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On Some Non-local Boundary Value and Internal Boundary Value Problems for the String Oscillation Equation

The work is devoted to the problem of setting new boundary and internal boundary value problems for hyperbolic equations. The consideration of these settings is given on the example of a wave equation. The research involves the d'Alembert method, the mean value theorem and the method of successive approximations. The paper formulates and studies a number of non-local problems summarizing the classical Goursat and Dardu tasks. Some of them are marginal, and the other part is internal-marginal, and in both cases both characteristic and uncharacteristic displacements are considered. It should also be noted that a number of problems discussed below arose as a special case in the construction of the theory of correct problems for the model loaded equation of string oscillation.

Keywords: Wave equation, general solution, Cauchy problem, Goursat problem, Darboux problem, problem with characteristic shift, problem with uncharacteristic displacement.

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Introduction

Boundary value problems for partial differential equations with nonlocal conditions present a class of problems solvability of which is important both for differential equations development and for their applications. They can be used in mathematical modeling as well as in the theory of loaded equations and coefficient inverse problems.

In 1969, in the work [1] A.M. Nakhushev proposed a number of problems of a new type, which entered the mathematical literature under the name of problems with displacement. These tasks were announced as part of the implementation of the problem of finding correctly posed problems for second-order mixed-type equations with two independent variables, put forward in the 60s of the last century by A.V. Bitsadze.

In accordance with the classification proposed by him in [2, 3], these problems, bounded by two intersecting characteristics of a given hyperbolic equation and one characteristic line, are non-local and with an edge offset. The bibliography of works devoted to regular local and nonlocal boundary value problems for strictly hyperbolic equations is very extensive and is most fully given in the monograph [4]. Let us note some of them that are closest to the problems discussed in this paper [1–11].

In this paper, a number of non-local problems generalizing the classical problems of Goursat and Darboux are formulated and investigated. Some of them are marginal, and the other part is internally marginal, and in both cases both characteristic and non-characteristic displacements are considered. It should also be noted that a number of the problems discussed below arose as a special case when constructing the theory of correct problems by analogy with [7] for the model loaded string oscillation equation.

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1 The main part

In this paper, the wave equation is considered as a model equation with second-order partial derivatives of two independent variables x and y a hyperbolic equation

$$u_{xx} - u_{yy} = 0. \tag{1}$$

Let Ω_1 be a simply connected domain of the plane of a complex variable $z = x + iy$, bounded by the characteristics AB, BC, CD and DA equation (1), coming out of the points, respectively $A\,(0,\,0)$, $B\left(\frac{1}{2},\,\frac{1}{2}\right)$, $C\,(1,\,0)$ and $D\left(\frac{1}{2},\,-\frac{1}{2}\right)$; $AC = \bar{J}$. By Ω_2 we denote the area bounded by the characteristics AB, BC and the segment AC of the straight line $y = 0$.

By the regular solution of the equation (1) we will understand any function $u\,(x, y) \in C\left(\bar{\Omega}_i\right) \cap C^2\left(\Omega_i\right),\ i = 1, 2$ satisfying equation (1).

2 Non-local problems with edge displacement

Problem 2.1. Find regular in the area Ω_1 decision $u\,(x, y)$ equation (1), satisfying conditions

$$u\left(\frac{x}{2}, \frac{x}{2}\right) + \alpha_1\,(x)\ u\left(\frac{1+x}{2},\ \frac{x-1}{2}\right) = \gamma_1\,(x)\,,\ \ x \in \bar{J},$$

$$u\left(\frac{x}{2}, -\frac{x}{2}\right) + \alpha_2\,(x)\ u\left(\frac{1+x}{2},\ \frac{1-x}{2}\right) = \gamma_2\,(x)\,,\ \ x \in \bar{J},$$

where

$$\alpha_1\,(0) \cdot \alpha_1\,(1) \neq \alpha_2\,(0) \cdot \alpha_2\,(1)\,, \tag{2}$$

$$\alpha_1\,(x) \cdot \alpha_2\,(x) \neq 1, \forall x \in \bar{J}, \tag{3}$$

$$\alpha_i\,(x)\,,\ \gamma_i\,(x) \in C\left(\bar{J}\right) \cap C^2\,(J)\,,\ \ i = 1, 2.$$

