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Метод вычисления уравнения колебания слоистой пластинки

Получены частотные уравнения собственных колебаний двухслойной пластинки при заданных механических и геометрических характеристиках, являющихся основными элементами сейсмостойкости строительных конструкций. Результаты данных исследований приносят огромную пользу при рассмотрении стационарных, нестационарных колебательных и волновых процессов в таких разделах науки, как гидродинамика, геофизика. Задача решена приближенным методом получения частотных уравнений на основе метода декомпозиции.

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Method of calculation of the equation of fluctuation of the layered plate

The frequency equations of natural vibrations of the 2-layer plate for given mechanical and geometrical characteristics are obtained, which are the main elements of the seismic stability of building structures. The results of these studies are extremely useful when stationary and non-stationary vibrational wave processes are considered in such branches of science such as fluid dynamics, geophysics.The problem is solved by the method of obtaining the approximate frequency equations based on the decomposition method.

UDC 510.67

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Ehrenfeucht theories with non-dense powerful digraphs*

All known Ehrenfeucht theories are based on powerful digraphs having the density property. In the paper, we construct examples of Ehrenfeucht theories with powerful digraphs having the non-density property, i. e., with covering elements for each vertex of digraph. The construction of required theories is realized by syntactic generic ones allowing to synthesize saturated models based on the class of finite structures with some type add-ins, closed under amalgams.

Key words: Ehrenfeucht theory, powerful digraph, density property, covering element, syntactic generic construction.

1 Introduction

In [1], the notion of powerful digraph is defined, its role in Ehrenfeucht theories (i. e., having finitely many but more than one pairwise non-isomorphic countable models) is clarified, and it is proven that the transitive closure of powerful digraph, being isomorphic to the dual one, forms a dense partial order or a partial order with infinitely many covering elements for each element. All known examples of Ehrenfeucht theories are constructed either on a base of powerful digraphs containing dense partial orders (dense linear orders [2–4] or dense ordered trees [5–8]) or on a base of powerful digraphs with unbounded lengths of shortest paths and with dense transitive closures [9].

In the paper, we construct examples of Ehrenfeucht theories with powerful digraphs having non-dense transitive closures.

We use without specifications the standard graph-theoretic and model-theoretic notions [10, 11], the system of notions in [1, 9, 12] as well as general principles for constructions of generic structures based on

* The work is supported by RFBR (grant 12-01-00460-a).

the syntactic approach [13]. All considered theories are first order, complete, countable, and without finite models.

2 Notions and overview

Definition [1]. Let $\Gamma=\langle X;Q\rangle$ be a graph, a be a vertex in Γ. The set $\nabla_Q(a)=\bigcup_{n\in\omega}Q^n(a,\Gamma)$ (respectively, $\Delta_Q(a)=\bigcup_{n\in\omega}Q^n(\Gamma,a)$) is called an *upper* (*lower*) Q*-cone* of a. We call the Q-cones $\nabla_Q(a)$ and $\Delta_Q(a)$ by *cones* and denote by $\nabla(a)$ and $\Delta(a)$ respectively, if Q is fixed.

A countable acyclic directed graph (i. e., a digraph) $\Gamma=\langle X;Q\rangle$ is said to be *powerful* if the following conditions hold:

(a) the automorphism group of Γ is *transitive*, that is, any two vertices are linked by an automorphism;
(b) the formula $Q(x,y)$ is equivalent in the theory $\mathrm{Th}(\Gamma)$ to a disjunction of principal formulas;
(c) $\mathrm{acl}(a)\cap\Delta(a)=\{a\}$ for each vertex $a\in X$;
(d) $\Gamma\models\forall x,y\exists z\big(Q(z,x)\wedge Q(z,y)\big)$ (the *pairwise intersection property*).

Clearly, in the classical examples of Ehrenfeucht theories [2], the countable graph with the relation $x<y$ of dense linear order is powerful. A rather rich class of powerful digraphs is formed by the acyclic digraphs $\langle P,Q\rangle=\langle P;\{(p,p')\mid p'=pg_0 \text{ on some line}\}\rangle$ corresponding to polygonometries $\mathrm{pm}\big(G,\langle P,L,\in\rangle,g_0\big)$ on projective planes [14].

