UDC 510.67

On similarities of Δ-PJ theories in enriched signature

Байытылған сигнатурадағы Δ-PJ теориялардың ұқсастықтары туралы

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Мақалада центрлік типтер арқылы байытылған сигнатурада позитивті йонсондық теориялардың кейбір модельді-теоретикалық қасиеттері қарастырылған. Осындай теорияларды семантикалық зерттеу тәсілі арқылы әр түрдегі ұқсастығы мен олардың центрлерінің арасындағы байланыс қарастырылған. Негізгі байқаумыз қарастырылған ұқсастықтар біраз классикалық модельді-теоретикалық қасиеттерін сақтайды.

В статье через центральные типы рассмотрены некоторые теоретико-модельные свойства позитивных йонсоновских теорий в обогащенной сигнатуре. С помощью семантического метода исследования йонсоновских теорий рассмотрены отношения различных типов подобий между теориями и их центрами. Замечено, что многие свойства теоретико-модельного характера являются инвариантными относительно таких подобий.

In this paper we consider the some positive case of Jonsson theories and their similarity in the enriched signature.We shall say that a property (or a notion) of theories (or models, or elements of models) is called Γ- semantic if and if it is invariantly regarding Γ-semantic similarity, where $\Gamma=\{J,\Delta-PJ,\Delta-PM\}$.

The first case J is usual jonssons' case. Other both are positive generalizations of on.

In the [1–3] was defined the classes of Δ-PJ and $\Delta-PM$ theories. Such theories are generalized the concept of Jonson's theories. We can continue to investigate the corresponding above considering concept in which the notion of P-λ-stable (see [4, 5]) and the notion of syntactical similar in sense [6] are replaced in some equivalent style in the class of $\exists^+$-complete perfect Δ-PJ and $\Delta-PM$ theories. Moreover we are considered some enrichment of signatures of such theories and we defined and considered the concept of central types of ones. This generalization led us to different questions of note of stability in enrich signature, for example like in [4, 5]. And finally we can to conclude that it is appropriate to consider and to investigate the Δ-PJ, $\Delta-PM$ analogues of some properties and notions from classical model theory in frames of Δ-PJ, $\Delta-PM$ theories. Let us to consider only Δ-PJ-case, because for $\Delta-PM$-case of such statement, we can do it easy by using of positive Morleyisation.

In [1] was defined the class of Δ-PJ theories. Such theories are generalized the concept of Jonson's theories. In this paper we investigate the corresponding concept in which the notion of P-λ-stable (in sense [7]) and the notion of syntactical similar in sense [8] are replaced in some equivalent style in the class of $\exists^+$-complete perfect Δ-PJ theories. Moreover we are considered some enrichment of signatures of such theories and we defined and considered the concept of central types of ones. This generalization led us to different questions of note of stability in enrich signature, for example like in [7]. And finally we can to conclude that it is appropriate to consider and to investigate the Δ-PJ-analogues of some properties and notions from classical model theory in frames of Δ-PJ- theories.

Let T is an arbitrary Δ-PJ theory in first order signature σ. Let C is a semantic model of T. $A\subseteq C$. Let $\sigma_\Gamma(A)=\sigma\cup\{c_a \mid a\in A\}\cup\Gamma$, where $\Gamma=\{P\}\cup\{c\}$. Let consider the following theory $T_\Gamma^{PJ}(A)=Th_{\forall\exists^+}(C,a)_{a\in A}\cup\{P(c_a)|a\in A\}\cup\{P(c)\}\cup\{"P\subseteq"\}$, where $\{"P\subseteq"\}$ is infinite set of the sentences, expressing fact, that the interpretation of the symbol P is existentially closed submodel in the signature σ. The requirement of existential closeness for a submodel is essential in that sense, that it should not be finite. This theory is not necessary complete. Through S_Γ^{PJ} denote a set of all $\exists^+$- completions of theory $T_\Gamma^{PJ}(A)$. Let λ is an arbitrary cardinal.

Δ-PJ theory T is J-P-λ-stable, if $\left|S_\Gamma^{PJ}\right|\le\lambda$ for any subset A of C, such that $|A|\le\lambda$.

Let consider all completions of T^* for T in σ_Γ, where $\Gamma = \{c\}$. Due to that T^* is Δ-PJ- theory, it has its centre and we call it as T^C. By restriction T^C till signature σ theory T^C became complete type. This type we call as central type of theory T. It will be noted that all semantic models are elementarily equivalent between each other.

That is why the definition of central type is correct.

Lemma. If T — perfect Δ-PJ theory, then $T_\Gamma^{PJ}(A)$ - perfect Δ-PJ theory.

