UDC 37.013

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On Teaching of Discrete Mathematics

This article is designed to help teachers make discrete mathematics more meaningful and understandable for the students. Teachers all over the world have developed different ways to teach discrete mathematics successfully in order to raise standards of achievement. This article brings together some of these tried and tested ideas from teachers, including the extensive experience of mathematical teachers and their colleagues working together in higher schools. The article is aimed at improvement of quality of teaching of discrete mathematics. In addition to understanding some issues that the teachers face in a mathematics classroom, we have included some teaching strategies of teaching mathematics.

Key words: discrete mathematics, set theory, combinatorics, graph theory, genetic approach to the teaching of discrete mathematics, Petri nets.

We all know that English is applied to almost all aspects of life: Music, Medicine, Communication, and Business — name it all. And of course, mathematics is included to add up the list. Today, our textbooks are translated in English and it is also taught using the English language as well. No wonder, even in Mathematics, English reigns for quality teaching because it allows mathematical terms to be expounded and expressed in a more understandable manner.

Mathematical education is considered to be the principal component of comprehensive education as well as the factor influencing the quality of education at a higher educational establishment on the whole run.

Yet regardless of how English is being used to explain Mathematics, there are some, or even most students who find Mathematics too difficult to understand. The dilemma is brought up particularly when school pupils encounter numbers that cannot be conceptualized as quantities in an obvious way. Therefore, «clarity is required in terms of the language used by both teachers and textbooks. One significant risk is that the explanatory models and metaphors used by teachers are over generalized by pupils or create contradictions».

This concern is actually taken heed by a thesis studied by Cecilia Kilhamn (University of Gothenburg). To be specific, she details on a risk that the explanatory models and metaphors used by teachers are over generalized by pupils that results to create contradictions. For example: The learner's insights such as zero in the context of Mathematics is visualized to be nothing when in fact, it is still a numerical figure with numerical value. What's worse is that with this mentality, young learners tend to have difficulties in understanding «how subtraction works and being able to deal with the number line are important prerequisites for negative numbers».

If learners have this kind of mentality, they will really have a hard time understanding Math. On second thought, students who excel more in English actually have a hard time coping up with Math; but it doesn't really justify the issue. If you were the teacher who sees this concern among your students, how are you going to work this out?

According to Kilhamn, it is important that the deficiencies and limitations of these metaphors are also made clear in teaching, and that logical mathematical reasoning is used in parallel with concretized models.

That's why it is important to learn language in the context. Allow learners to conceptualize the nature of Mathematics different from that of the native language. If teachers are able to convey this thought comprehensively, her students will know the difference. Besides, our English languages have enough words to be used to make Mathematics easier to understand. The ones who are tasked to convey this just have to convey it accurately.

Introduction.

For most students, the first and often only area of mathematics in college is calculus. And it is true that calculus is the single most important field of mathematics, whose emergence in the 17th century signalled the birth of modern mathematics and was the key to the successful applications of mathematics in the sciences.

But calculus (or analysis) is also very technical. It takes a lot of work even to introduce its fundamental notions like continuity or derivatives (after all, it took 2 centuries just to define these notions properly). To

get a feeling for the power of its methods, say by describing one of its important applications in detail, takes years of study.

If you want to become a mathematician, computer scientist, or engineer, this investment is necessary. But if your goal is to develop a feeling for what mathematics is all about, where is it that mathematical methods can be helpful, and what kind of questions do mathematicians work on, you may want to look for the answer in some other fields of mathematics.

There are many success stories of applied mathematics outside calculus. A recent hot topic is mathematical cryptography, which is based on number theory (the study of positive integers 1, 2, 3,. . .), nd is widely applied, among others, in computer security and electronic banking. Other important areas in applied mathematics include linear programming, coding theory, theory of computing. The mathematics in these applications is collectively called discrete mathematics. («Discrete» here is used as the opposite of «continuous»; it is also often used in the more restrictive sense of «finite».)