Definition [15]. A type $p\in S(T)$ is said to be *powerful* (in a theory *T*) if every model *M* of *T* realizing *p* also realizes every type $q\in S(T)$, that is, $M\models S(T)$.

Lemma 2.1 [15]. *Every Ehrenfeucht theory T has a powerful type.*

The following notion of semi-isolation is defined in [16]. A survey of related notions and results is represented in [17].

Definition. Let *M* be a model of a theory *T*, *a* and *b* be tuples in *M*, *A* be a subset of *M*.The tuple *a semi-isolates* the tuple *b* over the set *A* if there exists a formula $\varphi(a,y)\in\mathrm{tp}(b/Aa)$ for which $\varphi(a,y)\vdash\mathrm{tp}(b/A)$ holds. In this case we say that the formula $\varphi(a,y)$ (with parameters in *A*) *witnesses* that *b* is semi-isolated over *a* with respect to *A*.

Let *M* be a model of a theory *T*, $p(x)$ be a complete type of *T* over the empty set, $\psi(x,y)$ be a formula of *T*.Denote by $p(M)$ the set of realizations of $p(x)$ in *M*, and by $R^p_\psi(M)$ the binary relation $\{(a,b)\in(p(M))^2\mid M\models\psi(a,b)\}$. The relation $\{(a,b)\in(p(M))^2\mid a \text{ semi-isolates } b \text{ over } \varnothing\}$ is denoted by SI_p.

The following statement shows that the powerful digraphs reside «locally» in the structure of each non-principal powerful type.

Proposition 2.2 [1]. *If* $p(x)$ *is a non-principal powerful type of a theory T and M is a countable saturated model of T then for each formula* $\varphi(x)\in p(x)$, *there exists a formula* $\psi(x,y)$ *of T, satisfying the following conditions:*

(1) *for each* $a\in p(M)$, *the formula* $\psi(a,y)$ *is equivalent to a disjunction of principal formulas* $\psi_i(a,y)$, $i\le m$, *such that* $\psi_i(a,y)\vdash p(y)$, *and* $\models\psi_i(a,b)$ *implies that b does not semi-isolate a;*

(2) *for every* $a,b\in p(M)$, *there is a tuple c such that* $M\models\varphi(c)\wedge\psi(c,a)\wedge\psi(c,b)$.

We call the property (2) of Proposition 2.1, the *local pairwise intersection property* and denote it by (LPIP). If for the formula $\psi(x,y)$ the stronger property is true:

(2') for every $a,b\in p(M)$, there exists a tuple $c\in p(M)$ *such that* $M\models\psi(c,a)\wedge\psi(c,b)$, we then call it the *global pairwise intersection property* for $p(x)$ with respect to $\psi(x,y)$ and it will be denoted by (GPIP).

Whenever a formula $\psi(x,y)$ with properties 1 and 2' exist, call the digraph $\langle p(M);R_{\psi}^{p}(M)\rangle$ *prepowerful.*

Recall, that theories T_0 and T_1 of languages Σ_0 and Σ_1 respectively are said to be *similar* if for any models $M_i \models T_i$, $i=0,1$, there are formulas of T_i, defining in M_i predicates, functions and constants of language Σ_{1-i} such that the corresponding structure of Σ_{1-i} is a model of T_{1-i}.

A theory T is said to be (n,p)*-invariant* if for each formula $\psi(\bar{x})$ (where $l(\bar{x})=n$) of theory T_p (being a theory of restriction of ω-saturated model of T to the set of realizations of p), the restriction of the Morleyzation of T_p to the language $\{R_{\psi}\}$ (for the predicate with the set of solutions of $\psi(\bar{x})$) is similar to the restriction of the Morleyzation of the theory of the structure of some formula-definable set $\varphi(M)$ (where $\varphi\in p$, $M\models T$) to the same language.

Proposition 2.3 [1]. *If $p(x)$ is a non-principal powerful type of some $(2,p)$-invariant theory T and $\langle p(M);R_{\psi}^{p}(M)\rangle$ is a prepowerful digraph then for some formula $\theta(x,y)$ with $T\vdash\theta(x,y)\to\psi(x,y)$, the digraph $\langle p(M);R_{\theta}^{p}(M)\rangle$ is powerful.*

Recall that a partially ordered set $\langle X;\leq\rangle$ is *downward* (*upward*) *directed* if for each $x,y\in X$, there exists $z\in X$ such that $z\leq x$ and $z\leq y$ (respectively, $x\leq z$ and $y\leq z$).

Theorem 2.4 [1]. *Given a saturated powerful digraph $\Gamma=\langle X;Q\rangle$ in which $\mathrm{acl}(a)\cap\nabla(a)=\{a\}$ for each $a\in X$, the transitive closure $\mathrm{TC}(\Gamma)=\left\langle X;\bigcup_{n\in\omega}Q^n\right\rangle$ is isomorphic to a downward directed set with a transitive automorphism group and one of the following orders:*

(1_{α}) *a dense partial order with maximal antichains containing α elements, $\alpha\in(\omega+1)\setminus\{0\}$;*

(2) *a partial order with infinitely many covering elements for each element.*

Corollary 2.5 [1]. *If $\Gamma=\langle X;Q\rangle$ is a powerful digraph with a principal formula $Q(x,y)$ then the relation $\bigcup_{n\in\omega}Q^n$ is a dense partial order.*

Corollary 2.6 [1]. *Given a saturated powerful digraph $\Gamma=\langle X;Q\rangle$ with unbounded lengths of shortest paths such that $\mathrm{acl}(a)\cap\nabla(a)=\{a\}$ for each $a\in X$, its transitive closure $\mathrm{TC}(\Gamma)$ is isomorphic to a downward directed set with a transitive automorphism group that has one of the following orders:*

(1) *a dense partial order with infinite antichains;*

(2) *a partial order with infinitely many covering elements for each element.*

Definition [12, 17, 18]. A coloring $\mathrm{Col}\colon M\to\lambda\cup\{\infty\}$ (where λ is a cardinality) of a structure *M* is called *inessential* if each type $\mathrm{tp}(\bar{a})$ in the theory $\mathrm{Th}(\langle M,\mathrm{Col}\rangle)$ of the expansion of *M* by unary predicates $\mathrm{Col}_n=\{a\in M\mid\mathrm{Col}(\mathrm{a})=n\}$, $n\in\omega$, is implied by its restriction to the language of *M* united with its restriction to the predicates Col_n.

Let *M* be a model of a theory *T* and $\varphi(x,y)$ be a formula of *T*. A coloring $\mathrm{Col}\colon M\to\lambda\cup\{\infty\}$ (where λ is an infinite cardinality) is said to be φ*-ordered* if the following conditions hold:

(a) for any $\mu\leq\nu<\lambda$, there exist elements $a,b\in M$ such that $\models\mathrm{Col}_{\mu}(a)\wedge\mathrm{Col}_{\nu}(b)\wedge\varphi(a,b)$;

(b) if $\mu<\nu<\lambda$ then there are no elements $c,d\in M$ such that $\models\mathrm{Col}_{\mu}(c)\wedge\mathrm{Col}_{\nu}(d)\wedge\varphi(d,c)$.

Note that for each inessential coloring $\mathrm{Col}\colon M\to\lambda\cup\{\infty\}$ of model *M* of transitive theory $\mathrm{Th}(M)$ (with a unique 1-type), the set $\{\neg\mathrm{Col}_{\mu}(x)\mid\mu<\lambda\}$ implies a unique complete type $p_{\infty}(x)$.