Proof. First of all we need to note that adding the symbols of constants and one –placed predicate P does not spoiled of Δ-PJ-ness of T and T^*. The proof of this ones standart cheking of definition of Δ-PJ-ness. For proof of perfection of $T_\Gamma^{PJ}(A)$ it is enough to show, that $T_\Gamma^{PJ}(A)$ has the semantic model which will be saturated in its power. It is follows by definition $T_\Gamma^{PJ}(A)$. As the given model we take semantic model C of the theory T, and in depending on a subset A and interpretation one-placed predicate P in C is the model D= (C, M, a)$_{a\in A}$, where M is existentially closed submodel of C. It is easy to see that D will be saturated in its power, since C is existentially closed model itself, as semantic model of T.

Theorem 1. Let T be a $\exists^+$-complete perfect Δ-PJ theory. Then the following conditions equivalent:

theory T^c is P-λ-stable (in sense [2]);

theory T^* is J-P-λ-stable.

Proof. We can now show that from 1) to 2) the proof is trivial, since if it is no more all completions than λ, then in particularly $\exists^+$-completions no more than λ. Let's prove from 2) in 1). Let the theory T^* - J-P-λ-stable. It is equivalent to that, that $T_\Gamma^{PJ}(A)$ in the signature $\sigma_\Gamma(A) = \sigma \cup \{c_a \mid a \in A\} \cup \Gamma$ is equivalent correspondingly to the positive Kaiser's hull T^0 of the theory T. By perfection of theory T we have that $T^0 = T^*$ and hence $T_\Gamma^{PJ}(A) = T^0$ will be perfect Δ-PJ- theory. Let the theory T^0 has no more, than λ $\exists^+$-completions. The centre of the theory T in the new signature will be equaled $Th(C,a)_{a\in A} \cup \{P(c_a) \mid a \in A\} \cup \{"P\preceq"\} \cup \{P(c)\}$. Clear that $T^* = T^c$.We should be shown, that T* has no more then λ completions. That means that T* will be P-λ-stable. Let's understand due to what T* it is not complete in the new signature. Addition of constants gives only inessential expansions that will not change quantity types of existentially closed submodels C. The essential role is played the realizations of a predicate P. In this case realization of a predicate P will be some elementary submodel M of the model C. As C is the semantic model of T, this one is existentially closed and by sense of a predicate P in C ($M \preceq C$) follows, that $M \in E_T$. Let's consider any completion T' theories T^* in the new signature. By definition T^* there exist such model M from E_T^+, such that $T' = Th(C,M,a)_{a\in A}$, where M — interpretation of a predicate P in semantic model C. $T' = Th(C,M,a)_{a\in A}$ is Δ-PJ- theory. In this case T' is it positive model complete theory. And we have by positive model completeness T' that any formula in T' is equivalent to some positive existential formula in T'. Then by $\exists^+$-completeness of the theory T such completions by above mentioned are no more than λ. So, the statement is proved.

Let T is arbitrary Δ-PJ-theory, then $E^+(T) = \bigcup_{n<\omega} E_n^+(T)$, where $E_n^+(T)$ -is the lattice of positive existential formulas with exactly n free variables.

Let T_1 and T_2 — Δ-PJ-theories.

We shall say that, T_1 и T_2 -Δ- PJ-syntactically similar, if and only if there exist a bijection $f: E^+(T_1) \to E^+(T_2)$ such that the restriction of f up $E_n^+(T_1)$ is isomorphism of the $E_n^+(T_1)$ and $E_n^+(T_2)$, $n < \omega$;

$f(\exists v_{n+1}\varphi) = \exists v_{n+1} f(\varphi), \varphi \in E_n^+(T), n < \omega,$

$f(v_1 = v_2) = (v_1 = v_2).$

Theorem 2. Let T_1 and T_2 are $\exists^+$-complete perfect Δ-PJ theories. Then the following conditions equivalent:

1) T_1^* and T_2^* are J-syntactically similar in sense [3]

2) T_1^c and T_2^c are syntactical similar in sense [8]

