FORWARD AND INVERSE PROBLEMS FOR THE BARENBLATT-ZHELTOV-KOCHINA TYPE EQUATION WITH THE INITIAL CONDITION GIVEN BY A FRACTIONAL DERIVATIVE

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Let H be a separable Hilbert space and $A: H \to H$ be an arbitrary unbounded positive selfadjoint operator in$H$. We assume that A has compact inverse A^{-1}. Let λ_k and v_k be the eigenvalues and corresponding eigenfunctions of A:

$$Av_k = \lambda_k v_k,$$

where $0 < \lambda_1 \le \lambda_2 \cdots \to +\infty$ and thedomain of this operator has the form

$$D(A) = \{h \in H : \sum_{k=1}^{\infty} \lambda_k^2 |h_k|^2 < \infty\}.$$

Consider the following problem

$$\begin{cases} u_t(t) + A(u_t(t)) + Au(t) = 0, & 0 < t \le T; \\ B_t^{\rho} u(+0) = \varphi, \end{cases} \tag{1}$$

where $\varphi \in H$. These problems are also called *the forward problems*.

The initial condition is defined by B_t^{ρ}. In particular, initial conditions can be given through the one-sided Marchaud, Grunwald-Letnikov or Liouville-Weyl fractional derivatives. Note B_t^{ρ} satisfies the following property

$$B_t^{\rho} e^{-at} = a^{\rho} e^{-at}, \quad t > 0, \quad a > 0. \tag{2}$$

This formula played an essential used in our paper here.

In this paper, we prove the existence and uniqueness of a solution to the forwardproblem (1).

Theorem 1. *Let $\varphi \in D(A)$. Then there exists a unique solution of the forward problem* (1) *and it has the representation*

$$u(t) = \sum_{k=1}^{\infty} \frac{\varphi_k}{\mu_k^{\rho}} e^{-\mu_k t} v_k, \tag{3}$$

where $\mu_k = \dfrac{\lambda_k}{1+\lambda_k}$ *and* $\varphi_k = (\varphi, v_k)$.

Moreover, we study the inverse problem of finding the initial conditions of the problem (1). For this, we need an additional condition and as an additional condition, we will get the following condition:

$$u(\tau) = \Psi, \quad 0 < \tau \le T, \tag{4}$$

where τ – a fixed point.

Theorem 2. *Let $\varphi, \Psi \in D(A)$. Then the inverse problem* (1), (4) *has a unique solution* $\{u(t), \varphi\}$ *and this solution has the following form*

$$u(t) = \sum_{k=1}^{\infty} \frac{\varphi_k}{\mu_k^{\rho}} e^{-\mu_k t} v_k, \tag{5}$$

where

$$\varphi_k = \Psi_k \mu_k^{\rho} e^{\mu_k \tau},$$

and

$$\varphi = \sum_{k=1}^{\infty} \Psi_k \mu_k^{\rho} e^{\mu_k \tau} v_k. \tag{6}$$

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ON STABILIZATION PROBLEM FOR A LOADED HEAT EQUATION: THE TWO-DIMENSIONAL CASE

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Introduction. One of the important properties that characterize the behavior of solutions of boundary value problems for differential equations is stabilization, which has a direct relationship with the problems of controllability. The problems of solvability of stabilization problems of two-dimensional loaded equations of parabolic type with the help of feedback control given on the boundary of the region are investigated in the paper. These equations have numerous applications in the study of inverse problems for differential equations. The problem consists in the choice of boundary conditions (controls), so that the solution of the boundary value problem tends to a given stationary solution at a certain speed at $t\to\infty$. This requires that the control is feedback, i.e. that it responds to unintended fluctuations in the system, suppressing the results of their impact on the stabilized solution. The spectral properties of the loaded two-dimensional Laplace operator, which are used to solve the initial stabilization problem, are also studied. The paper presents an algorithm for solving the stabilization problem, which consists of constructively implemented stages. The idea of reducing the stabilization problem for a parabolic equation by means of boundary controls to the solution of an auxiliary boundary value problem in the extended domain of independent variables belongs to A.V. Fursikov [1]. At the same time, recently, the so-called loaded differential equations are actively used in problems of mathematical modeling and control of nonlocal dynamical systems.

Statement of the problem. Let $\Omega=\{x,y:-\pi/2<x,y<\pi/2\}$ be a domain with a boundary $\partial\Omega$. In the cylinder $Q=\Omega\times\{t>0\}$ with lateral surface $\Sigma=\partial\Omega\times\{t>0\}$ we consider the boundary value problem for the loaded heat equation

$$u_t-\Delta u+\alpha u(0,y,t)+\beta u(x,0,t)=0,\quad \{x,y,t\}\in Q, \tag{1}$$

$$u(x,y,0)=u_0(x,y),\quad \{x,y\}\in\Omega, \tag{2}$$

$$u(x,y,t)=p(x,y,t),\quad \{x,y,t\}\in\Sigma, \tag{3}$$

where $\alpha,\beta\in\mathbb{C}$ are given (in general case are complex) bounded constants, $u_0(x,y)$ is given function. The aim is to find a function $p(x,y,t)$ such that a solution of the boundary value problem (1)–(3) satisfies the inequality

$$\|u(x,y,t)\|_{L_2(\Omega)}\le C_0e^{-\sigma t},\quad \sigma>0,\quad t>0. \tag{4}$$

Note that here σ is a given constant and C_0 is an arbitrary bounded constant.

Equation (1) is called a loaded equation [2,3]. We note that problem (1)–(4) with a single load point was studied in [4], andwith a two-dimensional cases was studied in [5–7].