Definition [9]. Let $\Gamma_1=\langle X_1;Q_1\rangle$ be a (vertex) colored subgraph of an acyclic colored digraph $\Gamma_2=\langle X_2;Q_2\rangle$ with a coloring $\mathrm{Col}: X_2\to\omega\cup\{\infty\}$; *a* and *b* be vertices in X_1; *S* be an (a,b)-path, not entirely in Γ_1. The path *S* is *external* (over Γ_1) if only endpoints in *S* belong to X_1. Denote by $W(\Gamma_1,\Gamma_2)$ the set of triplets (a,b,n), $a,b\in X$, $n\in\omega\setminus\{0,1\}$, such that *a* and *b* are linked in Γ_2 by a shortest (a,b)-path of length *n*, and, moreover, every shortest (a,b)-path is external over Γ_1. A triplet $\langle X_1,Q_1,W_1\rangle$, where $W_1=W(\Gamma_1,\Gamma_2)$, is called a c_0-subgraph of the digraph Γ_2 if the vertex set X_1 is finite.

The relation «to be a c_0-subgraph» is denoted by $\subseteq_{c_0}$, that is, having the set W_1, we write $\langle\Gamma_1,W_1\rangle\subseteq_{c_0}\Gamma_2$. A structure $\langle\Gamma_1,W_1\rangle$ is often treated independently; we call $\langle\Gamma_1,W_1\rangle$ a *c-graph* and denote it also by $\langle X_1,Q_1,W_1\rangle$, where $\Gamma_1=\langle X_1;Q_1\rangle$. Here, X_1 is called the *universe* of the *c*-graph $\langle\Gamma_1,W_1\rangle$.

For a *c*-graph $\tilde\Gamma_1=\langle X_1,Q_1,W_1\rangle$, $\mathrm{cc}(\tilde\Gamma_1)$ denotes a minimal digraph Γ, $\tilde\Gamma_1\subseteq_{c_0}\Gamma$, which, for any triplet $(a,b,n)\in W_1$, contains a shortest (a,b)-path of length *n*, with every intermediate vertex being of degree 2 (and, by the definition, being in $\Gamma\setminus\Gamma_1$).

We define the relation $\subseteq_c$ on the class of *c*-graphs. Thus, the *c*-graph $\tilde\Gamma_1=\langle X_1,Q_1,W_1\rangle$ is called a *c-subgraph* of *c*-graph $\tilde\Gamma_2=\langle X_2,Q_2,W_2\rangle$, written $\tilde\Gamma_1\subseteq_c\tilde\Gamma_2$, if $X_1\subseteq X_2$, $Q_1=Q_2\cap(X_1)^2$, and $W_1=W\left(\tilde\Gamma_1,\mathrm{cc}\left(\tilde\Gamma_2\right)\right)$.

Clearly, the relation $\subseteq_c$ induces a partial ordering on any set of *c*-graphs.

Below the *c*-graphs as well as their universes are denoted by $A,B,\ldots$, possibly with indices. The empty set $\varnothing$, in this instance, is assumed to be the universe of a c-graph having the form $\langle\varnothing,\varnothing,\varnothing\rangle$. If $\langle A,Q,W\rangle$ is a *c*-graph, then the relations *Q* and *W* are denoted by Q_A and W_A, respectively. Also, we write $A\subseteq_c N$ in place of $A\subseteq_{c_0} N$.

Note that any *c*-graph *B* can be represented by the type $\Phi_c(B)$ of language $\{Q^{(2)}\}\cup\{\mathrm{Col}_n^{(1)}\mid n\in\omega\}$, without free variables and consisting of the diagram of the (vertex) colored graph $\langle B;Q_B\rangle$, as well as of all formulas describing for any subgraph $\langle A;Q_A\rangle$ of a graph $\mathrm{cc}(B)$ and for any vertices $a,b\in A\cap B$, the existence or the absence of only external over *A* shortest (a,b)-paths of length $n\geq 2$ (triplets $(a,b,n)\in W_A$ symbolize formulas describing the existence of these paths). By the definition of the relation $\subseteq_c$, we have $A\subseteq_c B$ if and only if $\Phi_c(A)\subseteq\Phi_c(B)$.

