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ON STABILIZATION PROBLEM FOR A LOADED HEAT EQUATION: THE TWO-DIMENSIONAL CASE

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Introduction. One of the important properties that characterize the behavior of solutions of boundary value problems for differential equations is stabilization, which has a direct relationship with the problems of controllability. The problems of solvability of stabilization problems of two-dimensional loaded equations of parabolic type with the help of feedback control given on the boundary of the region are investigated in the paper. These equations have numerous applications in the study of inverse problems for differential equations. The problem consists in the choice of boundary conditions (controls), so that the solution of the boundary value problem tends to a given stationary solution at a certain speed at $t \to \infty$. This requires that the control is feedback, i.e. that it responds to unintended fluctuations in the system, suppressing the results of their impact on the stabilized solution. The spectral properties of the loaded two-dimensional Laplace operator, which are used to solve the initial stabilization problem, are also studied. The paper presents an algorithm for solving the stabilization problem, which consists of constructively implemented stages. The idea of reducing the stabilization problem for a parabolic equation by means of boundary controls to the solution of an auxiliary boundary value problem in the extended domain of independent variables belongs to A.V. Fursikov [1]. At the same time, recently, the so-called loaded differential equations are actively used in problems of mathematical modeling and control of nonlocal dynamical systems.

Statement of the problem. Let $\Omega = \{x, y : -\pi/2 < x, y < \pi/2\}$ be a domain with a boundary $\partial\Omega$. In the cylinder $Q = \Omega \times \{t > 0\}$ with lateral surface $\Sigma = \partial\Omega \times \{t > 0\}$ we consider the boundary value problem for the loaded heat equation

$$u_t - \Delta u + \alpha u(0, y, t) + \beta u(x, 0, t) = 0, \quad \{x, y, t\} \in Q, \tag{1}$$

$$u(x, y, 0) = u_0(x, y), \quad \{x, y\} \in \Omega, \tag{2}$$

$$u(x, y, t) = p(x, y, t), \quad \{x, y, t\} \in \Sigma, \tag{3}$$

where $\alpha, \beta \in \mathbb{C}$ are given (in general case are complex) bounded constants, $u_0(x, y)$ is given function. The aim is to find a function $p(x, y, t)$ such that a solution of the boundary value problem (1)–(3) satisfies the inequality

$$\|u(x, y, t)\|_{L_2(\Omega)} \le C_0 e^{-\sigma t}, \quad \sigma > 0, \quad t > 0. \tag{4}$$

Note that here σ is a given constant and C_0 is an arbitrary bounded constant.

Equation (1) is called a loaded equation [2,3]. We note that problem (1)–(4) with a single load point was studied in [4], andwith a two-dimensional cases was studied in [5–7].

Auxiliary boundary value problem (BVP). Let $\Omega_1 = \{x, y : -\pi < x, y < \pi\}$ and $Q_1 = \Omega_1 \times \{t > 0\}$.

$$z_t - \Delta z + \alpha z(0, y, t) + \beta z(x, 0, t) = 0, \quad \{x, y, t\} \in Q_1, \tag{5}$$

$$z(x, y, 0) = z_0(x, y), \quad \{x, y\} \in \Omega_1, \tag{6}$$

$$\frac{\partial^j z(-\pi, y, t)}{\partial x^j} = \frac{\partial^j z(\pi, y, t)}{\partial x^j}, \quad \{y, t\} \in (-\pi, \pi) \times \{t > 0\},$$

$$\frac{\partial^j z(x, -\pi, t)}{\partial y^j} = \frac{\partial^j z(x, \pi, t)}{\partial y^j}, \quad \{x, t\} \in (-\pi, \pi) \times \{t > 0\}, \quad j = 0, 1. \tag{7}$$

The problem is to find an initial function $z_0(x, y)$ such that a solution of the BVP (5)–(7) satisfies the inequality

$$\|z(x, y, t)\|_{L_2(\Omega_1)} \le C_0 e^{-\sigma t}, \quad \sigma > 0, \quad t > 0. \tag{8}$$

We recall, as we indicated above, that here σ is a given constant and C_0 is an arbitrary bounded constant.

We will define the function $z_0(x, y)$ as a continuation of the function $u_0(x, y)$, which was given in the original domain Ω. Thus in the auxiliary boundary value problem (5)–(7) it is needed to find the function $z_0(x, y)$ on the square Ω_1, so that the requirement (8) is satisfied for a solution $z(x, y, t)$ of the problem (5)–(7). In this case the condition (4) holds for restriction $u(x, y, t)$ of $z(x, y, t)$ too and a required boundary control $p(x, y, t)$, $\{x, y\} \in \Sigma$ is defined as trace of function $z(x, y, t)$ for $\{x, y, t\} \in \Sigma$.

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A NONLOCAL PROBLEM FOR ESSENTIALLY LOADED DIFFERENTIAL EQUATIONS WITH INTEGRAL CONDITIONS

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We consider the following linear boundary value problem for systems of essentially loaded differential equations with integral conditions: