach to the synthesis of electrostatic fields with adjustable electron-optical parameters, ensuring compensation of angular aberrations of the second and higher orders, as well as increasing energy resolution.

3. The aim and objectives of the study

The aim of our work is to construct an electron-optical scheme for a quasi-spherical deflector-type energy analyzer based on a field generated by the superposition of an axisymmetric hexapole and a spherical field.

To achieve this aim, the following objectives were accomplished:

– to investigate the structure of the electrostatic field, which is a superposition of an axisymmetric hexapole and a spherical field;

– to determine the profile of the deflecting electrodes capable of inducing an electrostatic field with high focusing properties that can be used as effective energy analyzers for charged particle beams;

– to simulate the electron-optical scheme of an electrostatic quasi-spherical deflector-type energy analyzer based on a field formed by the superposition of an axisymmetric hexapole and a spherical field.

4. Materials and methods

The object of our study is electrostatic spherical deflector-type energy analyzers designed for energy analysis of charged particle beams.

The research hypothesis assumes that applying a multipole approach to the synthesis of the deflecting field of a spherical energy analyzer, namely, adding an axisymmetric hexapole component to the spherical electrostatic field, provides complete compensation of second-order quadratic angular aberrations, which leads to improved angular focusing and energy resolution of the energy analyzer without the use of additional corrective electrodes.

During the study, the following assumptions were adopted: the influence of fringe fields at the entrance and exit of the sector is negligibly small, and the movement of charged particles occurs in an ideal electrostatic field without taking into account space charge.

To ensure analytical solvability and computational stability of the problem, the following simplifications were accepted: an axisymmetric hexapole of the first type was considered, and the contribution of higher multipole components (above the sixth order) to the field structure was excluded from consideration.

An electrostatic potential field having an axis of symmetry and a plane of symmetry (or a plane of antisymmetry) perpendicular to the axis of symmetry is considered (Fig. 1) [24]. A circle of radius r_0, located in this plane, is chosen as the axial circle, with the field potential on this circle being taken as zero. In the case of a plane of antisymmetry, the field potential is zero throughout the entire plane.

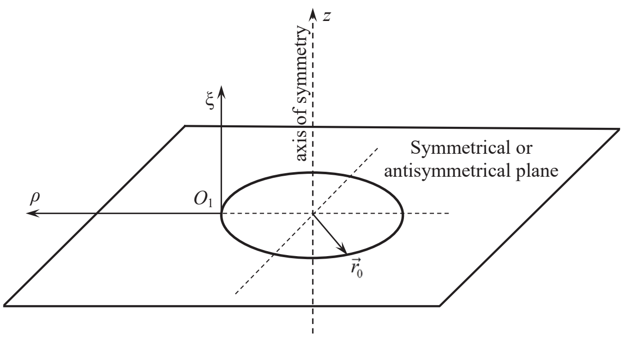


Fig. 1. Geometric elements explaining the description of an axisymmetric multipole

At an arbitrary point O_1 on the axial circle, the origin of the orthogonal coordinate system was aligned with this point. The following coordinates were introduced: $\rho = (r - r_0) / r_0$ – radial coordinate, s – linear or angular coordinate.

The axisymmetric field $U(\rho, \xi)$ satisfies Laplace's equation, the differential operator of which is the sum of two operators separated by coordinates ρ and ξ: $\Delta = T + \tau$ [25]. In the case of spherical coordinates:

$$T = \frac{d}{d\rho}\left[\left(1+\rho\right)^2 \frac{d}{d\rho}\right],$$

$$\tau = \frac{1}{\sin\theta}\frac{d}{d\theta}\left[\sin\theta \frac{d}{d\theta}\right], \tag{1}$$

$$s = \theta.$$

The algorithm for constructing a circular multipole was implemented as follows:

1) a circular multipole of order $2n$ is constructed as a sum, each term of which is the product of functions $f_{n-l}(\rho)$ and $\varphi_l(s)$ and transforms into $\rho^{n-l}s^l$ as ρ and s tend to zero. The functions $f_{n-l}(\rho)$ and $\varphi_l(s)$ constitute sets;

2) generation of sets of functions $f_n(\rho)$ and $\varphi_n(s)$, adopting the following generation rules:

$$Tf_n = -f_{n-1}, \tag{2}$$

$$\tau\varphi_n = \varphi_{n-1}, \tag{3}$$

where $n = 1, 2, 3, \ldots$

The solutions to the corresponding equations were used as basis functions

$$Tf_0 = 0, \tau\varphi_0 = 0. \tag{4}$$

The function of two variables $V_n(\rho, s)$, formed by the symmetrical summation of sets (2) and (3)

$$V_n\left(\rho, s\right) = \varphi_n f_0 + \varphi_{n-1} f_1 + \\ + \varphi_{n-2} f_2 + \ldots + \varphi_1 f_{n-1} + \varphi_0 f_n = \sum_{m=0}^{n} \varphi_{n-m} f_m, \tag{5}$$

is harmonic, i.e., it satisfies condition $\Delta V_n = 0$.