Based on the Asgeirsson principle for characteristic quadrilaterals with vertices respectively at point

$$(0, 0)\,, \left(\frac{x+1}{2},\ \frac{x-1}{2}\right), \left(\frac{x}{2},\ \frac{x}{2}\right), \left(\frac{1}{2},\ -\frac{1}{2}\right)$$

and

$$(0, 0)\,, \left(\frac{1+x}{2},\ \frac{1-x}{2}\right), \left(\frac{x}{2},\ -\frac{x}{2}\right), \left(\frac{1}{2},\ \frac{1}{2}\right)$$

it is easy to verify the equivalence of problem 2.1 to the following algebraic system

$$u\left(\frac{x}{2},\ \frac{x}{2}\right) + \alpha_1\,(x)\ u\ \left(\frac{x}{2}, -\ \frac{x}{2}\right) = \gamma_1\,(x) - \alpha_1\,(x)\ \left[u\left(\frac{1}{2},\ \frac{1}{2}\right) - u\,(0, 0)\right],$$

$$\alpha_2\,(x)\ \ u\ \left(\frac{x}{2},\ \frac{x}{2}\right) + u\ \ \left(\frac{x}{2}, -\ \frac{x}{2}\right) = \gamma_2\,(x) - \alpha_2\,(x)\ \left[u\left(\frac{1}{2}, -\frac{1}{2}\right) - u\,(0, 0)\right],$$

which by virtue of (2), (3) is unambiguously and unconditionally solvable in the class $C\left(\bar{J}\right) \cap C_2^2\,(J)$.

Therefore, problem 2.1 is reduced in an equivalent way to the Goursat problem with data on AB and AD, which is known to be correct.

Problem 2.2. Find a regular Ω_1 solution $u\,(x,\ y)$ of equation (1), the domain satisfying the conditions

$$u\ \left(\frac{x}{2},\ \frac{x}{2}\right) + \alpha_1\,(x)\ u\ \left(\frac{1+x}{2},\ \frac{1-x}{2}\right) = \gamma_1\,(x)\,,\ \ \ x \in \bar{J},$$

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$$u\left(\frac{x}{2}, -\frac{x}{2}\right) + \alpha_2(x)\, u\left(\frac{x+1}{2}, \frac{x-1}{2}\right) = \gamma_2(x), \quad x \in \bar{J},$$

where

$$\alpha_1(0) \cdot \alpha_1(1) \neq \alpha_2(0) \cdot \alpha_2(1), \tag{4}$$

$$1 + \alpha_1(x) \neq 0, 1 + \alpha_2(x) \neq 0 \ \forall x \in \bar{J} \tag{5}$$

$$\alpha_i(x),\, \gamma_i(x) \in C\left(\bar{J}\right) \cap C^2(J),\ i = 1, 2.$$

Based on the Asgeirsson principle for characteristic quadrilaterals with vertices respectively at points

$$\left(\frac{x}{2}, \frac{x}{2}\right), (1, 0), \left(\frac{x+1}{2}, \frac{x-1}{2}\right), \left(\frac{1}{2}, \frac{1}{2}\right)$$

and

$$\left(\frac{x}{2}, -\frac{x}{2}\right), (1, 0), \left(\frac{1+x}{2}, \frac{1-x}{2}\right), \left(\frac{1}{2}, -\frac{1}{2}\right)$$

we are convinced of the equivalence of problem 2.2 to the following two algebraic systems for finding $u\left(\frac{x}{2}, \frac{x}{2}\right)$, $u\left(\frac{x}{2}, -\frac{x}{2}\right)$, $u\left(\frac{x+1}{2}, \frac{x-1}{2}\right)$ and $u\left(\frac{1+x}{2}, \frac{1-x}{2}\right)$

