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INFORMATION-ENTROPY METHOD FOR DETECTING GRAVITATIONAL WAVE SIGNALS

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The detection of gravitational waves came from a pair of merging black holes marked the beginning of the era of GW astronomy. Traditionally, to extract gravitational wave signals from experimental data, the scientific collaborations use the standard matched filtering technique. The matched filtering technique relies on the existing waveform templates, that makes it difficult to find gravitational wave signals that go beyond theoretical expectations. Moreover, the computational cost of matched filter is very high, as it depends on the number of templates used. In this article, we propose a new information-entropy method for gravitational waves detection that does not require a theoretical bank of signal templates. To demonstrate the reliability of our method we conducted an analysis using simulated and real data. Through this study, we revealed that our measure of conditional information detects the gravitational wave signals and can be used along with the matched filtering method.

Keywords: gravitational waves, information-entropy, detection, nonlinear process.

Introduction

The first direct detection of gravitational waves (GWs) by the advanced LIGO observatory proved the fundamental predictions of Einstein's theory of General Relativity and started the era of GW astronomy. The registration of first GW, the so-called GW150914, was realized due to the merger of two binary black holes with estimated masses of 29 M⊙ and 36 M⊙ [1]. During three observing sessions (O1, O2, O3) the LIGO and VIRGO collaborations recorded 90 GW signals produced by the coalescence of compact objects, mainly pairs of black holes with a small fraction of neutron star [1-7].

The detection of GWs is one of the most difficult tasks faced in fundamental science since the GW signal is much weaker compared to typical noise levels. To detect GW signals from the experimental data, the LIGO and Virgo collaborations mainly use the matched filtering method [8-10]. This method convolves a set of precalculated template waveforms with the measured data, where each template represents a source with different components such as masses, spins, etc. For each template waveform, a signal-to-noise ratio (SNR) time series is calculated, and candidates are determined according to the peak of the SNR time series. The matched filtering method is optimal for signal detection in Gaussian noise, where it yields the most statistically significant detection candidates [11]. However, this method, in order to match the signal, does a full search in the bank of templates, which in turn can slow down the data processing speed [12]. Furthermore, the premise of matched filtering method requires an accurate theoretical template. If GWs are beyond theoretical expectations, this may lead to the fact that gravitational wave signals not being detected [10].

Currently, information-theoretic approaches have found wide application in modern signal processing problems. Information-entropy technique quantifies the degree of complexity and irregularity of a signal. It is known that information is a measure of certainty, and entropy is a measure of uncertainty or disorder (noise). These two characteristics are analytically related and can provide a theoretical basis for describing signal characteristics. In papers [13-15], information entropy detection methods have been successfully applied to signal recognition. In this paper, we propose a new information-entropy method for gravitational-wave data analysis. The novelty of the method lies in the use of conditional information, defined as the difference between the joint and conditional entropy. In addition, this method does not require the construction of a theoretical bank of signal templates.

The contents of this paper are structured as follows. In section 1, we introduce our measure of conditional information by comparing it with the usual definition of Shannon's mutual information. In

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section 2, we present the data generation for testing our method and describe our algorithm for determining the difference in conditional information. In Section 3, we present a result of numerical analysis of model and real gravitational wave signals. In section 4, we make a conclusion.

1 Conditional information defined through the entropy difference

Let us consider discrete time series of signals $X = \{x[n]\}$, $Y = \{y[n]\}$ given through samples with numbers *n*, which have a statistical relationship, forming an ensemble (X, Y). Assume that X is the received signal, $Y = \{\Delta x[n]\}$ is the signal interference. By the signal interference we mean pulsed, chaotic and noise disturbances of the signal $\Delta x[n]$ caused by nonlinear distortions of the signal itself and external influences.

Let us take the notation $\Delta x[n]$, n =1,2,3 – numbers of the samples, $y[n] = \Delta\ x[n]$- the interval of deviation from $x[n]$ caused by interference. The deviation interval is defined by the central (symmetric) difference $\Delta x[n] = (x[n+1] + x[n-1])/2 - x[n]$. Using this form of $\Delta\ x[n]$ instead of the one-sided difference $x[n+1] - x[n]$ allows us to consider the second derivative of X, i. e. the wave shape. The second derivative of any nonlinear function is nonzero except for its zero value at the inflection point. Near the inflection point, the function changes impulsively, which gives a reason to use a neutral difference to detect GW in the form of nonlinear bursts of the signal disturbances. At the point of inflection, the required information will be minimal.

