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## On the solvability of the boundary value problems for the elliptic equation of high order on a plane

For the elliptic equation of  $2l$ -th order with of constant (and only) real coefficients we consider boundary value problem of the normal derivatives  $(k_j - 1)$  order,  $j = 1, \dots, l$ , where  $1 \leq k_1 < \dots < k_l \leq 2l - 1$ . When  $k_j = j$  it moves into the Dirichlet problem, and when  $k_j = j + 1$  it moves into the Neumann problem. In this paper, the study is carried out in space  $C^{2l, \mu}(\bar{D})$ . We found the condition for Fredholm solvability of this problem and computed the index of this problem.

*Keywords:* elliptic equation, boundary value problem, Dirichlet problem, Neumann problem, solvability of BVP.

### Introduction

From the viewpoint of an explicit description of the conditions of solvability of Fredholm and of index for this problem has been studied [1] in the class

$$u \in C^{2l}(D) \cap C^{2l-1, \mu}(\bar{D}), \quad \sum_{0 \leq r \leq 2l} a_{r, 2l} \frac{\partial^{2l} u}{\partial x^{2l-r} \partial y^r} \in C^\mu(\bar{D}).$$

In this paper, under the assumption that  $\Gamma \in C^{2l, \mu}$  obtained in the paper [1] results extend to a standard class  $C^{2l, \mu}(\bar{D})$ , which no longer depends on the equation (1).

In [2–8], an explicit form of the Green function of the Dirichlet problem for a polyharmonic equation in a multidimensional ball is constructed. The paper [9, 10] is devoted to the investigation of the solvability of various boundary value problems for a polyharmonic equation in a multidimensional ball. In this paper we obtain a necessary and sufficient condition for the problem to be Fredholm in terms of the original data, that is, from the right-hand side of the inhomogeneous polyharmonic equation and from the right-hand sides of the inhomogeneous boundary conditions. The correct restrictions of the stationary Navier-Stokes equation in a three-dimensional cube are described in [11], and the correct boundary conditions for the pressure in the medium are determined. In [12], initial-boundary value problems for the equations of motion of a viscous heat-conducting gas are studied with allowance for a magnetic field with cylindrical and spherical symmetry. In this paper, we prove theorems on the existence and uniqueness of solutions as a whole with respect to the time of initial-boundary value problems. In [13], a brief summary of the theory of extensions and contractions of operators in Hilbert space is given, and certain classes of well-posed boundary value problems for the bi-Laplace operator are written out. The Green function of the Neumann problem for the Poisson equation in a multidimensional ball is constructed in [14].

### Formulation of the problem

In simply connected region  $D$  in the plane bounded by a simple smooth contour  $\Gamma$ , we consider the elliptic equation

$$\sum_{0 \leq r \leq k \leq 2l} a_{rk}(z) \frac{\partial^k u}{\partial x^{k-r} \partial y^r} = g(z), \quad z = x + iy \in D, \quad (1)$$

with real coefficients  $a_{rk} \in C^\mu(\bar{D})$ ,  $0 < \mu < 1$ , constant at  $k = 2l$ . Without loss of generality we can assume that  $a_{2l, 2l} = 1$ .

The Generalized Dirichlet - Neumann problem for this equation is determined by the boundary conditions

$$\frac{\partial^{k_j-1}u}{\partial n^{k_j-1}} \Big|_{\Gamma} = f_j, \quad j = 1, \dots, l, \quad (2)$$

where  $1 \leq k_1 < k_2 < \dots < k_l \leq 2l$ ,  $n = n_1 + in_2$  means the unit external normal and under normal derivative  $k$ -th order we mean the expression

$$\left( n_1 \frac{\partial}{\partial x} + n_2 \frac{\partial}{\partial y} \right)^k u = \sum_{r=0}^k \binom{k}{r} n_1^r n_2^{k-r} \frac{\partial^k u}{\partial x^r \partial y^{k-r}}.$$

*Fredholm solvability of the problem*

As usual Fredholm property and the index of the problem are understood in relation toward its restricted operator

$$C^{2l,\mu}(\overline{D}) \rightarrow C^\mu(\overline{D}) \times \prod_{j=1}^l C^{2l-k_j+1,\mu}(\Gamma). \quad (3)$$