$$\begin{cases} u\left(\frac{x}{2}, \frac{x}{2}\right) + \alpha_1(x)\, u\left(\frac{x+1}{2}, \frac{x-1}{2}\right) = \gamma_1(x), \\ u\left(\frac{x}{2}, \frac{x}{2}\right) - u\left(\frac{x+1}{2}, \frac{x-1}{2}\right) = u\left(\frac{1}{2}, \frac{1}{2}\right) - u(1, 0) \end{cases}$$

and

$$\begin{cases} u\left(\frac{x}{2}, -\frac{x}{2}\right) + \alpha_2(x)\, u\left(\frac{1+x}{2}, \frac{1-x}{2}\right) = \gamma_2(x), \\ u\left(\frac{x}{2}, -\frac{x}{2}\right) - u\left(\frac{1+x}{2}, \frac{1-x}{2}\right) = u\left(\frac{1}{2}, -\frac{1}{2}\right) - u(1, 0), \end{cases}$$

which, by virtue of conditions (4), (5) are uniquely solvable.

Therefore, problem 2.2 is reduced in an equivalent way, as in the case of problem 2.1, to the Goursat problem.

Problem 2.3. Find a regular Ω_1 solution $u(x, y)$ of equation (1), in the domain of equation (1), satisfying the conditions

$$u\left(\frac{x}{2}, \frac{x}{2}\right) + \alpha_1(x)\, u\left(\frac{1-x}{2}, -\frac{1-x}{2}\right) = \gamma_1(x),\ x \in \bar{J}, \tag{6}$$

$$u\left(\frac{x}{2}, -\frac{x}{2}\right) + \alpha_2(x)\, u\left(\frac{1-x}{2}, \frac{1-x}{2}\right) = \gamma_2(x),\ x \in \bar{J}, \tag{7}$$

where

$$1 - \alpha_1(1-x)\ \alpha_2(x) \neq 0, x \in \bar{J}, \tag{8}$$

$$\alpha_i(x),\, \gamma_i(x) \in C\left(\bar{J}\right) \cap C^2(J),\ i = 1, 2.$$

Replacing in (6) everywhere x by $1 - x$, to find $u\left(\frac{x}{2}, -\frac{x}{2}\right)$ we obtain the following algebraic system

$$\alpha_1(1-x)\ u\left(\frac{x}{2}, -\frac{x}{2}\right) + u\left(\frac{1-x}{2}, \frac{1-x}{2}\right) = \gamma_1(1-x),$$

$$u\left(\frac{x}{2}, -\frac{x}{2}\right) + \alpha_2(x)\, u\ \left(\frac{1-x}{2}, \frac{1-x}{2}\right) = \gamma_2(x).$$

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Similarly, replacing in (7) x by $1-x$, to find $u\left(\frac{x}{2},\,\frac{x}{2}\right)$ we get

$$u\left(\frac{x}{2},\,\frac{x}{2}\right)+\alpha_1\left(x\right)u\left(\frac{1-x}{2},-\frac{1-x}{2}\right)=\gamma_1\left(x\right),$$

$$\alpha_2\left(1-x\right)\,u\left(\frac{x}{2},\,\frac{x}{2}\right)+u\left(\frac{1-x}{2},\,-\frac{1-x}{2}\right)=\gamma_2\left(1-x\right).$$

The last two systems are unconditionally and unambiguously solvable under the conditions (8). Therefore, problem 2.3 is reduced equivalently, as in the case of problem 2.1, to the Goursat problem.

Problem 2.4. Find regular in the area Ω_1 solution $u\left(x,y\right)$ of equation (1), satisfying conditions

$$u\left(\frac{x}{2},\,\frac{x}{2}\right)+\alpha_1\left(x\right)\,u\left(\frac{2-x}{2},\,\frac{x}{2}\right)=\gamma_1\left(x\right),\ x\in\bar{J},$$

$$u\left(\frac{x}{2},-\frac{x}{2}\right)+\alpha_2\left(x\right)u\left(\frac{2-x}{2},-\frac{x}{2}\right)=\gamma_2\left(x\right),\ x\in\bar{J},$$

where $\alpha_i\left(x\right),\,\gamma_i\left(x\right)\in C\left(\bar{J}\right)\cap C^2\left(J\right),\ i=1,2.$