Further, we consider the general concepts of mutual information and entropy, followed by the introduction of our measure. It is known that mutual information measures the nonlinear relationship between two random variables. Moreover, mutual information can show us how much information can be obtained from one random variable by observing a second random variable. Mutual information has a close relationship with the concept of entropy. Because in some cases, when one of the variables is known, mutual information to some extent can reduce the uncertainty of another random variable. Thus, this means that a high value of mutual information indicates a large reduction of uncertainty, and a small reduction if the value is low. In cases where the mutual information is zero, this means that the two random variables are independent [16-18].

The commonly used mutual information $I(Y;X)$ transmitted over the communication channel X is determined by the difference between the one-dimensional and conditional Shannon entropies $H(Y)$, $H(Y|X)$ [18-20]. The relationship between these values is shown in the following formula:

$$I(Y;X) = H(Y) - H(Y|X), \tag{1}$$

As explained before, mutual information $I(Y;X)$ is related to entropy and to understand what $I(Y;X)$ actually means, we need to define entropy and conditional entropy. It is known that entropy of a random variable is the average level of "uncertainty" inherent to the variable's possible outcomes. For example, if we have a discrete random variable Y, with possible outcomes x_1,x_2, x_n which happen with probability p_1, p_2, the entropy of Y is defined as following:

$$H(Y) = -\sum_{i=1}^{N}\sum_{j=1}^{M} p(x[i], y[j])\, log_2\, p\,(y[j]). \tag{2}$$

Entropy $H(Y)$ can measure the level of expected uncertainty in a random variable. This means that $H(Y)$ is roughly how much information can be learned of the random variable Y by observing just one sample. Conditional entropy can measure how much uncertainty has the random variable Y, when we know the value X. And we can define conditional entropy according to the following formula:

$$H(Y|X) = -\sum_{i=1}^{N}\sum_{j=1}^{M} p(x[i], y[j])\, log_2\, p\,(y[j]|x[i]), \tag{3}$$

where,

$$p(y[j]|x[i]) = p(x[i], y[j]) \,/\, p(x[i]). \tag{4}$$

And $p(x[i], y[j])$, $p(y[j])$, $p(y[j]|x[i])$ are the joint, one-dimensional, and conditional probabilities of the points of the phase space (*X*; *Y*) falling into squares with a relative size δ<<1. In formula (2), the equal to 1 sum over *i* is left for convenience of further analysis [21-23].

From formulas (1)-(3) follows

$$I(Y;X) = \sum_{i=1}^{N} \sum_{j=1}^{M} p(x[i], y[j])\, log_2 \frac{p(x[i],y[j])}{p(x[i])p(y[j])}. \quad (5)$$

In the absence of a mutual correlation between $x[i], y[j]$, there is $p(x[i], y[j])$ $=p(x[i])$, hence, $I(Y;X) = 0$. From the structure of formula (5), mutual information is symmetric with respect to permutation of variables $X \rightleftarrows Y: I(Y;X) = I(X;Y)$, the roles of the signal interference are not different. For these reasons, for the set goal (X and Y should be different variables), it is necessary to use another measure of certainty - conditional information, which we will determine through the difference of the entropies.

$$I(Y|X) = H(X,Y) - H(Y|X). \quad (6)$$

In formula (6) $H(X;Y)$ is a joint entropy of the ensemble [24-26]:

$$H(X,Y) = \sum_{i=1}^{N} \sum_{j=1}^{M} p(x[i],\ y[j]) log_2 p(x[i], y[j]) \quad (7)$$

When we consider together two random variables the joint entropy measures the uncertainty. In contrast to mutual information, conditional information is asymmetric with respect to the permutation of the variables $X \rightleftarrows Y$ since $H(Y|X) \neq H(Y|X)$. Dividing formula (6) by (7), we derive a kind of well-known conservation law for the normalized values of conditional information and entropy.

$$\tilde{I}(Y|X) + \widetilde{H}(Y|X) = 1, \qquad \widetilde{H} = H/H(X,Y) \qquad \tilde{I} = I/H(X,Y). \quad (8)$$

The relationship between information and entropy in the form of formula (8) is known for the Boltzmann entropy for an equilibrium state, or in the case of choosing Y as constant parameters. We choose the condition Y in the form of characteristic features of the desired signal determined from the experimental data.

From formula (8) we get the difference of conditional information

$$\Delta\tilde{I} = \tilde{I}(Y|X) - \tilde{I}(X|Y) = \widetilde{H}(X|Y) - \widetilde{H}(Y|X). \quad (9)$$

When the variables $X \rightleftarrows Y$ are permuted, conditional information and entropy acquire different meanings. For example, $\tilde{I}(Y|X)$ determines information about a burst of nonlinear disturbances Y (about the presence of GW) in a known noise signal X. The corresponding decrease in entropy describes $H(Y(X)) - H(Y|X)$. The decrease in information corresponds to an increase in entropy $\widetilde{H}(X|Y)$. If $\Delta\tilde{I} > 0$, the signal is detected, while if $\Delta\tilde{I} < 0$ there is no signal. This criterion can also have positive values in cases with rearranged variables $(X;Y)$. These cases correspond to a sharp decline in Y near the inflection point. The use of $\Delta\tilde{I}$ increases the reliability of signal analysis.

Assume that the calculated variable Y has a systematic error $\Delta Y (Y = Y[0] + \Delta Y)$. Limiting ourselves to the first term of the Taylor series expansion at the point $Y[0]$, we get the function $\Delta\tilde{I}$

$$\Delta\tilde{I} = \Delta\tilde{I}(Y[0]) + \frac{d\Delta I}{dY}\Big|_{Y=Y[0]} \Delta Y \quad (10)$$

When calculating the derivative in (10) through finite differences, ΔY falls out. For weak signals with a signal-to-noise ratio of the order of one, a small increment from $\Delta\tilde{I}$ may not arise as a signal. Thus, a weak signal with a systematic error ΔY of the individual terms in (9) may be absent when calculating $\Delta\tilde{I}$.

2 Algorithm for determining the difference of conditional information

2.1 Data generation

To thoroughly test our algorithm, we use PyCBC package [27] to generate the GW signals. The parameters of the model signals are selected according to the article [9]. The GW signal is determined by the component masses m1, m2 randomly selected in the range from 10 to 50M⊙ and phases in the range φ0 from 0 to 2 pi. In accordance with the selected target SNR, the amplitude and distance of the source are determined. We assume the presence of one LIGO Hanford detector, whose inclination and polarization

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parameters are equal to 0, since any change in these parameters can be completely absorbed by changes in the amplitude and phase of the signal. The signals are whitened by dividing the Fourier transformed signal by the square root of the power spectral density (PSD) to reduce power at frequencies within the sensitivity of the detector. Noise modeling and whitening is also done using the same PSD that was used to whiten the signals. The colored noise whitening eliminates error sources, and it is applicable to real noise. The simulated signals are randomly placed in a time series (-16:16 s) with the condition that the peak amplitude of each signal is randomly located in the time series range from 0.75 to 0.95. After that, to achieve the optimal signal-to-noise ratio (SNR), the signal amplitude is scaled. And the optimal SNR ρ_{opt} can be determined by [9]

$$\rho_{opt}^2 = 4Re\left[\int df\,\frac{\tilde{h}(f)\tilde{h}^*(f)}{S_n(f)}\right], \tag{11}$$

where $\tilde{h}$ is the frequency domain representation of the GW strain, $\tilde{h}^*$ is its conjugate, S_n is the PSD and Re extracts the real part of the complex number. A data set was generated for each predefined optimal SNR value in the range from 2 to 10 with integer steps.

2.2 Algorithm realization

From the discrete signal of the GW , ΔX is calculated by the formula:

$$\Delta x[n] = \frac{x[n+1] + x[n-1]}{2} - x[n] \tag{12}$$

Using the well-known Sliding Window algorithm, we divide a given discrete time series $x[n]$ and $\Delta x[n]$ into windows with a length of L points. If we assume that the time delay parameter of the analysis window is 1, then the signal sequence $X[n]$ can be divided into segments with the number $N - L$, which form the following matrices $x[n] \rightarrow X$ and $\Delta x[n] \rightarrow \Delta X$.

$$X = \begin{vmatrix} x[1] & x[2], \ldots & X[L] \\ x[2], & x[3], \ldots & x[L+1] \\ X[N-L], & \ldots, & x[N] \end{vmatrix}$$

and

$$\Delta X = \begin{vmatrix} \Delta x[1], & \Delta x[2], \ldots & \Delta x[L] \\ \Delta x[2], & \Delta x[3], \ldots & \Delta x[L+1] \\ \Delta x[N-L], & \ldots, & \Delta x[N] \end{vmatrix}$$

To determine the difference of conditional information, the plane $X[n]$ and $\Delta X[n]$. is constructed and divided into cells (i x j). Next, the probability of each cell is determined and the probability matrix $P(x_i, y_j)$is obtained. From the probability matrix, we determine the difference of conditional information by the (9).

3 Results of numerical analysis of gravitational wave signals

In our method, when we detect the GW signal, the difference of conditional information $\Delta\tilde{I}$ increases, while the noise fluctuates around zero. In order to define the window size and threshold value of conditional information $\Delta\tilde{I}$, we calculate the *false positive ratio* (FPR) (Fig. 1).