Based on expressions (2) and (3), the following relationship was obtained

$$\Delta V_n = \left(T+\tau\right)V_n = \sum_{m=0}^{n} \varphi_{n-m} T \cdot f_m + \sum_{m=0}^{n} f_m \cdot \tau\left(\varphi_{n-m}\right) = \\ = -\sum_{m=1}^{n} \varphi_{n-m} f_{m-1} + \sum_{m=0}^{n-1} \varphi_{n-m-1} \cdot f_m, \tag{6}$$

Here, $\varphi_{-1} = f_{-1} = 0$. After shifting the summation index m in the first sum by one, it is established that the sums in expression (6) are equal in magnitude and opposite in sign.

The constructions of the field thus depend on the choice of the basis functions $f_0(\rho)$ and $\varphi_0(s)$, and the boundary conditions that the functions $f_n(\rho)$ and $\varphi_n(s)$ must satisfy. The basis function $f_0(\rho)$ is a fundamental solution to the Laplace equation satisfying the condition $f_0(0) = 0$. The basis function $\varphi_0(s)$ is a solution to equation (4), or is equal to unity, or some odd function of s, determined individually for the chosen coordinate system.

5. Results of constructing an electron-optical scheme for a sector quasi-spherical deflector-type energy analyzer

5. 1. Investigating the structure of the electrostatic field induced by the superposition of an axisymmetric hexapole and a spherical field

The coordinate $s = \theta$ is the polar angle in the spherical coordinate system; in the plane of field symmetry, $\theta = \pi/2$. The operators T and τ are defined from formulas (1). The fundamental solution to the Laplace equation in this coordinate system is the function $f_0 = 1 - (1 / (1 + \rho))$, which is equal to zero on the sphere $\rho = 0$.

By choosing the basis functions $f_0(\rho) = \varphi_0(\theta) = 1$ in accordance with rules (2) to (5), a spherical quadrupole and octupole are constructed. The consideration is limited to structures symmetrical with respect to the plane $\theta = \pi/2$:

$$U_q\left(\rho,\theta\right) = \ln\left(\sin\theta\right) + 1 - \ln\left(1+\rho\right) - \frac{1}{1+\rho}, \tag{7}$$

$$U_{oct}\left(\rho,\theta\right) = \frac{1}{2}\left[\ln\left(\sin\theta\right)\right]^2 - \frac{1}{2}\left[\ln\left(\tan\frac{\theta}{2}\right)\right]^2 + \\ + \left[\ln\left(1+\rho\right) + \frac{1}{1+\rho} - 2\right]\ln\left(\sin\theta\right) + \\ + \frac{1}{2}\left[\ln\left(1+\rho\right)\right]^2 - 2\ln\left(1+\rho\right) - \frac{1}{2\left(1+\rho\right)^2} - \frac{1}{1+\rho} + \frac{3}{2}. \tag{8}$$

For the basic functions $f_0 = 1 - (1/(1 + \rho))$, $\varphi_0(\theta) = 1$, a spherical hexapole is obtained

$$U_h\left(\rho,\theta\right) = \left[2 - \ln\left(\sin\theta\right)\right]\left[1 - \frac{1}{1+\rho}\right] - \\ - \ln\left(1+\rho\right)\left[1 + \frac{1}{1+\rho}\right]. \tag{9}$$

Fig. 2 shows the family of equipotential lines of a circular spherical hexapole $U_h(\rho, \theta)$. For convenience of plotting, Cartesian coordinates $x = (1 + \rho)\sin\theta$, $y = (1 + \rho)\cos\theta$ were used.

Here, in the cross-section of the azimuthal plane, three branches of zero potential (aO_1b, cO_1d, eO_1f), intersecting at nodal point O_1, delineate the field into six regions with potentials of opposite signs.

The structure of combined fields is considered, provided that when they are superimposed, the central circle of the multipole coincides with the axial circle of the energy analyzer scheme. The potential of the total field, which is a superposition of a spherical hexapole (9) and a spherical field, is determined from the following formula

$$U(x,y)=U_h(x,y)+\mu\left[\frac{1}{\sqrt{x^2+y^2}}-1\right], \tag{10}$$

where μ is a coefficient that defines the weighting contribution of the spherical field.

