ON SOLVABILITY OF A NONLOCAL BOUNDARY VALUE PROBLEM

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We consider the nonlocal boundary value problem with integral condition on $\overline{\Omega}=\{(x,t): t\le x\le t+\omega, 0\le t\le T\}, T>0,\ \omega>0$ for the system of partial differential equations

$$D\left[\frac{\partial}{\partial x}u\right]=A(x,t)\frac{\partial u}{\partial x}+S(x,t)u+f(x,t),\qquad u\in R^n, \tag{1}$$

$$B(x)\frac{\partial u}{\partial x}(x,0)+C(x)\frac{\partial u}{\partial x}(x+T,T)+\int_0^T K(x,s)\frac{\partial u}{\partial x}(x,s)ds=d(x), \tag{2}$$

$$u(t,t)=\Psi(t),\qquad t\in[0,T]. \tag{3}$$

Here $u(x,t)=col(u_1(x,t),u_2(x,t),...,u_n(x,t))$ is unknown function; $D=\frac{\partial}{\partial t}+\frac{\partial}{\partial x}$; $(n\times n)$ are matrices $A(x,t)$, $S(x,t)$, $K(x,t)$, n is vector-function $f(x,t)$, $(n\times n)$ are matrices $B(x)$, $C(x)$, n is vector-function $d(x)$ continued and is function $\Psi(t)$ is continuously differentiable on $\overline{\Omega}$, $[0,\omega]$, $[0,T]$ accordingly.

Let $C(\overline{\Omega},R^n)$ be a space of functions $u:\overline{\Omega}\to R^n$, are continuous on $\overline{\Omega}$, with norm

$$\|u\|_0=\max_{(x,t)\in\overline{\Omega}}\|u(x,t)\|;\ \|A\|=\max_{(x,t)\in\overline{\Omega}}\|A(x,t)\|=\max_{(x,t)\in\overline{\Omega}}\max_{i=\overline{1,n}}\sum_{j=1}^{n}|a_{ij}(x,t)|,\ \|d\|_1=\max_{x\in[0,\omega]}\|d(x)\|,\ \|\Psi\|_2=\max_{t\in[0,T]}\|\Psi(t)\|.$$

In the present work are investigated a questions of solvability to wide extent of the boundary value problem with integral condition (1)-(3).

Used the work's idea [1]-[4] introduce new unknown functions $v(x,t)=\frac{\partial u}{\partial x}(x,t)$ and investigation problem is reduced to the equivalent problem for the system of hyperbolic first-order equations

$$Dv=A(x,t)v+S(x,t)u+f(x,t),\qquad (x,t)\in\overline{\Omega},\qquad v\in R^n, \tag{4}$$

$$B(x)v(x,0)+C(x)v(x+T,T)+\int_0^T K(x,s)v(x,s)ds=d(x), \tag{5}$$

$$u(x,t)=\Psi(t)+\int_t^x v(\eta,t)d\eta,\qquad t\in[0,T]. \tag{6}$$

A pair $(v(x,t),u(x,t))$ of continuous functions on $\overline{\Omega}$ is called a solution to boundary problem for the system of hyperbolic first-order equations (4)-(6) to wide extent of Friedrichs if the function

$v(x,t) \in C(\overline{\Omega}, R^n)$ has a continuous derivative with respect to t along characteristic and satisfies family of ordinary differential equations, and condition (5), in which the functions $u(x,t)$ and $v(x,t)$ by the functional relation (6).

Using method of the characteristic receive in the $\overline{\mathrm{H}} = \{(\xi,\tau): 0 \le \xi \le \omega, 0 \le \tau \le T\}$, $T > 0$, $\omega > 0$ boundary value problem for the ordinary differential equations:

$$\frac{\partial \tilde{v}}{\partial \tau} = \tilde{A}(\xi,\tau)\tilde{v} + \tilde{S}(\xi,\tau)\tilde{u}(\xi,\tau) + \tilde{f}(\xi,\tau), \qquad \tau \in [0,T], \tag{7}$$

$$\tilde{B}(\xi)\tilde{v}(\xi,0) + \tilde{C}(\xi)\tilde{v}(\xi,T) + \int_0^T \tilde{K}(\xi,\tau)\tilde{v}(\xi,\tau)d\tau = \tilde{d}(\xi), \quad \xi \in [0,\omega], \tag{8}$$

$$\tilde{u}(\xi,\tau) = \Psi(\tau) + \int_\tau^{\xi+\tau} \tilde{v}(\varsigma,\tau)d\varsigma, \qquad \tau \in [0,T]. \tag{9}$$

For the finding solution of boundary value problem (7)-(9), an algorithm is offered.

Step-0: in (7) accepting $\tilde{u}(\xi,\tau) = \Psi(\tau)$, and solved boundary value problem (7)-(8) we shall define initial approach $\tilde{v}^{(0)}(\xi,\tau)$. Using the $\tilde{v}(\xi,\tau) = \tilde{v}^{(0)}(\xi,\tau)$ from correlation (9) finding $\tilde{u}^{(0)}(\xi,\tau)$.

Step-1: we shall take in right part (7) $\tilde{u}(\xi,\tau) = \tilde{u}^{(0)}(\xi,\tau)$, and solving boundary value problem (7)-(8) we shall define initial approximation $\tilde{v}^{(1)}(\xi,\tau)$. Substituting in (9) the function $\tilde{v}^{(1)}(\xi,\tau)$ found, finding $\tilde{u}^{(1)}(\xi,\tau)$.

And so on.

On step-k: continuing this process we shall get $\left(\tilde{v}^{(k)}(\xi,\tau), \tilde{u}^{(k)}(\xi,\tau)\right)$.

On each step of the offered algorithm using the parameterization method [1].

By fixed $\tilde{u}(\xi,\tau)$, $\xi \in [0,\omega]$ the problem (7)-(8) will be to family two points of boundary value problem for the ordinary differential equations

To family two points of boundary value problem for the ordinary differential equations using the parameterization method.

Theorem. *Let the function* $\tilde{u}(\xi,\tau), \xi \in [0,\omega]$ *be fixed and the family of two-point boundary value problem (7)-(8) for differential equations is solvable. Then following approximate* $\left(\tilde{v}^{(k)}(\xi,\tau), \tilde{u}^{(k)}(\xi,\tau)\right)$ *converges to the unique solution of the problem (7)-(9) and nonlocal boundary value problem (1)-(3) there is solvability in the wide extent.*

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