The FPR is one of the important standard metrics, that is used to evaluate the detection of GWs. The FPR can be defined as follows

$$FPR = \frac{FP}{FP + TN}, \tag{13}$$

where FP is the number of false positive predictions, TN is the number of true negatives.

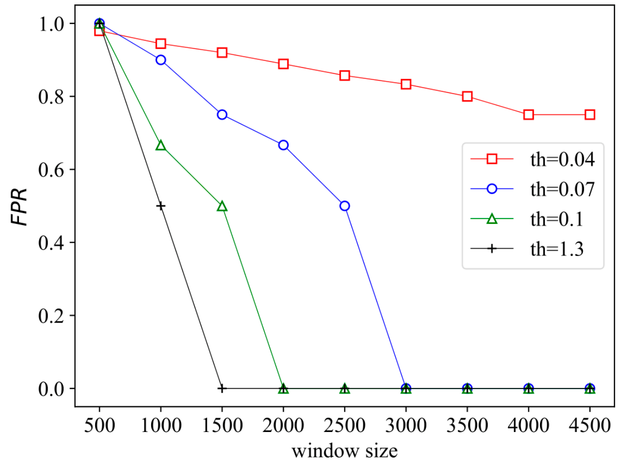

Fig. 1. The plot shows the dependence of the FPR on the window size, where, the threshold (*th*) value changes in the range from 0.04 to 0.1 with a step of 0.02. We calculated the FPR for the one GW model signal, whose SNR=10.

According to the figure 1, when the window size is L>2000 and the threshold value is th>0.1, the signal is detected without any errors. From a data set consisting of 10 000 signals at each SNR, the number of detected signals was calculated using our method. Further, to calculate the efficiency (fig.2), we choose the window size of 3000 and a threshold value of 0.1 for the validation accuracy.

The efficiency is then determined by

$$efficiency = \frac{N_{\Delta\tilde{I}>th}}{N_s}, \qquad (14)$$

where $N_{\Delta\tilde{I}>th}$ is the number of detected signals, N_s is the total number of signals.

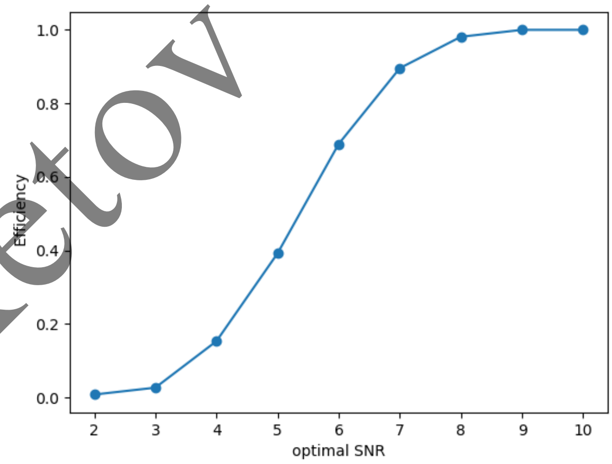

Fig. 2. The efficiency as a function of optimal SNR

The results in Fig. 2 show that the efficiency increases with the optimal SNR and achieves 100 % at an SNR of 9. To demonstrate the feasibility of our method, we conducted analysis using real LIGO data. Fig. 3 (b, c, d, e) shows the values $\tilde{I}(Y;X)$,$\tilde{I}(Y|X)$,$\tilde{I}(X|Y)$, $\Delta\tilde{I}$ for the GW150914 event, calculated by formulas (1), (8), (9), correspondingly.

From Fig. 3 (b) it can be seen that the value of mutual information $\tilde{I}(Y;X)$ fluctuates around the same level and does not changed over the entire time interval and shows nearly 0.19. While the value of conditional information $\tilde{I}(Y|X)$ and the difference of conditional information $\Delta\tilde{I}$ increased at the time of detection of the desired signal nearly from 0.7 to 0.9 and from 0 to 0.6, respectively. As for the value of conditional information $\tilde{I}(X|Y)$, it decreased at the time of detection of the gravitational wave from 0.75 to 0.25 approximately.