Based on Asgeirsson's principle for characteristic quadrilaterals with vertices respectively at points

$$(0,0)\,,\left(\frac{1}{2},\frac{1}{2}\right),\left(\frac{2-x}{2},\frac{x}{2}\right),\left(\frac{1-x}{2},-\frac{1-x}{2}\right)$$

and

$$(0,0)\,,\left(\frac{1}{2},\,-\frac{1}{2}\right),\left(\frac{2-x}{2},\,-\frac{x}{2}\right),\left(\frac{1-x}{2},\,\frac{1-x}{2}\right),$$

we are convinced of the equivalence of problem 2.4 to the following algebraic system

$$u\left(\frac{x}{2},\,\frac{x}{2}\right)+\alpha_1\left(x\right)\,u\left(\frac{1-x}{2},-\frac{1-x}{2}\right)=\gamma_1\left(x\right)-\alpha_1\left(x\right)\,\left[u\left(\frac{1}{2},\,\frac{1}{2}\right)-u\left(0,0\right)\right],$$

$$u\left(\frac{x}{2},-\frac{x}{2}\right)+\alpha_2\left(x\right)\,u\left(\frac{1-x}{2},\,\frac{1-x}{2}\right)=\gamma_2\left(x\right)-\alpha_2\left(x\right)\,\left[u\left(\frac{1}{2},-\frac{1}{2}\right)-u\left(0,0\right)\right].$$

Therefore, problem 2.4 is equivalently reduced to problem 2.3.

Problem 2.5. Find regular in the area Ω_1 solution $u\left(x,y\right)$ of equation (1), satisfying conditions

$$u\left(\frac{x}{2},\,\frac{x}{2}\right)+\alpha_1\left(x\right)\,u\,\left(\frac{2-x}{2},\,-\frac{x}{2}\right)=\gamma_1\left(x\right),\quad x\in\bar{J},$$

$$u\left(\frac{x}{2},\,-\frac{x}{2}\right)+\alpha_2\left(x\right)\,u\,\left(\frac{2-x}{2},\,\frac{x}{2}\right)=\gamma_2\left(x\right),\quad x\in\bar{J},$$

where

$$\alpha_1\left(0\right)\neq\alpha_2\left(0\right),\,\alpha_1\left(1\right)\,\cdot\alpha_2\left(1\right)\neq1, \tag{9}$$

$$\alpha_1\left(x\right)\cdot\alpha_1\left(1-x\right)\neq1,\alpha_2\left(x\right)\cdot\alpha_2\left(1-x\right)\neq1\ \forall\,x\in\bar{J}, \tag{10}$$

$$\alpha_i\left(x\right),\,\gamma_i\left(x\right)\in C\left(\bar{J}\right)\cap C^2\left(J\right),\ i=1,2.$$

Using the same reasoning as in problem 2.4, we are convinced that problem 2.5 is equivalent to the following algebraic system

$$u\left(\frac{x}{2},\,\frac{x}{2}\right)+\alpha_1\left(x\right)\,u\left(\frac{1-x}{2},\,\frac{1-x}{2}\right)=\gamma_1\left(x\right)-\alpha_1\left(x\right)\,\left[u\left(\frac{1}{2},-\frac{1}{2}\right)-u\left(0,0\right)\right],$$

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$$u\left(\frac{x}{2},-\frac{x}{2}\right)+\alpha_2(x)\,u\left(\frac{1-x}{2},\frac{x-1}{2}\right)=\gamma_2(x)-\alpha_2(x)\,\left[u\left(\frac{1}{2},\frac{1}{2}\right)-u(0,0)\right],$$