The aim of this article is not to cover «discrete mathematics» in depth (it should be clear from the description above that such a task would be ill-defined and impossible anyway). Rather, we discuss a number of selected results and methods of teaching discrete mathematics , mostly from the areas of combinatory, graph theory, and combinatorial geometry, with a little elementary number theory.

At the same time, it is important to realize that mathematics cannot be done without proofs. Merely stating the facts, without saying something about why these facts are valid, would be terribly far from the spirit of mathematics and would make it impossible to give any idea about how it works. Sometimes this is not possible; quite simple, elementary facts can be extremely difficult to prove, and some such proofs may take advanced courses to go through.

Another important ingredient of mathematics is problem solving . You won't be able to learn any mathematics without dirtying your hands and trying out the ideas you learn about in the solution of problems. To some, this may sound frightening, but in fact most people pursue this type of activity almost every day: everybody who plays a game of chess, or solves a puzzle, is solving discrete mathematical problems. Treat it as puzzle solving, and if you find some idea that you come up with in the solution to play some role later, be satisfied that you are beginning to get the essence of how mathematics develops.

We hope that we can illustrate that mathematics is a building, where results are built on earlier results, often going back to the great Greek mathematicians; that mathematics is alive, with more new ideas and more pressing unsolved problems than ever; and that mathematics is an art, where the beauty of ideas and methods is as important as their difficulty or applicability.

Discrete mathematics is the study of mathematical structures that are fundamentally discrete. In contrast to real numbers that have the property of varying «smoothly», the objects studied in discrete mathematics – such as integers, graphs, and statements in logic – do not vary smoothly in this way, but have distinct, separated values. Discrete mathematics therefore excludes topics in «continuous mathematics» such as calculus and analysis. Discrete objects can often be enumerated by integers. More formally, discrete mathematics has been characterized as the branch of mathematics dealing with countable sets (sets that have the same cardinality as subsets of the natural numbers, including rational numbers but not real numbers). However, there is no exact definition of the term «discrete mathematics». Indeed, discrete mathematics is described less by what is included than by what is excluded: continuously varying quantities and related notions [1].

How can you explain the meaning of Discrete Mathematics in layman's terms? Courses in Discrete Math started a few decades ago when computer usage became common. Universities found that the typical math sequence that leads to calculus courses did not sufficiently cover the math needed by computer scientists. So they put together the extra math topics needed into a course, now called Discrete Mathematics.

The calculus sequence dealt with real-valued functions quite well. But it did not deal with much math other than the mathematics of real numbers. Since the real numbers are continuous, this mostly left areas of math that dealt with discrete as opposed to continuous sets of values.

For example, logic deals with two values, true and false.

Number theory deals with natural numbers or integers.

Here are some topics covered in Discrete Mathematics, and some examples as to their importance:

1) Logic. Used in proofs to show that one step follows from the previous step. Surprisingly, logic gates are used in the hardware of all modern computers

2) Set theory. This is an unexpectedly complicated theory, given that we mostly think of a set as a collection of unique objects. One fun and important topic in set theory is that some infinite sets are bigger than others.

3) Combinatorics (counting). Do you know how to count? If so, if I have a class of 50 students, in how many ways can I choose 5 of them to help me move a table? If that's easy for you, try this. McDonalds has 40 items on its menu. How many food orders with 8 items are there? (Keep in mind that repetition is allowed here. For instance, you can get 5 McRibs, and 3 large fries. A perfect meal.)

4) Recurrence relations and recursion. Recursion is a fascinating topic, and its importance in mathematics cannot be overstated. Here's a simple question that is more easily expressed using a recurrence relation. The half-life of Cesium-137 is 30 years. If you start with 1 Kg. of Cesium-137, how much will be left after 1000 years?

5) Graph Theory. So many aspects of real life can be modeled with graphs that graph theory experts are usually focused on just a few areas. Here's one important question that can be answered by graph theory. Given 20 cities, and the distance between them, what is the shortest route from one of the cities to another of them?