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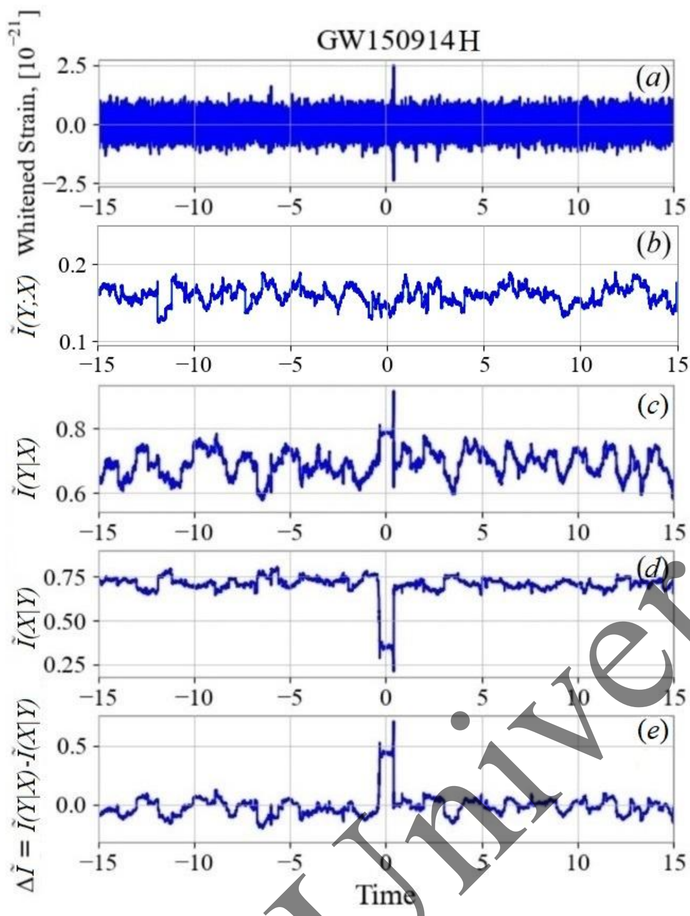

Fig. 3. Event GW150914H: whitened signal GW (a), mutual information $\tilde{I}(Y;X)$ (b), conditional information $\tilde{I}(Y|X)$ (c), conditional information $\tilde{I}(Y|X)$ (d), difference of conditional information $\Delta\tilde{I}$ (e), where number of samples L=3000

The validity of using the information-entropy method proposed by us is shown in Table 1.

Table 1. Results of information-entropy analysis of GW signals from the Handford (H) and Livingston (L) detectors. M/M_⊙ is the total mass of black holes relative to the mass of the Sun, SNR is the signal-to-noise ratio, R is the distance to the source according to [1-5, 28].

Event	$\tilde{I}(Y\|X)$	$\tilde{I}(X\|Y)$	$\Delta\tilde{I}$	SNR	$M/M_{\odot}$	R
[1], GW150914H GW150914L	+ +	+ +	+ +	24.4	62	410 Mpc
[2], GW151226H GW151226L	+ (minimum) -	- F-	+ $(Y\|X \rightarrow X\|Y)$ -	13.1	21.4	4292 Mpc
[3], GW170104H GW170104L	+ +	+ +	+ +	13.0	51.1	880 Mpc
[4], GW170608H GW170608L	- -	- -	- -	14.9	18.5	340 Mpc
[5], GW170814H GW170814L	+ +	+ +	+ +	15.9	56.0	5518 Mpc
[6], GW170817H GW170817L	+ +	+ +	+ +	33.0	2.73	40 Mpc

The following Table 1 presents an analysis of the mutual $\tilde{I}(Y;X)$, conditional information $\tilde{I}(Y|X)$ of six gravitational wave (GW) events recorded by the Hanford (H) and Livingston (L) detectors. The plus (+) symbols mean the detection of a signal with highlighted positive values $\tilde{I}(Y|X)$, $\tilde{I}(X|Y)$, $\Delta\tilde{I}$. Minus (–) symbols mean that there is no highlighted maximum or minimum in the difference of conditional information. The GW151226, GW170608L signals have a low signal-to-noise ratio and total masses of black holes relative to other signals (Table 1).

Conclusion

From the above study, it can be seen that the proposed information-entropy method demonstrates the good performance in GW data recognition. According to the obtained results, in all six GW events the values of mutual information $\tilde{I}(Y;X)$ do not changed over the entire time interval, while the difference of conditional information $\Delta\tilde{I}$ increases sharply at the time of gravitational waves detection. It clear that the new measure introduced by us – conditional information $\Delta\tilde{I}$ detects a signal, and the known measure – mutual information $\tilde{I}(Y;X)$ does not reveal a signal. According to the results obtained, the method can serve as an addition to the existing methods for analyzing GW signals.

Furthermore, this method in comparison with the well-known matched filtering method can discover signals beyond the existing templates. We believe that this method will undoubtedly play an important role in searching GW signals beyond what we have in the existing template bank.

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