For derivatives of  $v \in C^{r,\mu}(\Gamma)$ ,  $1 \leq r \leq 2l - 1$ , with respect to the parameter arc length we have the expression

$$\left( \frac{d}{ds} \right)^r v = \frac{\partial^r v}{\partial e^r} + \dots$$

where  $e = e_1 + ie_2 = -in$  is the unit tangent vector to the contour  $\Gamma$ , tangential derivative of  $r$ - order  $\partial^r v / \partial e^r$  is understood as analogous (2) and the dots denote a linear differential operator of order  $r - 1$ , whose coefficients are expressed through the function  $e_1, e_2$  and derivatives of order  $r - 1$  inclusive. In virtue of the assumptions about the smoothness of the contour  $\Gamma$  coefficients of the operator belong to the class  $C^{2l-r,\mu}(\Gamma)$ . Therefore, similar to [1] boundary conditions (2) can be rewritten in the equivalent form

$$\left( e_1 \frac{\partial}{\partial x} + e_2 \frac{\partial}{\partial y} \right)^{2l-k_j} \left( n_1 \frac{\partial}{\partial x} + n_2 \frac{\partial}{\partial y} \right)^{k_j-1} u + L_j^0 u = f_j^0, \quad 1 \leq j \leq l, \quad (4)$$

with the right-hand side

$$f_j^0 = f_j^{(2l-k_j)} + \int_{\Gamma} f_j(t) d_1 t,$$

where the symbol  $d_1 t$  is an element of arc length, and operators

$$L_j^0 u = \sum_{0 \leq r \leq s \leq 2l-2} a_{j,rs} \frac{\partial^s u}{\partial x^{s-r} \partial y^r} + \int_{\Gamma} \frac{\partial^{k_j-1} u}{\partial n^{k_j-1}} d_1 t,$$

with some coefficients  $a_{j,rs}(z) \in C^{1,\mu}(\Gamma)$ . It is clear that the operator  $L^0 = (L_1^0, \dots, L_l^0)$  is compact  $C^{2l,\mu}(\overline{D}) \rightarrow C^{1,\mu}(\Gamma)$ .

Consider the map

$$\mathcal{D}u = (U_1, \dots, U_{2l}), \quad U_j = \frac{\partial^{2l-1} u}{\partial x^{2l-j} \partial y^{j-1}},$$

that acts from  $C^{2l,\mu}(\overline{D})$  in the space  $C^{1,\mu}(\overline{D})$  of vector-functions satisfying the relations

$$\frac{\partial U_j}{\partial y} = \frac{\partial U_{j+1}}{\partial x}, \quad 1 \leq j \leq 2l - 1. \quad (5)$$

The core of this operators  $\ker \mathcal{D}$  is the class  $P_{2l-2}$  of all polynomials of degree at most  $2l - 2$ , which is equal to the dimension of  $l(2l - 1)$ .

As in [1] introduce the right-hand operator  $\mathcal{D}^{(-1)}$ , so that any function  $u \in C^{2l,\mu}(\overline{D})$  uniquely represented in the form

$$u = \mathcal{D}^{(-1)}U + p, \quad p \in P_{2l-2}, \quad (6)$$

where the vector-function  $U \in C^{1,\mu}(\overline{D})$  satisfying the relations (5).

Substituting this representation in (1) and using (4), from the elliptic equation can come to the equivalent first order system

$$\frac{\partial U}{\partial y} - A \frac{\partial U}{\partial x} + L^1(\mathcal{D}^{(-1)}U + p) = g^1 \quad (7)$$

with  $2l \times 2l$ - matrix

$$A = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \cdot & \cdot & \cdot & \dots & \cdot \\ 0 & 0 & 0 & \dots & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{2l-1} \end{pmatrix}, \quad a_r = a_{r,2l},$$

with the right-hand side  $g^1 = (0, \dots, 0, g)$  and the operator

$$L^1 v = (0, \dots, 0, L_{2l}^1 v), \quad L_{2l}^1 v = \sum_{0 \leq r \leq k \leq 2l-1} a_{rk} \frac{\partial^k v}{\partial x^{k-r} \partial y^r}.$$

Note that the operator  $L^1$  is compact  $C^{2l, \mu}(\bar{D}) \rightarrow C^\mu(\bar{D})$ .