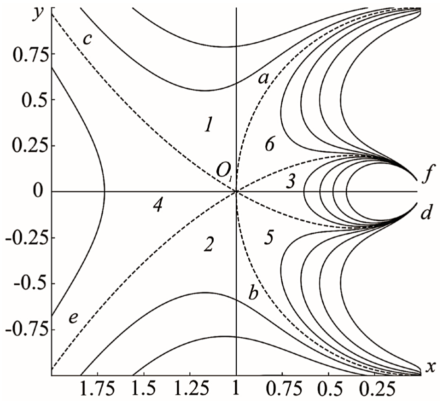


Fig. 2. Family of equipotential lines of a spherical hexapole $U_h(x,y)$: 1–6 – regions of potentials of opposite signs

The dynamics of changes in the structure of the total field (14) were investigated when the value of coefficient μ, which defines the weighting contribution of the spherical field, was changed. Families of equipotential of the total field (10), formed by the superposition of an axisymmetric hexapole and a spherical field, were calculated (series of Fig. 3, 4).

Analysis of the dynamics of changes in the structure of the electrostatic field formed by the superposition of an axisymmetric hexapole and a spherical field revealed that the nature of the equipotential distribution significantly depends on the value of coefficient μ, which determines the weighting contribution of the spherical field. At small values of coefficient μ (for example, $\mu = 0.1$), the equipotentials retain a pronounced multipole symmetry. With an increase in coefficient μ, a gradual deformation of the equipotential lines is observed, and a tendency towards increased radial symmetry becomes apparent. At $\mu = 1$, the spherical field makes a decisive contribution, which leads to a transformation of the potential distribution pattern towards an approximation of a quasi-spherical type.

At negative values of coefficient μ, a fundamentally different dynamic of the transformation of the total field structure is observed. Negative values of coefficient μ cause the destruction of the regular multipole structure and lead to the formation of asymmetric distributions.