6) Tree theory. Did you know that the files and folders on you C: drive can be modeled with a tree? Or that a binary search tree can find a value in a sorted structure with 1 trillion elements before you can blink your eyes?

The article of A.V. Kalinkin outlines the issues of setting he content of special courses in higher mathematics for undergraduate students of technical specialties. He presents the experience and innovations in teaching methods, based on using model calculations that include standard problems in educational process. He gives sample calculations for the students' homework on the equations of mathematical physics, methods of optimization, discrete mathematics, probability theory and mathematical statistics. In particular, he describes the methodological impact of modern computer-assisted construction of the analytical solutions of mathematical problems on the students' learning process. He discusses the psychological aspects of the teachers' work with the students. The problem of improving the existing technology training is common to the disciplines of physical and mathematical profile and the higher school [2].

The article of I. Safuanov is devoted to genetic approach to the teaching of discrete mathematics. As noted earlier (Safuanov, 2005), the principle of the genetic approach in teaching mathematics requires that the method of the teaching of a subject should be based, as far as possible, on natural ways and methods of knowledge inherent in the science. The teaching should follow the path of the development of knowledge. We will call the teaching of a mathematical discipline genetic if it follows natural ways of the creation and application of the mathematical theory. Genetic teaching gives the answer to the question: how can the development of the contents of the mathematical theory be explained?

Genetic teaching of mathematics at universities should have the following properties:

– it should be based on students' previously acquired knowledge, experience and level of thinking;
– for the study of new themes and concepts, the problem situations and wide contexts (matching the experience of students) of non-mathematical or mathematical content should be used;
– in teaching, various problems and naturally arising questions are widely used, which should be answered by students independently with minimal necessary effective help of the teacher;
– strict and abstract reasoning's should be preceded by intuitive or heuristic considerations; construction of theories and concepts of a high level of abstraction can be properly carried out only after accumulation of sufficient (determined by thorough analysis) supply of examples, facts and statements at a lower level of abstraction;
– students' mental and cognitive activity should be stimulated: they should be constantly motivated;
– the gradual enrichment of mathematical objects through the study of interrelations with other objects, through consideration of the objects themselves and by looking at results from new angles and in new contexts should be undertaken.

The genetic approach to the teaching of mathematical disciplines has various aspects: historical, logical-epistemological, psychological, and sociocultural. The basic themes of a course of discrete mathematics for pedagogical universities provide rich opportunities for using all these aspects, beginning with a historical one. Furthermore, one can also apply the principle of the concentrated teaching in this course.

The program of a course includes the following themes: Fibonacci numbers. Transformations of the sums of degrees of natural numbers. The Euler's summation formula. Elements of coding theory. Codes detecting and correcting errors. Hamming codes. Elements of combinatorics. Newton's binomial theorem. Pascal's triangle. The basic concepts of the graph theory. Eulerian graphs. Hamiltonian graphs. Planar graphs. The theorem of Euler about polyhedra. The nonplanar graphs of Kuratowski - Pontryagin. Bipartite graphs. The theorem of Koenig. The problem of Four colours.

It is expedient to begin the course with acquainting students with the method of mathematical induction. One can introduce the principle and method of mathematical induction informally, proceeding from reasons of common sense.

It is appropriate to demonstrate the first applications of the method of mathematical induction on the example of summation of consecutive natural numbers (or of their degrees). From these tasks, one can proceed to the proof of the Euler's summation formula.

As other applications of the method of mathematical induction it is possible to consider Catalan numbers, elements of combinatorics with Newton's binomial theorem and Pascal's triangle and, finally, Fibonacci numbers. The Fibonacci numbers have huge number of beautiful properties, many connections to other sections of mathematics; they have a lot of applications. The applications go as far as to the theoretical foundations of the stock and currencies exchange activities. The connections of Fibonacci numbers with gold section allow to consider the numerous applications in art – in architecture, music, together with applications in other spheres of the human activity - construction, botany etc.

Certainly, such concepts connected with names of famous scientists, as the Fibonacci numbers, the Pascal triangle, Newton's binomial formula, allow us to consider in detail both historical sources and preconditions of their origination. Therefore, it is possible to create fruitful problem situations for introduction and construction of these concepts.

These problem situations will promote also the development of motivation of learning. Elements of the coding theory (codes correcting errors, Hamming codes) constitute an applied theme intensively developed mainly during the second half of twentieth century.

One can easily find both practical tasks, which have resulted in creation of the appropriate theories, and connections of these theories with such important sections of theoretical mathematics as linear algebra, theory of groups, theory of polynomials and theory of fields.

The great place in the course of discrete mathematics is occupied by elements of graph theory – one of major theoretical bases of modern applied mathematics which is distinct from traditional sections connected with concepts of limit and continuity.

The graph theory also has been intensively developed mainly in 20th century. However, the first sources of graph theory are in 18th century, when Leonhard Euler was the first to pose and solve the famous problem on Seven Bridges of Koenigsberg.

This problem, which until now has not lost its importance as an entertaining task for the capable pupils, results in the important concept of the Eulerian graph (graph containing an Eulerian cycle that is a cycle in a graph which visits each edge exactly once). It is possible to introduce also other concepts and results of graph theory (for example, Hamiltonian, planar and bipartite graphs) using a history of their origination from practical tasks and even from entertaining puzzles.

For example, the concepts of planar and nonplanar graphs can be developed from the consideration of the famous problem of Three Houses and Three Wells, On the other hand, the famous Four Colour problem («whether four coluors are sufficient to colour every planar map in such a way that regions sharing a boundary are coloured in different colours») gives rise to the demonstration of the complexity of graph theory. The simplified version of the problem where the number of colours is five can be solved more elementarily and its solution can be presented in a class.

Our experience of teaching discrete mathematics has contained rich manifestations of principle of concentrated teaching: anticipation, repetition, combination of functions and linkage (Verzahnung). For example, such topics as recurrences and coding theory are linked to many concepts of fundamental mathematics such as infinite series, groups, polynomials and matrices. Throughout the course, the ideas of induction and recurrence are repeatedly developed and used. Entertaining problems (Koenigsberg Bridges, Three Houses and Three Wells, Horse Tour on Chessboard) anticipate important concepts of graph theory. More globally, the course of discrete mathematics as a whole anticipates further, more sophisticated courses in Computer Science.

Thus, it is possible and useful to apply the genetic approach (combined with the principle of concentrated teaching) to the teaching of discrete mathematics [3].

The article f N.V. Zolotova and of R.S. Ismagilov [4] is devoted to the presentation of one of the numerous applications of General concepts of Discrete Mathematics. As an example the beginnings of the theory of Petri networks are outlined. The main concepts of this theory are defined and the work of Petri

networks is described. Such a description is given, first, the language of the theory of graphs (visual description); then the work of the Petri network is described in the language of linear operations on vectors with integer coordinates. The Graph theory and trees markings are affected. The problem of the algorithmic solvability of the tasks associated with graphs markings is touched. It is explained how Petri networks are used for description of complex systems, in particular to describe the working of the system of interacting devices. An example of Petri networks of this kind is considered in detail. The exposition is closed in itself and does not require prior knowledge on the subject. For perception of the stated one needs only basic information on graph theory and the beginning of linear algebra. The article material can be introduced into the teaching, for example, the topics of students' studies [5].

There are many teaching methods in education that enhance the learning process of the students [6]. We list some teaching methods, which are successfully used for teaching discrete mathematics.

To achieve the goal of teaching, the teacher must adopt effective teaching methods. The teacher has many options to choose from, different techniques, designed specifically for teaching and learning. Writing lesson plans is a foremost thing that a teacher must do, before executing any teaching strategy in the class. The teaching method should be adopted on the basis of certain criteria like the knowledge of the students, the environment, and the set of learning goals decided in the academic curriculum.