With respect to the matrix  $C = (C_{jk}) \in C^{2l-1, \mu}(\Gamma)$ , the elements of which are defined by the relations

$$\sum_{k=1}^{2l} C_{jk}(t) z^{k-1} = [e_1(t) + e_2(t)z]^{2l-k_j} [-e_2(t) + e_1(t)z]^{k_j-1}, \quad 1 \leq j \leq l, \quad (8)$$

the boundary conditions (4) can be written in the form

$$CU^+ + L^0(\mathcal{D}^{(-1)}U + p) = f^0, \quad (9)$$

where the symbol  $+$  indicates the limit value functions. Recall that appearing here the operator  $L\mathcal{D}^{(-1)}$  is compact  $C^{1, \mu}(\bar{D}) \rightarrow C^{1, \mu}(\Gamma)$ .

We write the characteristic polynomial equation (1) in the form

$$\sum_{r=0}^{2l} a_{r,2l} z^r = \prod_{k=1}^m [(z - \nu_k)(z - \bar{\nu}_k)]^{l_k}, \quad \text{Im } \nu_k > 0, \quad (10)$$

and with each vector-function  $g(z) = (g_1(z), \dots, g_n(z))$ , analytic in the neighborhood of the point  $\nu_1, \dots, \nu_m$ . We introduce block  $n \times l$ - matrix

$$W_g(\nu_1, \dots, \nu_m) = (W_g(\nu_1), \dots, W_g(\nu_m)),$$

where the matrix  $W_g(\nu_k) \in \mathbb{C}^{n \times l_k}$  is composed of column - vectors

$$g(\nu_k), g'(\nu_k), \dots, \frac{1}{(l_k - 1)!} g^{(l_k-1)}(\nu_k).$$

We introduce block  $2l \times 2l$ - matrix

$$\begin{aligned} \tilde{B} &= (B, \bar{B}), \quad B = W_h(\nu_1, \dots, \nu_m) \in \mathbb{C}^{2l \times l}, \\ \tilde{J} &= \text{diag}(J, \bar{J}) \quad J = \text{diag}(J_1, \dots, J_m), \end{aligned} \quad (11)$$

where  $h(z) = (1, z, \dots, z^{2l-1})$  and

$$J_k = \begin{pmatrix} \nu_k & 1 & 0 & \dots & 0 \\ 0 & \nu_k & 1 & \dots & 0 \\ \cdot & \cdot & \cdot & \dots & \cdot \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & \nu_k \end{pmatrix} \in \mathbb{C}^{l_k \times l_k}$$

is a Jordan cell, corresponding to the eigenvalue  $\nu_k$ .

As shown in [1], the matrix  $\tilde{B}$  is reversible and transfers in  $A$  to Jordan form  $\tilde{J}$ , i.e. we have the equality

$$\tilde{B}^{-1} A \tilde{B} = \tilde{J}.$$

Obviously, the operation of multiplication by a matrix  $\tilde{B}^{-1}$  transforms real  $2l$ - vector-functions  $U$  in the complex vector-function  $\tilde{\phi}$  block form  $(\phi, \bar{\phi})$ . Wherein

$$(\tilde{B}^{-1}L_A\tilde{B})\tilde{\phi} = (L_J\phi, \overline{L_J\phi}), \tag{12}$$

where for brevity

$$L_A = \frac{\partial}{\partial y} - A\frac{\partial}{\partial x}, \quad L_J = \frac{\partial}{\partial y} - J\frac{\partial}{\partial x}.$$

Recall that the operator  $\mathcal{D}^{-1}$ , appearing in (7), (8), is defined on  $2l$ - the vector-functions  $U \in C^{1,\mu}(\overline{D})$ , satisfying the conditions (5). In terms of projector  $Q$ , acting according to the formula

$$(QU)_j = \begin{cases} U_j, & 1 \leq j \leq 2l-1; \\ 0, & j = 2l, \end{cases}$$

these conditions can be described in the form  $QL_AU = 0$ . As shown in [1], there is limited to  $C^{1,\mu}(\overline{D})$  projector  $P$  with the image  $\text{im } P = \{U \in C^{1,\mu}(\overline{D}), QL_AU = 0\}$ . This operator is constructed as follows [1].

We choose  $\rho$  so large that the closed region  $\overline{D}$  is contained in the disc  $D_0 = \{|z| < \rho\}$ . Then there is a bounded operator  $C^\mu(\overline{D}) \rightarrow C^\mu(\overline{D}_0)$  continuation, denoted by  $\varphi \rightarrow \hat{\varphi}$ , with properties

$$\hat{\varphi}|_D = \varphi, \quad \hat{\varphi}|_{\partial D_0} = 0.$$

To every non-zero complex number  $z = x + iy$  we associate an invertible matrix  $zJ = x1 + yJ$ , where  $1$  is a single  $l \times l$ - matrix. We introduce the integral operator

$$(I^1\varphi)(z) = \frac{1}{\pi i} \int_D (t-z)_J^{-1} \hat{\varphi}(t) d_2t, \quad z \in D,$$

where  $d_2t$  is the area element. This expression is the bounded mapping  $C^\mu(\overline{D}) \rightarrow C^{1,\mu}(\overline{D})$  and is a right-hand inverse of  $L_J$ , i.e.

$$L_J I^1 \varphi = \varphi. \tag{13}$$

Taking into account

$$(\tilde{B}^{-1}I\tilde{B})\tilde{\varphi} = (I^1\varphi, \overline{I^1\varphi}), \quad \tilde{\varphi} = (\varphi, \bar{\varphi}),$$

obtain an operator  $I$ , acting in the space  $C^\mu(\overline{D})$  of real  $2l$ - vector-functions, which in view of (12) has a similar property in relation to  $L_A$ . In our notation the desired projector  $P$  is defined by  $P = 1 - IQL_A$ .

As in [1] via this projector from (7), (8) we can move on to the problem

$$L_AU + L^1(\mathcal{D}^{-1})PU + p = g^1, \quad CU^+ + L^0(\mathcal{D}^{-1})PU + p = f^0, \tag{14}$$

which is already considered in the whole space  $C^{1,\mu}(\overline{D})$ . Since  $QL^0 = 0$ , from the first equation of this problem it follows  $QL_AU = Qf^1$ . Therefore, if the right side  $f^1$  has the property  $Qf^1 = 0$ , i.e.  $f_j^1 = 0$ ,  $1 \leq j \leq 2l-1$ , then any solution  $U$  problems (14) satisfies the condition (5). In other words, for the given right-hand side  $f^1$  problem (14) is equivalent to (7), (8).

We use further substitution

$$U = \tilde{B}\tilde{\phi}, \quad \tilde{\phi} = (\phi, \bar{\phi}), \tag{15}$$

according to which we introduce the operators  $L^{(1)} : C^{1,\mu}(\overline{D}) \times P_{2l-2} \rightarrow C^\mu(\overline{D})$  and  $L^{(0)} : C^{1,\mu}(\overline{D}) \times P_{2l-2} \rightarrow C^\mu(\Gamma)$ , acting according to the formulas

$$(L^{(1)}(\phi, p), \overline{L^{(1)}(\phi, p)}) = \tilde{B}^{-1}L^1(\mathcal{D}^{-1})P\tilde{B}\tilde{\phi} + p, \quad L^{(0)}(\phi, p) = L^0(\mathcal{D}^{-1})P\tilde{B}\tilde{\phi} + p.$$

Then, taking into account (11), (12) the substitution of (15) leads (14) to the following equivalent problem

$$L_J\phi + L^{(1)}(\phi, p) = f^1, \quad 2\text{Re}(CB\phi) + L^{(0)}(\phi, p) = f^0, \tag{16}$$

where we put  $\tilde{B}^{-1}g^1 = (f^1, \overline{f^1})$ , which is considered in the class  $C^{1,\mu}(\overline{D})$   $l$ - complex vector-functions  $\phi$ .