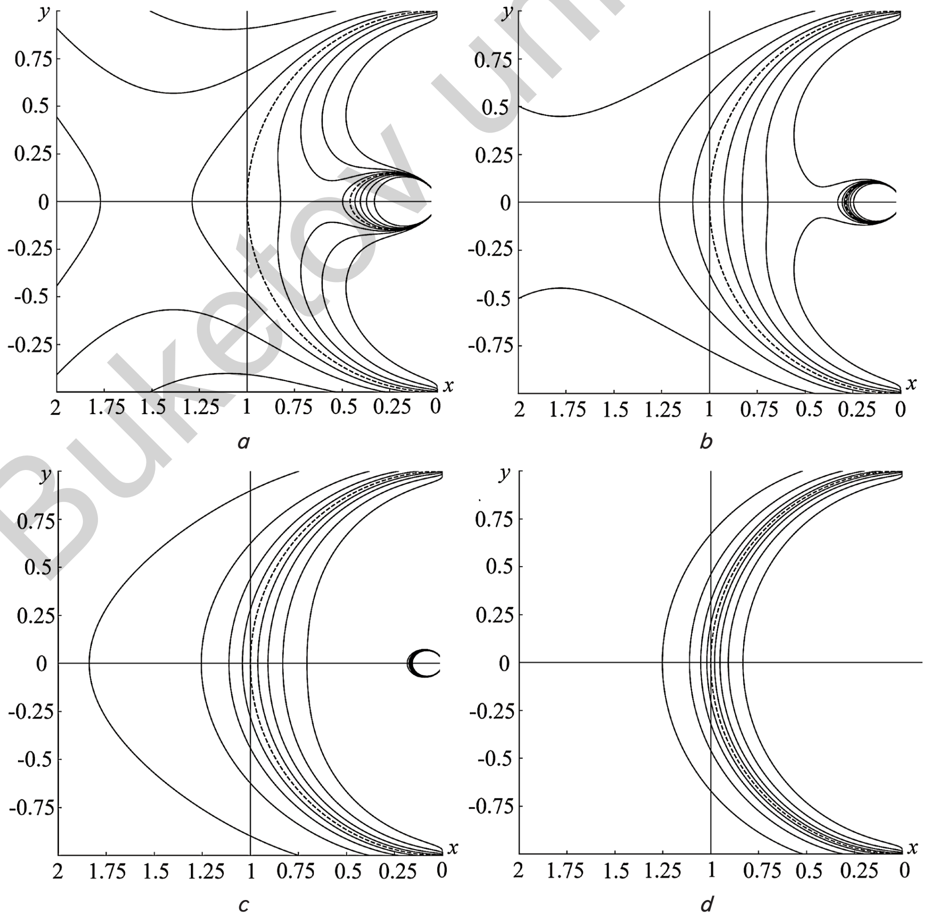
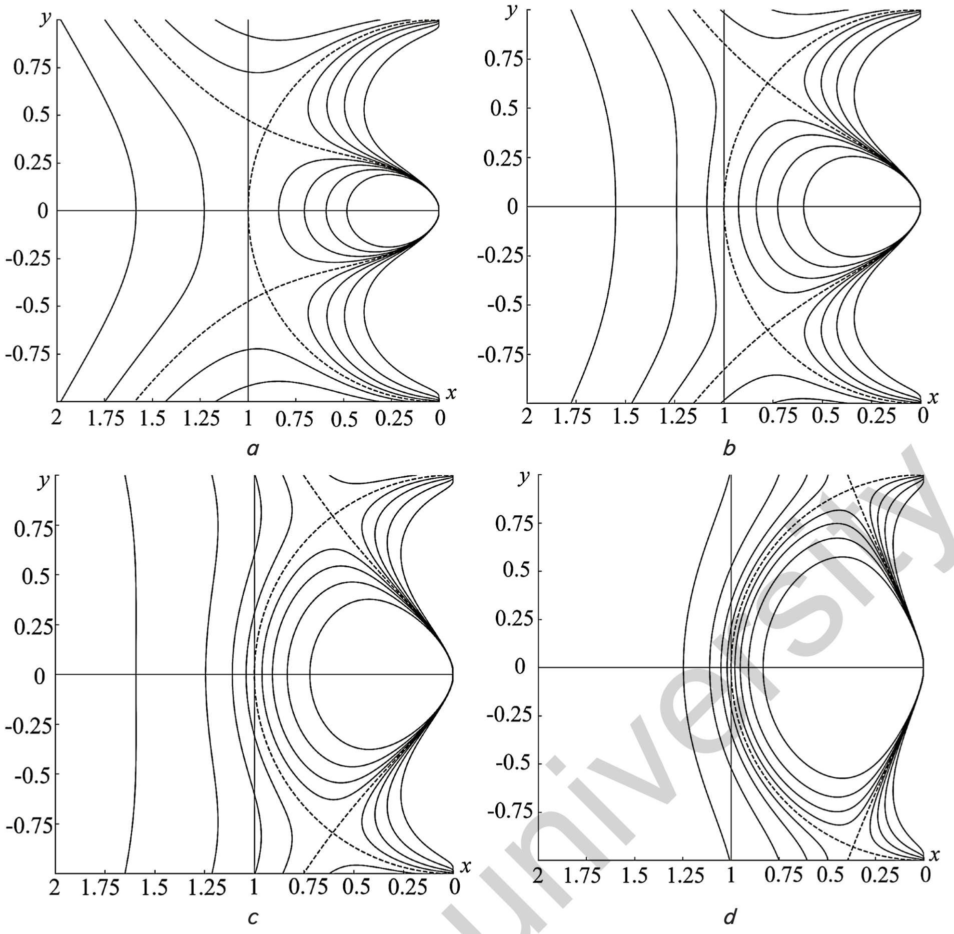


Fig. 3. Equipotential lines of the total field $U(x,y)=U_h(x,y)+\mu\left[\frac{1}{\sqrt{x^2+y^2}}-1\right]$: $a - \mu = 0.1$; $b - \mu = 0.25$; $c - \mu = 0.5$; $d - \mu = 1$



a b c d

Fig. 4. Equipotential lines of the total field $U(x,y)=U_h(x,y)+\mu\left[\frac{1}{\sqrt{x^2+y^2}}-1\right]$:

$a-\mu=-0.1$; $b-\mu=-0.25$; $c-\mu=-0.5$; $d-\mu=-1$

5. 2. Applying an axisymmetric hexapole for the synthesis of deflecting fields in electrostatic sector energy analyzers of the deflector type

One of the areas for applying a circular multipole is related to sector energy analyzers of the deflector type for charged particle beams.

The electron-optical parameters of an electrostatic sector energy analyzer depend on the structure of the deflecting field, concentrated in a narrow region localized near the axial orbit r_0, in which the components of the electric field vector are represented by the following series:

$$E_\rho=E_0\begin{bmatrix}1+\varepsilon_1\rho+\varepsilon_2\rho^2+\\+1/2\left(1-\varepsilon_1-2\varepsilon_2\right)\xi^2+\varepsilon_3\rho^3+\ldots\end{bmatrix},$$

$$E_\xi=E_0\left[-\left(1+\varepsilon_1\right)\xi+\left(1-\varepsilon_1-2\varepsilon_2\right)\xi\rho+\ldots\right], \quad (11)$$

where $\rho=(r-r_0)/r_0$, $\xi=z/r$.

The differential equation of motion for charged particles in a deflecting electrostatic field takes the following form:

$$r''-2\frac{r'^2}{r}-r=\frac{e}{m}E_r\left(r,z\right)\left(\dot{\varphi}\right)^{-2},$$

$$z''-2\frac{r'z'}{r}=\frac{e}{m}E_z\left(r,z\right)\left(\dot{\varphi}\right)^{-2}, \quad (12)$$

where e and m are the charge and mass of the electron.

The differential equation of motion of charged particles (12) in the field defined by expansion (11) is solved by the method of successive approximations. The electron-optical characteristics of the energy analyzer, determined in the first approximation, include the parameters of first-order angular focusing, linear energy dispersion, and linear magnification coefficient. These parameters depend on the expansion coefficient ε_1 (11), while coefficients ε_2 and ε_3 affect aberrations of the second and higher orders. It is possible to realize the given electron-optical parameters of the energy analyzer only at strictly defined values of the expansion coefficients ε_n. However, to construct an energy analyzer with the required parameters, it is necessary to know the field distribution and specify the shape of the deflecting electrodes in a volume significantly exceeding the region in which the analyzed beam is localized. Restoring the potential of an axisymmetric field in all space from expansion (11) is very difficult, while the multipole approach significantly simplifies solving this problem.

Connecting multipole components to the basic deflecting field of the energy analyzer, whose axial circular orbit of radius r_0 is aligned with the axial circumference of the multipole, allows for the formation of complex fields. These fields have constituent components (quadrupole, hexapole, etc.) that selectively influence the parameters of the energy analyzer, in particular the aberration coefficients of various orders, thereby making it possible to eliminate them. This approach provides a new way to solve the problem of synthesizing a deflecting field that provides the specified electron-optical parameters of the energy analyzer, completely eliminating the difficulties of analytically defining the field in the entire space and finding the configuration of the deflecting electrodes.

The energy analyzer field is considered as a superposition of constituent fields

$$\frac{U(\rho,\xi)}{U_0}=g(\rho,\xi)=U_0(\rho,\xi)+qU_q(\rho,\xi)+$$
$$+\beta U_h(\rho,\xi)+\omega U_{oct}(\rho,\xi)+dU_d(\rho,\xi)+..., \tag{13}$$

where $U_0(\rho, \xi)$ is the base field, $U_q(\rho, \xi)$, $U_h(\rho, \xi)$, etc. are the components of the axisymmetric multipole; q, β, ω, d are the weighting components of the multipole components.

A relationship has been established between the weighting components of sum (13) and the expansion coefficients (11). The coefficients of these expansions represent the contribution of each multipole component to ε_n and are given in Table 1 along with the expansion coefficients of the base spherical field intensity. It was taken into account in the calculations that $\theta = \pi/2$.

Table 1

Spherical field and spherical multipoles

ε_i	$1-\frac{1}{1+\rho}$	$q\left.\frac{\partial U_q}{\partial \rho}\right\|_{\rho=0}$	$\beta\left.\frac{\partial U_h}{\partial \rho}\right\|_{\rho=0}$
ε_1	–2	$-q$	0
ε_2	3	$2q$	$-s/2$
ε_3	–4	$-3q$	$4s/3$
ε_4	5	$4q$	$-29s/12$

Table 1 shows that connecting the quadrupole component to the basic field affects the parameters of the energy analyzer calculated in the first approximation, while connecting the hexapole component affects the parameters starting from the second approximation.

The goal of the next stage of consideration was the theoretical justification of the possibility of improving the quality of angular focusing of sector-type deflector energy analyzers built on the basis of a synthesized field (13).

The point source and its image are located on the axial trajectory in the energy analyzer field; the conditional boundaries of the sector are the axial planes in which the source and image are located. This assumption made it possible to disregard the influence of fringe fields at the sector boundaries.

The profile of the deflecting electrodes of a quasi-spherical energy analyzer, designed to compensate for quadratic angular aberrations when appropriate potentials are applied to them, has been determined. An angular focusing scheme in an energy analyzer whose deflecting field is given by superposition (13) is considered.

In an electrostatic energy analyzer, spatial focusing of a beam of charged particles is carried out. The quadratic aberration coefficient with respect to α at $\varphi = 180°$ is $A_{II}(\varphi) = 0$. A hexapole component has been added to the basic electrostatic spherical field

$$g(\rho,\xi)=1-\frac{1}{1+\rho}+\beta U_h. \tag{14}$$

The coefficient of quadratic angular aberrations that blur the image of the source in the radial plane

$$A_{II}(\varphi)=-\frac{K}{2\eta^2}\left[1-\cos\sqrt{\eta}\varphi\right]+$$
$$+\frac{1+\frac{K}{2\eta}}{3\eta}\left[\cos\sqrt{\eta}\varphi-\cos 2\sqrt{\eta}\varphi\right], \tag{15}$$

where $\eta = 3 + \varepsilon_1$, $K = 6 + 4\varepsilon_1 + \varepsilon_2$, φ is the angular coordinate measured from the point source in the direction of the beam, $\eta = 1$, $K = 1 - (\beta / 2)$.