3 Construction and results

In this Section, we define a modification of construction in [9], producing a powerful digraph $\hat\Gamma=\langle\hat X;\hat Q\rangle$ having the transitive closure being a partial order with infinitely many covering elements for any element in $\hat X$.

The relation $\hat Q$ is a disjoint union of relations $\hat Q_0$ and $\hat Q_1$ such that $\hat Q_0(x,y)$ and $\hat Q_1(x,y)$ are principal formulas satisfying the following:

$$\vdash\hat Q_i(x,y)\leftrightarrow\hat Q(x,y)\wedge\left(\exists z\left(\hat Q(x,z)\wedge\hat Q(y,z)\right)\right)^{1-i}\wedge\left(\exists u\left(\hat Q(x,u)\wedge\hat Q(u,y)\right)\right)^{i},\ i=0,1.$$

Here, $\hat Q_0(a,\hat\Gamma)$ is the set of successors for *a* in $\mathrm{TC}(\hat\Gamma)$.

Denote by $\hat K^*$ the class of all *c*-graphs $\langle A,\hat Q,W\rangle$, $\hat Q=\hat Q_0\dot\cup\hat Q_1$, such that $\models\hat Q_0(a,b)$ implies the absence of (c,b)-paths for any $c\in\hat Q(a,A)\setminus\{b\}$, and $\models\hat Q_1(a,b)$ implies the absence of (b,c)-paths for any $c\in\hat Q(a,A)\setminus\{b\}$. Here, if $(a,b)\in\hat Q_i$ then the index *i* is the *arc color* of (a,b).

Below, considering c-graphs in $\hat{K}^*$, we distinguish colors of arcs and use the language $\{\hat{Q}_0, \hat{Q}_1\}$ instead of (or as an addition to) $\hat{Q}$ in digraphs and c-graphs.

If A, $\langle B, \hat{Q}_B, W_B\rangle$, and $\langle C, \hat{Q}_C, W_C\rangle$ are c-graphs, and $A = B \cap C$, then we call the c-graph $\langle B \cup C, \hat{Q}_B \cup \hat{Q}_C, W_B \cup W_C\rangle$ the *free c-amalgam* of the c-graphs B and C over A and denote it by $B *_A C$.

Clearly, the free c-amalgam $B *_A C$ exists for any c-graphs A, B, C with $A = B \cap C$. In this case, c-graphs A, B, and C are c-subgraphs of the c-graph $B *_A C$.

If $A \subseteq_c B$ then the c-graph A is denoted by $B|A$.

A one-to-one map $f: A \to B$ is called a *c-embedding* of the c-graph $\langle A, \hat{Q}_A, W_A\rangle$ into the c-graph $\langle B, \hat{Q}_B, W_B\rangle$ (written $f: A \to_c B$) if f is an embedding of the colored graph $\langle A; \hat{Q}_A\rangle$ into the colored graph $\langle B; \hat{Q}_B\rangle$ such that $W_{B|f(A)} = \{(f(a_1), f(a_2), n) \mid (a_1, a_2, n) \in W_A\}$ and

$$\{(f(a_1), f(a_2)) \mid (a_1, a_2) \in \hat{Q}_{A,i}\} \subseteq \hat{Q}_{B,i}, i = 0,1.$$

We say that c-graphs A and B are *c-isomorphic* if there is a c-embedding $f: A \to_c B$ with $f(A) = B$. In this event, f is called a *c-isomorphism between A and B*, and c-graphs A and B are called *c-isomorphic copies.*

A one-to-one map $f: A \to N$ is a *c-embedding* of the c-graph A into the digraph N (written $f: A \to_c N$) if f is a c-embedding of the c-graph A onto the c_0-subgraph $f(A)$ of N.