which is apparently equivalently reduced to the next two algebraic systems

$$\begin{cases} u\left(\frac{x}{2},\frac{x}{2}\right)+\alpha_1(x)\,u\left(\frac{1-x}{2},\frac{1-x}{2}\right)=\gamma_1(x)-\alpha_1(x)\,\left[u\left(\frac{1}{2},-\frac{1}{2}\right)-u(0,0)\right], \\ \alpha_1(1-x)\,u\left(\frac{x}{2},\frac{x}{2}\right)+u\left(\frac{1-x}{2},\frac{1-x}{2}\right)=\gamma_1(1-x)-\alpha_1(1-x)\,\left[u\left(\frac{1}{2},-\frac{1}{2}\right)-u(0,0)\right] \end{cases}$$

and

$$\begin{cases} u\left(\frac{x}{2},-\frac{x}{2}\right)+\alpha_2(x)\,u\left(\frac{1-x}{2},\frac{x-1}{2}\right)=\gamma_2(x)-\alpha_2(x)\,\left[u\left(\frac{1}{2},\frac{1}{2}\right)-u(0,0)\right], \\ \alpha_2(1-x)\,u\left(\frac{x}{2},-\frac{x}{2}\right)+u\left(\frac{1-x}{2},\frac{x-1}{2}\right)=\gamma_2(1-x)-\alpha_2(1-x)\,\left[u\left(\frac{1}{2},\frac{1}{2}\right)-u(0,0)\right]. \end{cases}$$

If the conditions (9) and (10) are met, the last two systems are unambiguously and unconditionally solvable. Therefore, problem 2.5, as in the case of problem 2.1, is equivalently reduced to Goursat problem.

3 Non-local problems with intra-boundary displacement

Problem 3.1. Find regular in the area Ω_1 solution $u(x,y)$ of equation (1), satisfying conditions

$$u\left(\frac{x}{2},\frac{x}{2}\right)+\alpha\,u\left(\frac{1}{2},\frac{1}{2}-x\right)=\gamma(x),\ x\in\bar{J},$$

$$u(x,0)=\tau(x),\ x\in\bar{J}, \tag{11}$$

where

$$\alpha\neq\frac{1}{2},\ \alpha\left[\gamma(1)-\alpha\tau(1)\right]=\left(\alpha^2+\alpha-1\right)\tau(0)-(1-\alpha)\,\gamma(0), \tag{12}$$

$$\tau(x),\ \gamma(x)\in C\left(\bar{J}\right)\cap C^2(J). \tag{13}$$

Based on Asgeirsson's principle for characteristic quadrilaterals with vertices respectively at points

$$\left(\frac{x}{2},\frac{x}{2}\right),\left(\frac{1}{2},\frac{1}{2}-x\right),\left(\frac{1-x}{2},\frac{1-x}{2}\right)\ \text{and}\ (x,0),$$

seeing the equivalence of problem 3.1 to the following algebraic system

$$u\left(\frac{x}{2},\frac{x}{2}\right)+\alpha u\left(\frac{1}{2},\frac{1}{2}-x\right)=\gamma(x),$$

$$u\left(\frac{x}{2},\frac{x}{2}\right)+u\left(\frac{1}{2},\frac{1}{2}-x\right)=u\left(\frac{1-x}{2},\frac{1-x}{2}\right)+u(x,0).$$

From where

$$(1-\alpha)\,u\left(\frac{x}{2},\frac{x}{2}\right)+\alpha\,u\left(\frac{1-x}{2},\frac{1-x}{2}\right)=\gamma(x)-\alpha\,\tau(x). \tag{14}$$

Changing everywhere x on $1-x$, from the last equation we get

$$\alpha u\left(\frac{x}{2},\frac{x}{2}\right)+(1-\alpha)\,u\left(\frac{1-x}{2},\frac{1-x}{2}\right)=\gamma(1-x)-\alpha\,\tau(1-x). \tag{15}$$

From the system (14), (15) we find that

$$u\left(\frac{x}{2},\frac{x}{2}\right)=\frac{1-\alpha}{1-2\alpha}\left[\gamma(x)-\alpha\tau(x)\right]-\frac{\alpha}{1-2\alpha}\left[\gamma(1-x)-\alpha\tau(1-x)\right]. \tag{16}$$

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Consequently, the solution of problem 3.1 is equivalently reduced to the problem of Darboux (11), (16), the regular solution of which, when the conditions (12), (13) are met, exists only. Next, obtained in the area Ω_2 the solution can be naturally continued throughout Ω_1.