According to Table 1: $\varepsilon_1 = -2$, $\varepsilon_2 = 3 - (\beta/2)$, $\varepsilon_3 = -4 + (4\beta/3)$. From expression (15) it follows that condition $A_{II} = 0$ is achieved by choosing the weighting coefficient $\beta = 3$.

According to (9) and (11), the equation of equipotentials for a spherical deflector-type energy analyzer takes the following form

$$\frac{U}{U_0}=\frac{1}{\sqrt{x^2+y^2}}-1+$$
$$+\beta\left\{\begin{array}{l}\left(2-\ln\frac{x}{\sqrt{x^2+y^2}}\right)\left(\frac{1}{\sqrt{x^2+y^2}}-1\right)+\\+\ln\frac{1}{2}\left(x^2+y^2\right)\left(\frac{1}{\sqrt{x^2+y^2}}+1\right)\end{array}\right\}. \tag{16}$$

The zero equipotential line is the circle $(1 + \rho)^2 + \xi^2 = 1$.

Fig. 5 shows the equipotentials for $U/U_0 = \pm 0.5$, $\varphi = 180°$: 1–1′ – field of a spherical deflector-type energy analyzer ($\beta = 0$), in which $A_{II}(\varphi) = -2$; 2–2′ – field (10), constructed on the basis of a superposition of a spherical field and a circular hexapole ($\beta = 3$), $A_{II}(\varphi) = 0$. The profile of the deflecting electrodes of the proposed quasi-spherical sector deflector-type energy analyzer coincides with the equipotential 2–2′.

The dashed line indicates the zero equipotential in the form of a circle of unit radius, which hosts the axial trajectory of the beam, intersecting the plane of the figure at point O_1. The radial coordinates of extreme points a and b on the surface of deflecting electrodes are $r_a / r_0 = 2.45$, $r_b / r_0 = 0.95$.

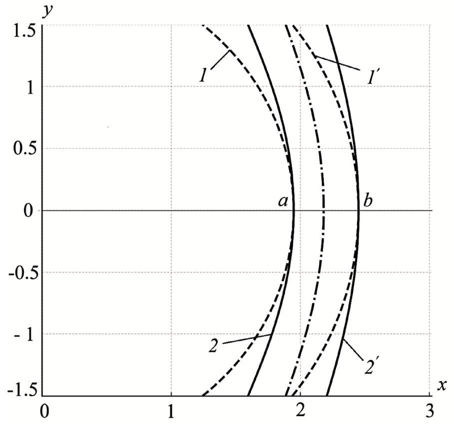


Fig. 5. Profile of deflecting electrodes for sector-type energy analyzers: 1–1′ – for a spherical field ($\beta = 0$); 2–2′ for a synthesized field (10) ($\beta = 3$)

5. 3. Modeling the electron-optical scheme of a quasi-spherical deflector-type energy analyzer

Numerical modeling of the electron-optical scheme of a quasi-spherical deflection-type energy analyzer was performed. The modeling was carried out using the numerical program "Focus" [26], designed for modeling axially symmetric corpuscular-optical systems.

Fig. 6 shows the distribution of the electrostatic field in the electron-optical scheme of the quasi-spherical energy analyzer. Here, the potential values at the grid nodes of the region subdivision were calculated, and the field distribution was visualized using a color scale, where higher potential values correspond to "warm" shades.

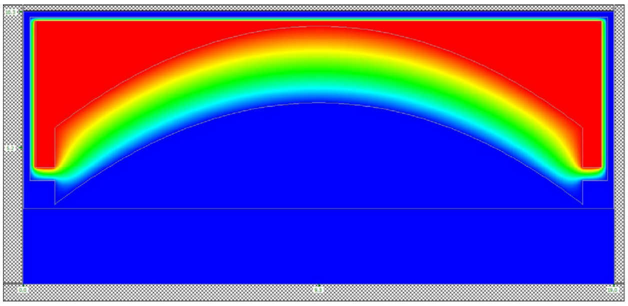


Fig. 6. Distribution of the electrostatic field in the electron-optical scheme of a quasi-spherical energy analyzer

Fig. 7 shows the electron-optical scheme of a quasi-spherical energy analyzer. The field is formed between electrodes 2 and 3, and a deflecting potential U is applied to the outer electrode. According to the diagram, charged particles originate from a point source 1, then enter the field, and under the action of the potential on the outer electrode 3, are deflected back and focused into an annular image 5. The ratio of the kinetic energy of the charged particle to the potential of the outer electrode is $E / V = 1.6$ eV/V.