Denote by $\hat{K}_0^*$ the subclass of $\hat{K}^*$ generated by stated below c-graphs $\langle A, \hat{Q}_0, \hat{Q}_1, W\rangle$ by taking of c-subgraphs, c-isomorphic copies, free c-amalgams, *routings* (allowing for any chosen pair of vertices $a \neq b$, that are not linked by (a,b)-paths, where $\mathrm{Col}(a) \leq \mathrm{Col}(b)$ and there are no vertices with $(c,b) \in \hat{Q}_0$ and $a \in \Delta_{\hat{Q}}(c)$ or with $(c,a) \in \hat{Q}_0$ and $b \in \Delta_{\hat{Q}}(c)$, to extend records W by an information on shortest (a,b)-paths of arbitrary length m, exceeding $\frac{k(k-1)}{2}$, where $k = |\mathrm{cc}(A)|$), and *deroutings* (allowing to remove the information above):

(a) $\Gamma_{\alpha,\beta,\gamma,0} = \langle\{0,1,2\}, \{(0,1),(1,2)\}, \{(0,2)\}, \varnothing\rangle$;

(b) $\Gamma_{\alpha,\beta,\gamma,1} = \langle\{0,1,2\}, \{(0,1)\}, \{(0,2),(1,2)\}, \varnothing\rangle$;

(c) $\Gamma_{\alpha,\beta,\gamma,s} = \langle\{0,1,2\}, \{(0,1)\}, \{(0,2)\}, W\rangle$, where $W = \{(1,2,s)\}$, $2 \leq s < \omega$, $\mathrm{Col}(0) = \alpha$, $\mathrm{Col}(1) = \beta$, $\mathrm{Col}(2) = \gamma$, $\alpha \leq \beta \leq \gamma$, $\gamma \in \omega \cup \{\infty\}$.

Lemma 3.1 ($\hat{\ }$-amalgamation Lemma). *The class $\hat{K}_0^*$ has the $\hat{\ }$-amalgamation property, that is, for any c-embeddings $f_0: A \to_c B$ and $g_0: A \to_c C$, where $A, B, C \in \hat{K}_0^*$, there exist a c-graph $D \in \hat{K}_0^*$ and c-embeddings $f_1: B \to_c D$ and $g_1: C \to_c D$ such $f_0 \circ f_1 = g_0 \circ g_1$.*

Proof is obvious. □

Denote by $\hat{K}_0$ the class of colored acyclic digraphs in which every finite subgraph forms a c-graph in $\hat{K}_0^*$.

Theorem 3.2. *There exists countable, colored, saturated digraph $\hat{M} \in \hat{K}_0$ satisfying the following:*

(1) *if $f: A \to_c \hat{M}$ and $g: A \to_c B$ are c-embeddings and $B \in \hat{K}_0^*$, then there exists a c-embedding $h: B \to_c \hat{M}$ such that $f = g \circ h$;*

(2) *if A and B are c-isomorphic c-subgraphs of $\hat{M}$, then $tp_{\hat{M}}(A) = tp_{\hat{M}}(B)$;*

(3) *the coloring of restriction* $\hat{M}\big|_{\hat{Q}}$ *of* $\hat{M}$ *to the graph language* $\Sigma = \{\hat{Q}\}$ *is inessential and* $\hat{Q}$*-ordered;*

(4) *the formula* $\hat{Q}(x,y)$ *is equivalent in* $\mathrm{Th}\left(\hat{M}\big|_{\hat{Q}}\right)$ *to the disjunction of principal formulas* $\hat{Q}_0(x,y)$ *and* $\hat{Q}_1(x,y)$.

Proof is similar to the proof of [9, Theorem 1.3]. The construction of the saturated structure $\hat{M}$ repeats steps of construction for the structure *M* in the proof of [9, Theorem 1.3] with replacing of K_0^* by the class $\hat{K}_0^*$ and of K_0 by $\hat{K}_0$. □

Elements in $\hat{Q}_0(a,\hat{M})$ are successors of *a* in the digraph $\mathrm{TC}(\hat{\Gamma})$, where $\hat{\Gamma} = \hat{M}\big|_{\hat{Q}}$, since by construction, there are no elements $b \in \hat{Q}(a,\hat{M})$ such that there exists a (b,c)-path with $c \in \hat{Q}_0(a,\hat{M}) \setminus \{b\}$. Thus, the partial order is not dense in $\mathrm{TC}(\hat{\Gamma})$.