Proof. We can now show from 1) to 2).We have that for any $n<\omega$ $E_n^+(T_1)$ is isomorphic to $E_n^+(T_2)$. Let this isomorphism is making by f_{1n}.Under conditions of theorem and perfection for any $n<\omega$ $E_n(T_1)$ and $E_n(T_2)$ are Boolean algebras. But with perfection of T_1 и T_2 we have that T_1^* и T_2^* are positive model complete and so for any $n<\omega$, $\varphi(\overline{x}) \in F_n(T_1^*)$ there exist $\psi(\overline{x})$ из $E_n^+(T_1^*)$ так, что в $T_1^* \models \varphi \leftrightarrow \psi$. And in power of T_1 is complete for positive existential sentences and $E_n^+(T_1) \subseteq E_n^+(T_1^*)$ (in power of $T_1 \subseteq T_1^*$), we have that $E_n^+(T_1) = E_n^+(T_1^*)$. With the same argument we have that $E_n^+(T_2) = E_n^+(T_2^*)$. For any $n<\omega$, $\varphi_1(\overline{x}) \in F_n(T_1^*)$ we are defining the following map between $F_n(T_1^*)$ and $F_n(T_2^*)$ by next way $f_{2n}(\varphi_1(\overline{x}))= f_{1n}(\psi_1(\overline{x}))$, where in $T_1^* \models \psi_1 \leftrightarrow \varphi_1$, for $\psi_1 \in E_n^+(T_1)$. It is easy to note that under properties of f_{1n} and above mentioned f_{2n} is a bijection which giving to us isomorphism between $F_n(T_1^*)$ и $F_n(T_2^*)$. Hence, T_1^* и T_2^* — syntactically similar in sense [4]. But from previously theorem 1 under consideration of central types of Δ-PJ-theory, since $T^* = T^c$, we have that 1)⇒2) of theorem 2 is proved.

2)⇒1). It is trivial, since $F_n(T_1^*)$ is isomorphic to $F_n(T_2^*)$ for any $n<\omega$, and in power of conditions of theorem this isomorphism is be able to go on to all subalgebras.

The following definitions led us to other kind of similarity, this one weaker than syntactical similarity. All definitions are taken from [4].

(1) By a pure triple we mean $\langle A, \Gamma, M \rangle$, where A *is not empty set,* Γ is a permutation group on A and M is a family of subsets of A such that

$$M \in M \Rightarrow g(M) \in M \text{ for every } g \in \Gamma.$$

(2) If $\langle A_1, \Gamma_1, M_1 \rangle$ and $\langle A_2, \Gamma_2, M_2 \rangle$ are pure triples, and $\psi : A_1 \to A_2$ is a bijection, then ψ is an *isomorphism*, if

(i) $\Gamma_2 = \{\psi g \psi^{-1} : g \in \Gamma_1\}$;

(ii) $M_2 = \{\psi(E) : E \in M_1\}$.

The pure triple $\langle |C|, G, N \rangle$ is called the *semantically triple* of T (abbreviated s.t.), where $|C|$ is the universe of C, $G = Aut(C)$ and N is the class of all subsets of $|C|$ which are universes of suitable elementary submodels of C.

Complete theories T_1 and T_2 are *semantically similar* is and only if their semantic triples are isomorphic.

Very interesting one can to consider this result with the following:

Proposition 1 [8]. If T_1 and T_2 are syntactically similar, then T_1 and T_2 are semantically similar.

A property (or a notion) of theories (or models, or elements of models) is called semantic if and if it is invariant relative to semantic similarity.

It is turned out that a lot important notion from classical model theory belongs to next list.

Proposition 2 [8]. The following properties and notions are semantic:

type;
forking;
λ-stability;
Lascar rank;
Strong type;
Morley sequence;
Orthogonality, regularity of types;
$I(\aleph_\alpha, T)$ — the spectrum function.

By virtue of this notice we can say that all above mentioned properties and notions from Proposition 2 in the class of centers of $\exists^+$-complete perfect Δ-PJ theories are semantic. Moreover if we are consider above mentioned enrichments of signatures of such theories and we will consider central types of ones we got that the situation will not change. And finally it is appropriate to consider the Δ-PJ-analogues of the list of semantic properties and notions from classical model theory.

And finally we have the following result:

Theorem 3. Let T_1 and T_2 - $\exists^+$-complete perfect Δ-PJ theories and Γ-syntactically similar, and both ones satisfied for corresponding conditions of perfection, completeness and jonssonness as above.

Then points 1), 2), 3), 8) from Proposition 2 [4] for theories T_1^*, T_2^* are Γ- semantic as above, but in the point 8) we will consider only existentially closed models.

Proof. Completely clear, that for complete theories T_1^c and T_2^c or in our case to the relevant central types is executed as in [8].

It is sufficiently to show for case when Γ = J. The other cases follows immediately from [5] and after replacing of J to $\Delta - PM$ and remembering previously results concerning $\Delta - PM$ and $\Delta - PJ$ *cases*. So, let T_1 and T_2 are $\exists -$ complete perfect Jonsson's theories. Then from previously results regarding Jonsson's theories the centres of them syntactically similar between each other. But they are complete theories and hence for them is truth that they semantic similar between each other. And hence we have statements about points 1), 2), 3). For proving the point 8), we should to use perfection. We know that in perfect case of Jonsson's theory the class of existentially closed models of one is equal to the class of models of its center.

All unknown notions and results which we used in this article one can find out in [4].

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