The divergence of the charged particle beam in this case is limited by angles (51–63)°, which corresponds to a light-gathering power (solid angle) of 17.5% of 2π. Thus, the proposed scheme provides angular focusing of the "axis-ring" type.

Table 2 gives focusing properties of a quasi-spherical energy analyzer under the "axis-to-ring" angular focusing mode.

Fig. 8 shows the instrumental function of the quasi-spherical energy analyzer. The exit aperture was located at the exit focal point. The relative energy resolution is $R(\%) = \Delta E / E = 1.6\%$.

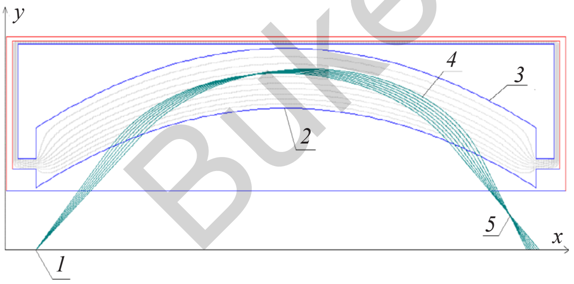


Fig. 7. Electron-optical diagram of a quasi-spherical energy analyzer: 1 — point source of charged particles; 2 — inner electrode; 3 — outer electrode; 4 — trajectories of charged particles; 5 — output focus

Table 2

Focusing properties of a quasi-spherical energy analyzer

Focusing type	"Axis-to-ring"
Focusing order	2
Central focusing angle	57°
X coordinate of the focal point	16.97
Y coordinate of the focal point	1.5
Reflection parameter	1

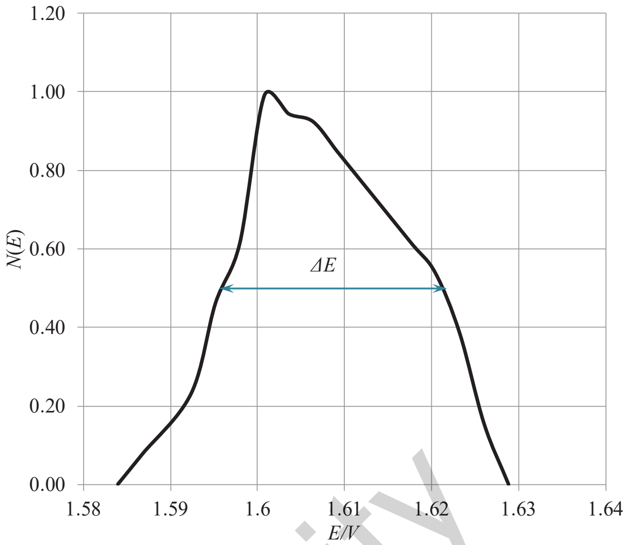


Fig. 8. Instrumental function of a quasi-spherical energy analyzer

Thus, numerical modeling showed that the relative energy resolution of the energy analyzer is 1.6% at an aperture of 17.5% of 2π.

6. Results of calculating and modeling the electron-optical scheme of a quasi-spherical deflector-type energy analyzer: discussion

The results of calculating the equipotential families of the total field, shown in Fig. 3, 4, demonstrate a regular change in the structure of the electrostatic field when varying the value of coefficient μ, which determines the weighted contribution of the spherical component. The calculation of equipotential families showed that at small values of μ (0.1–0.25), the field retains a pronounced multipole symmetry characteristic of a hexapole, while with an increase in μ to 1, a transition to a radially symmetric potential distribution corresponding to a quasi-spherical type of field is observed. The nature of the electrostatic field structure is of fundamental importance since the geometry of the deflecting electrodes of the energy analyzer replicates one of the equipotentials of the calculated field. Thus, the shape and symmetry of the equipotentials directly determine the shape of the working electrodes of the energy analyzer, providing the required potential distribution and specified focusing properties. Therefore, the change in the field structure when varying the coefficient μ not only reflects the dynamics of potential redistribution but also provides the basis for the practical implementation of the optimal electrode profile that compensates for aberrations.

According to the calculation results given in Table 1 and expressions (11) to (15), the addition of a spherical multipole to the main field of the spherical energy analyzer leads to a change in the expansion coefficients (11), starting from the quadratic terms. The choice of values for coefficient β, which defines the weighting contribution of the axisymmetric hexapole, allows for targeted control over the magnitudes of the second-order aberration coefficients and thus correction of the angular focusing of charged particle beams.