Problem 3.2. Find regular in the area Ω_2 solution $u(x,y)$ of equation (1), satisfying conditions

$$u\left(\frac{x}{2},\frac{x}{2}\right)+\alpha u\left(\frac{1}{2},\left|\frac{1}{2}-x\right|\right)=\gamma(x),\ x\in\bar{J},$$

$$u(x,0)=\tau(x),\ x\in\bar{J}.$$

By virtue of the Asgeirsson principle $0<x^*<\frac{1}{2}$ fair ratio

$$u\left(\frac{x^*}{2},\ \frac{x^*}{2}\right)+u\left(\frac{1}{2},\ \left|\frac{1}{2}-x^*\right|\right)=u(x^*,0)+u\left(\frac{1-x^*}{2},\ \frac{1-x^*}{2}\right).$$

For a symmetric point with respect to zero, the following ratio is valid

$$u\left(\frac{x^*}{2},\ \frac{x^*}{2}\right)+u\left(\frac{1}{2},\ \left|\frac{1}{2}-x^*\right|\right)=u(1-x^*,0)+u\left(\frac{1-x^*}{2},\frac{1-x^*}{2}\right).$$

This suggests that the mean theorem, which is valid for all $x\in\bar{J}$ we can only use it when

$$u(x,0)=u(1-x,0).$$

Theorem 3.1. Let $\tau(x),\gamma(x)\in C\left(\bar{J}\right)\cap C^2(J)$ and the conditions are met

$$\alpha\neq 1,\tau(x)\equiv\tau(1-x)\ \ \forall x\in\bar{J}. \tag{17}$$

Then there is only one regular in the field Ω_2 problem solving 3.2.

When the conditions (17) are met, it is sufficient to repeat the same reasoning and calculations as in the study of problem 3.1 to prove the theorem 3.1.

Problem 3.3. Find a regular solution in the region of Ω_2 of the $u(x,y)$ equation (1) satisfying the conditions

$$u\left(\frac{x}{2},\frac{x}{2}\right)+\beta u\left(\frac{1}{2},\frac{x}{2}\right)=\gamma(x),\ x\in\bar{J}, \tag{18}$$

$$u(x,0)=\tau(x),\ x\in\bar{J}. \tag{19}$$

Theorem 3.2. Let $|\beta|<1$, $\tau(x),\gamma(x)\in C\left(\bar{J}\right)\cap C^2(J)$ and the condition is fulfilled

$$\tau(0)+\beta\tau\left(\frac{1}{2}\right)=\gamma(0).$$

Then problem 3.3 is uniquely solvable, its solution is represented as

$$\begin{gathered}u(x,y)=\tau(x-y)+\beta\left[\tau\left(\frac{1-x+y}{2}\right)-\tau\left(\frac{1-x-y}{2}\right)\right]-\gamma(x-y)+\gamma(x+y)-\\-\sum_{j=1}^{\infty}\sum_{i=j}^{\infty}(-1)^i\beta^i\left[\gamma\left(\frac{(2j-1)+x-y}{2^i}\right)-\gamma\left(\frac{(2j-1)+x+y}{2^i}\right)-\right.\\-\gamma\left(\frac{(2j-1)-x+y}{2^i}\right)+\gamma\left(\frac{(2j-1)-x-y}{2^i}\right)+\\+\beta\tau\left(\frac{2(2j-1)-1+x-y}{2^{i+1}}\right)-\beta\tau\left(\frac{2(2j-1)+1+x+y}{2^{i+1}}\right)-\\\left.-\beta\tau\left(\frac{2(2j-1)-1-x+y}{2^{i+1}}\right)+\beta\tau\left(\frac{2(2j-1)+1-x-y}{2^{i+1}}\right)\right].\end{gathered} \tag{20}$$

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Proof. It is known that any regular solution of the equation (1) can be represented as

$$u(x,y) = f(x-y) + g(x+y), \tag{21}$$

where $f(x)$, $g(x) \in C^2(\Omega) \cap C(\bar{\Omega})$.

Satisfying (21) conditions (18), (19) for finding $f(x)$ and $g(x)$ we obtain the following system of functional equations

$$f(x) = \tau(x) - g(x), \tag{22}$$

$$g(x) - \beta g\left(\frac{1-x}{2}\right) + \beta g\left(\frac{1+x}{2}\right) = \gamma(x) - \beta\tau\left(\frac{1-x}{2}\right). \tag{23}$$

The solution of equation (23) under the condition $|\beta| < 1$ can be constructed by the iteration method. Indeed

$$g_0(x) = \gamma(x) - \beta\tau\left(\frac{1-x}{2}\right),$$

$$g_1(x) = \gamma(x) - \beta\tau\left(\frac{1-x}{2}\right) + \beta\gamma\left(\frac{1-x}{2}\right) - \beta\gamma\left(\frac{1+x}{2}\right) - \\ -\beta^2\tau\left(\frac{1+x}{4}\right) + \beta^2\tau\left(\frac{3-x}{4}\right)$$

etc.

Continuing this process indefinitely, we get

$$g(x) = \gamma(x) - \beta\tau\left(\frac{1-x}{2}\right) + \\ + \sum_{j=1}^{\infty}\sum_{i=j}^{\infty}(-1)^i\beta^i\left[\gamma\left(\frac{(2j-1)+x}{2^i}\right) - \gamma\left(\frac{(2j-1)-x}{2^i}\right) + \right. \\ \left. + \beta\tau\left(\frac{2(2j-1)-1+x}{2^{i+1}}\right) - \beta\tau\left(\frac{2(2j-1)+1-x}{2^{i+1}}\right)\right]. \tag{24}$$

It is easily verified that the series itself (24) the series obtained after differentiation converge uniformly. Substituting (24) into (22) we find $f(x)$, and hence $u(x,y)$ by formula (21).

It should be noted that formula (20) in the case of $\beta = 0$ coincides with the formula for solving the Darboux problem.

Conclusion

In this paper, all problems are considered in the characteristic quadrilateral; therefore, the choice of an inner manifold, the points of which are associated with the boundary manifold, is small. In the case of sufficiently derived domains (for example, a rectangular noncharacteristic domain), a problem arises both in the nonlocal condition itself and in the corresponding inner manifold. This method is of particular interest for weakly hyperbolic equations.

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Ішек тербелісінің теңдеуі үшін кейбір бейлокал шеттік және ішкі шеттік есептер

Жұмыста классикалық Гурса және Дарбу есептерін жалпылайтын бірқатар бейлокал есептер тұжырымдалған және зерттелген. Олардың кейбіреулері шеттік, ал екінші бөлігі ішкі шеттік болып табылады және екі жағдайда да сипаттамалық және сипаттамалық емес ығысулар қарастырылған. Сондай-ақ, мақалада қарастырылған бірқатар есептер ішек тербелісінің модельдік жүктелген теңдеуі үшін корректілі қойылған есептер теориясын құруда ерекше жағдай ретінде туындағанын атап өткен жөн.

Кілт сөздер: толқындық теңдеу, жалпы шешім, Коши есебі, Гурса есебі, Дарбу есебі, сипаттамалық ығысу есептері, сипаттамалық емес ығысу есептері.

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О некоторых нелокальных краевых и внутренних краевых задачах для уравнения колебания струны

В работе сформулированы и исследованы некоторые нелокальные задачи, обобщающие классические задачи Гурса и Дарбу. Часть из них являются краевыми, а другая — внутренними краевыми, причем в обоих случаях рассмотрены как характеристические, так и нехарактеристические смещения. Следует также отметить, что ряд задач, рассмотренных в статье, возникли как частный случай при построении теории корректных задач для модельного нагруженного уравнения колебания струны.

Ключевые слова: волновое уравнение, общее решение, задача Коши, задача Гурса, задача Дарбу, задача с характеристическим смещением, задача с нехарактеристическим смещением.

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