Similarly Corollaries 1.4–1.6 in [9], we state that the relation $\mathrm{SI}_{p_\infty(x)}$ of semi-isolation is non-symmetric, the digraph $\langle p_\infty(\hat{M}); R_{\hat{Q}}^{p_\infty}(\hat{M})\rangle$ is powerful, and the theory $\mathrm{Th}(\hat{M})$ is not simple.

Notice that constructions, described in Sections 1 and 2 of [9], can be applied to $\hat{M}$ and also produce a non-ω-categorical, non-simple theory, for which any non-principal type is powerful (and therefore there are exactly two isomorphism types of prime models over finite sets). Thus, we obtain the following theorem.

Theorem 3.3. *There exists a generic theory T satisfying the following conditions:*

(1) $|\mathrm{RK}(T)| = 2$;

(2) *the structure of realizations of some powerful type* $p \in S(T)$ *contains a powerful digraph* Γ *with unbounded lengths of shortest paths and with infinitely many covering elements for each element in the transitive closure* $\mathrm{TC}(\Gamma)$.

Repeating the construction in [9, Section 3] we obtain an Ehrenfeucht theory with three countable models, based on the powerful digraph $\hat{\Gamma}$:

Theorem 3.4. *There exists a generic Ehrenfeucht theory T with* $I(T,\omega) = 3$ *and having a structure of powerful type with a powerful digraph* $\hat{\Gamma}$, *whose transitive closure forms a non-dense partial order*.

Repeating the proof of Theorem 4.1 in [9], based on the powerful digraph $\hat{\Gamma}$, we obtain the basic theorem of [9] with an additional property that the structure of powerful digraph is not dense:

Theorem 3.5. *For any finite preordered set* $\langle X;\le\rangle$ *with the least element* x_0 *and the greatest class* $\tilde{x}_1$ *in the ordered factor set* $\langle X;\le\rangle/\!\sim$ *with respect to* ~ (*where* $x \sim y \Leftrightarrow x \le y$ *and* $y \le x$), *and for any function* $f\colon X \to \omega$ *satisfying the conditions* $f(\tilde{x}_0) = 0$, $f(\tilde{x}_1) > 0$ *for* $|X| > 1$, *and* $f(\tilde{y}) > 0$ *for* $|\tilde{y}| > 1$, *there exist a complete theory T, with a structure of non-dense transitive closure of a powerful digraph, and an isomorphism* $g\colon \langle X;\le\rangle \tilde{\to} \mathrm{RK}(T)$ *such that* $\mathrm{IL}(g(\tilde{y})) = f(\tilde{y})$ *for any* $\tilde{y} \in X/\!\sim$.

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Тығыз емес үстем орграфты эренфойхт теориялары

Барлық белгілі эренфойхтық теорияларды құрудың негізі тығыздық қасиетіне ие үстемді орграфтар болып табылады. Мақалада тығыздық қасиеті жоқ үстемді орграфты эренфойхтық теорияның мысалдары берілген, яғни әрбір төбесі толығымен құрсауланатын орграфтың бар болатындығы көрсетілген. Ізделініп отырған теорияны құру синтаксистік генерикалық құрылымдардың негізінде жүзеге асады. Бұл кейбір типтік алғашқы құрылымдардың ақырлы құрылымдар класынан шығып, қаныққан модельдерді синтездеуге мүмкіндік берді.

С.В.Судоплатов

Эренфойхтовы теории с неплотными властными орграфами

Основой построения всех известных эренфойхтовых теорий являются властные орграфы, обладающие свойством плотности. В статье даны примеры эренфойхтовых теорий с властными орграфами, обладающими свойством неплотности, т.е. наличия покрывающих элементов у каждой вершины орграфа. Построение искомых теорий проводится на основе синтаксических генерических конструкций, позволяющих синтезировать насыщенные модели, исходя из класса конечных структур с некоторыми типовыми надстройками, замкнутого относительно взятия амальгам.