The calculation results obtained by the perturbation method showed that the additive addition of a hexapole to the main deflecting spherical field affects the angular focusing of the energy analyzer. Unlike classical spherical deflector-type energy analyzers [4–6], in which compensation of quadratic angular aberrations is impossible without additional corrective electrodes, the proposed method allows this to be achieved due to the new field structure formed by multipole superposition.

The aberration coefficients, which determine the quality of angular focusing, characterize the image blurring of the second and higher orders with respect to φ. The aberration coefficients of the energy analyzer depend on coefficient ε_2 and other expansion coefficients (11), which in turn depend on coefficient β, which defines the weighting contribution of the axisymmetric hexapole. Thus, an additional field parameter β is introduced, determined from the requirement that the coefficient of the angular quadratic aberration with respect to φ is equal to zero.

The improvement in angular focusing quality in the proposed quasi-spherical energy analyzer compared to the classical spherical deflector-type energy analyzer has made it possible to increase the luminosity several times by increasing the beam divergence angle from the axial orbit plane φ. This also contributes to the symmetrization of the energy analyzer's instrumental function under conditions of angular focusing of a higher order than the first.

The results of numerical modeling performed using the "Focus" software package (Fig. 6–8, Table 2) confirm the effectiveness of the proposed electron-optical scheme of the quasi-spherical energy analyzer. The calculations showed that the implemented electrostatic field distribution provides a second-order angular focusing mode of the "axis-ring" type. With an optimal ratio of parameters $E/V = 1.6$ eV/V, high focusing quality is achieved without the introduction of additional corrective electrodes. Our calculations demonstrate that the relative energy resolution of the instrument is 1.6% at a luminosity of 17.5% of 2π.

The limitation of this study is that the influence of fringe fields at the boundaries of the sector energy analyzer was not taken into account when considering the energy analyzer scheme. A disadvantage of our analysis is the use of an approximate field model; however, this aspect can be eliminated in further research.

Subsequent studies are planned to take into account the influence of fringe field effects and the search for other angular focusing modes in the electron-optical scheme of the proposed quasi-spherical deflector-type energy analyzer.

7. Conclusions

1. The structure of the electrostatic field obtained by summing an axisymmetric hexapole and a spherical electrostatic field has been theoretically investigated. It is shown that changing coefficient μ, which determines the contribution of the spherical field, leads to a transformation of the synthesized field. At small values of the weighting coefficient of the spherical field $\mu = 0.1$, the equipotentials of the complex field retain six-lobed multipole symmetry; at $\mu \approx 0.5$, deformation of the equipotential branches and compression of the field along the axis are observed. At $\mu = 1$, an almost radially symmetric potential distribution is formed, close to a quasi-spherical type. Our results are attributed to the redistribution of the potential in the region of the axial orbit.

2. It has been established that in an electrostatic quasi-spherical sector energy analyzer of the deflector type, built on the basis of the superposition of a spherical deflecting field and an axisymmetric hexapole, the choice of coefficient β, which defines the weighting contribution of the axisymmetric hexapole, makes it possible to control second-order aberrations and ensure compensation of the angular quadratic aberration. Based on analytical calculations, the profile of the deflecting electrodes of the sector quasi-spherical energy analyzer has been determined, enabling elimination of second-order angular aberrations. The improvement in the quality of angular focusing compared to the classical spherical energy analyzer is explained by the fact that the addition of the hexapole component changes the potential distribution, improving the focusing properties of the deflecting field. At a value of $\beta = 3$ for the weighting contribution of the spherical hexapole, an angular focusing mode is realized in which the second-order angular aberration is completely compensated, $A_{II}(\varphi) = 0$ at $\varphi = 180°$.

3. Our simulations performed using the "Focus" software package confirmed the feasibility of an electron-optical scheme for a quasi-spherical deflector-type energy analyzer based on a superposition field of an axisymmetric hexapole and a spherical field, providing a second-order angular focusing mode "axis-to-ring" in the absence of quadratic angular aberration. Achieving this focusing mode is ensured by the very structure of the synthesized field, which eliminates the need for corrective electrodes and simplifies the instrument's design. Numerical modeling showed that the relative energy resolution is 1.6% at an aperture of 17.5% of 2π, which confirms effectiveness of the proposed scheme compared to a classical spherical deflector-type energy analyzer. The results obtained could be used in the design of real high-resolution electron spectrometers and instruments for analyzing charged particle flows, where high-quality angular focusing of particles is required.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

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Data availability

All data are available, either in numerical or graphical form, in the main text of the manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

Authors' contributions

Zhanar Kambarova: investigation, writing – review & editing, funding acquisition;

Serik Kassymov: conceptualization, methodology; formal analysis.

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