Students respond differently to different methods. Also, the students have their unique way of demonstrating the knowledge acquired, and absorbing the information that is imparted. So, to aid this process of demonstrating the knowledge, the teacher has to adopt a technique that assists the students in retaining the information and increasing their understanding. There are many techniques to teach students like questioning, modeling, demonstrating, collaborating, and explaining. All of these have been discussed here.

Specific Methods and Strategies.

Questioning. Testing and questioning are always known to be effective methods due to its interactive nature. The questions are asked by the teacher with an intention to know what the student has learned from earlier discussions. It helps in deciding what should be taught further.

This can even be vice-versa — students questioning the teachers to clarify the doubts that would enhance their understanding of the subject. The inquisitive instinct of the students evoke them to ask questions and satiate their query.

The teacher should encourage this in a positive way so that the student's critical thinking is developed. Testing differs in one aspect from questioning. A test is taken in order to know about the previous knowledge, what has been already taught to the students.

Explaining. Sometimes, the experiences can also be shared as a part of knowledge that would work as a source of inspiration for the students. While adopting this method, the teacher should give an introduction and a proper summary. Make sure that the information is specific to the audience.

The explanation should be accompanied with suitable examples for a better understanding of the students. It is like a discourse on a particular subject or topic that is for the entire class or public. Explaining can be clubbed with the modeling process to be more effective, and to have a long-lasting effect on the pupils.

Modeling. Modeling is a type of visual aid for teaching as well as learning. It is a known fact that the human brain absorbs more, and understands better, when visual aid facilitates explanation. This method works on three criteria - observing, retaining, and replicating. The students learn more by observing the things, and acquire it by imitating it time and again.

This is also known as reinforced behavior. This type of learning has a very important role to play in the learning process, especially during the student period, though it can happen in any stage of life. This helps the students to visualize things and, then hypothesize the solution.

Demonstrating. With the help of demonstrative teaching, students get an opportunity to explore the various aspects and understand the theory from a different perspective. Demonstration is a step-by-step explanation, along with their reasons, and significance for the better understanding of the student.

It enhances the student's understanding by practically applying the knowledge, and sharpen their skills and hence, help them in identifying and organizing the subject matter in a more efficient way. Practical experimentation is a very good method used for demonstrating the subject.

Collaborating. Teamwork is a contemporary form of collaboration. The students are taught to work in a group that makes the instructing easier for the teacher. This method of teaching promotes a sense of mutual responsibility among the students. They learn to put in more effort to research for the topic, and apply effec-

tive techniques to get the result. This inculcates patience and develops an ability to critically analyze a subject. It gives an opportunity to the students to solve the problem by a healthy discussion and co-operation. This is what we call 'group discussions' which motivates the students to perform in a team, show leadership skills, and enhances the presentation capabilities as well. This is one of the best, direct instructional methods.

These techniques for special education is a little different from the methods and theories for others. The education is imparted to these students based on their strengths and weaknesses. The teachers cater to the special needs of the students like modification in the regular teaching program, use of supplementary aids that allows students to participate in the learning process. Different effective teaching strategies are adopted on the basis of the disabilities. Four kinds of provisions are adopted in special education and they are inclusion, mainstream, segregation, and exclusion.

Apart from these defined methods, nowadays many other methods are being adopted to give quality education. Methods like role-play, story or games, seminars, presentations, workshops, conferences, brainstorming, case study, educational trips and modern audio-visual aids like documentary films, computers, Internet, etc., have been introduced in education. These new methods have increased the pace of learning and understanding of discrete mathematics. This also enhances the capability of the students to research and think logically about a given problem. We hope teachers will use the ideas in this article help students increase their mathematical knowledge and skills.

References

1 *Белоусов А.И., Ткачев С.Б.* Дискретная математика. — М.: МГТУ им. Н.Э. Баумана, 2004. — 743 с.

2 *Калинкин А.В*. Методические особенности преподавания специальных математических дисциплин на старших курсах // Гуманитарный вестн. — М.: МГТУ им. Н.Э. Баумана (электронный журн.), 2014. — № 2(16). — [ЭР]. Режим доступа: http: //hmbul.ru/articles/178/178.pdf

3 *Сафонов И.* Генетический подход к преподаванию дискретной математики /28 PME конференция: Междунар. группа по психологии матем. образования. 14–18 июля. — Берген, Норвегия, 2004. — Т. 4. — С. 153–160.

4 *Золотова Н.В., Исмагилов Р.С.* Опыт преподавания дискретной математики: сети Петри /Инженерный журнал: наука и инновации. — М.: МГТУ им. Н.Э. Баумана (электронный журн.), 2013. — № 5(17). — [ЭР]. Режим доступа: http: //engjournal.ru/articles/738/738.pdf)

5 *Питерсон Дж.* Теория сетей Петри и моделирование систем. — М.: Мир, 1984. — 264 с.

6 *Есенбаева Г.А., Копжасарова У.И., Денисарова Н.В*. Об особенностях преподавания математики на английском языке / Вестн. Караганд. ун-та. Сер. Педагогика. — Караганда: Изд-во КарГУ, 2014. — № 3 (75). — С. 52–58.

Г.А. Есенбаева, О.И. Ульбрихт

Дискреттік математиканы оқыту туралы

Мақала оқытушыларға көмек ретінде дискреттік математиканы студенттерге аса мағыналы және түсінікті жеткізу үшін арналған. Жетістіктер деңгейін арттыру мақсатында бүкіл әлем оқытушылары дискреттік математиканы оқытудың түрлі тәсілдерін ойлап тапты. Бұл мақалада оқытушылардың, сондай-ақ жоғарғы оқу орындарында жұмыс істейтін коп тәжірибелі математика пәнінің оқытушылары мен олардың әріптестерінің кейбір тексерілген және зерттелген пікірлері жиналған. Авторлар дискреттік математиканы оқытуда сапа деңгейін арттыруға атсалысқан. Сонымен қатар математикалық сыныптарда кездесетін кейбір сұрақтарды түсінуге қосымша ретінде математиканы оқытудың кейбір оқыту стратегияларын енгізген.

Г.А.Есенбаева, О.И.Ульбрихт

О преподавании дискретной математики

Цель статьи — улучшить качество преподования дискретной математики, сделать ее более осмысленной и понятной для студентов. Авторами проанализирован обширный опыт работы преподавателей математики высших учебных заведений и общеобразовательных школ. Рассмотрены различные способы обучения дискретной математике в целях повышения уровня учебных достижений. Разработаны некоторые обучающие стратегии преподавания математики по отдельным вопросам, с которыми сталкиваются преподаватели в математических классах.

References

1 Belousov A.I., Tkachev *S.B. Discrete Mathematics,* Moscow: MSTU named after N.E. Bauman, 2004, 743 p.

2 Kalinkin A.V. *Humanitarian Bulletin,* Moscow: MSTU named after N.E. Bauman (electronic journal), 2014, 2 (16), [ER]. Access mode: http: //hmbul.ru/articles/178/178.pdf

3 Safuanov I. *Genetic approach to the teaching of discrete mathematics /28th PME Conferences: International Group for the Psychology of Mathematics Education,* Bergen, Norway, July 14–18, 2004, Vol. 4, p. 153–160.

4 Zolotova N.V., Ismagilov R.S. *Experience of teaching discrete mathematics:* Petri networks, Moscow: MSTU named after N.E. Bauman (electronic journa). 2013, 5(17), [ER]. Access mode http: //engjournal.ru/articles/738/738.pdf

5 Peterson J. *The theory of Petri nets and modeling systems,* Moscow: Peace, 1984, 264 p.

6 Yessenbayeva G.A., Kopzhasarova U.I., Denivarova N.V. Bulletin of the Karaganda University. Pedagogy series, 2014, 3 (75), p. 52–58.