So far all reviews have been carried out in the same way as [1] with the difference that in this work problem (16) is considered in the class of functions  $\phi \in C^\mu(\bar{D}) \cap C^1(D)$ , for which  $L_J\phi \in C^\mu(\bar{D})$ . Following [2], we introduce the generalized Cauchy type integrals

$$(I^0\psi)(z) = \frac{1}{2\pi i} \int_{\Gamma} (t-z)_J^{-1} dt_J \psi(t), \quad z \in D,$$

with a density  $\varphi \in C^{1,\mu}(\Gamma)$ , where with respect to the point  $t = t_1 + it_2$  on the curve  $dt_J$  is a complex matrix differential  $dt_1 + dt_2 J$  and contour  $\Gamma$  positively oriented with respect to  $D$ . It is important to note that it has the property

$$L_J I^0 \varphi^0 = 0. \tag{17}$$

The Cauchy type integrals answer corresponding singular integral

$$(S^0\psi)(t_0) = \frac{1}{\pi i} \int_{\Gamma} (t-t_0)_J^{-1} dt_J \psi(t), \quad t_0 \in \Gamma,$$

which is understood in the sense of the Cauchy principal value. Note that in the case of a scalar matrix  $J = i$  the operator  $S^0$  becomes classic singular Cauchy operator, denoted by  $S$ . As shown in [3], operators  $S$  and  $S^0$  are bounded in the spaces  $C^\mu(\Gamma)$ ,  $C^{1,\mu}(\Gamma)$ , and the difference  $S^0 - S$  is a compact operator. In addition, by the differentiation formulas given in [3] the operator  $L^0$  is bounded  $C^{1,\mu}(\Gamma) \rightarrow C^{1,\mu}(\bar{D})$  and just corresponds to an analogue of Sokhotskii - Plemelj

$$(I^1\varphi)^+ = (\varphi + S^1\varphi)/2. \tag{18}$$

Based on these results, similarly to the classical theory of singular operators [4] we show that under the assumption of

$$\det[C(t)B] \neq 0, \quad t \in \Gamma, \tag{19}$$

the operator

$$N^0\psi = \text{Re}[CB(\psi + S^0\psi)], \tag{20}$$

acting in the space of real  $l$ - of vector-functions  $\psi \in C^{1,\mu}(\Gamma)$ , is Fredholm and its index is given by

$$\text{ind } N^0 = -\frac{1}{\pi} [\arg \det(CB)]|_{\Gamma}. \tag{21}$$

Further arguments are similar to those given in [1]. As this paper shows any function  $\phi \in C^{1,\mu}(\bar{D})$  can be uniquely represented in the form

$$\phi = I^1\varphi^1 + I^0\varphi^0 + i\xi, \quad \xi \in \mathbb{R}^l,$$

with some complex  $l$ - vector-function  $\varphi^1 \in C^\mu(\bar{D})$  and real  $\varphi^0 \in C^{1,\mu}(\Gamma)$ . The substitution of this representation in (16) given (13), (17), (18) reduces the problem to an equivalent system of integral equations

$$\begin{aligned} \varphi^1 + L_J(I^0\varphi^0 + i\xi) + L^{(1)}(I^1\varphi^1 + I^0\varphi^0 + i\xi, p) &= f^1; \\ \text{Re}[CB(\varphi^0 + S^0\varphi^0)] + 2\text{Re}[CB(I^1\varphi^1 + i\xi)] + L^{(0)}(I^1\varphi^1 + i\xi, p) &= f^0. \end{aligned}$$

In the notation (20) we write it briefly in the operator form

$$N^0\varphi^0 + M^{00}\varphi^0 + M^{01}\varphi^1 + T^0(p, \xi) = f^0, \quad \varphi^1 + M^{10}\varphi^0 + M^{11}\varphi^1 + T^1(p, \xi) = f^1, \tag{22}$$

with the relevant operators  $T^i$  and

$$\begin{aligned} M^{00}\varphi^0 &= L^{(0)}I^0\varphi^0, \quad M^{01}\varphi^1 = 2\text{Re}(CB I^1\varphi^1) + L^{(0)}I^1\varphi^1, \\ M^{10}\varphi^0 &= L^{(1)}I^0\varphi^0, \quad M^{11}\varphi^1 = L^{(1)}I^1\varphi^1. \end{aligned}$$

Since the operators  $L^{(0)}$  and  $L^{(1)}$  are compact, in the operator matrix

$$M = \begin{pmatrix} M^{00} & M^{01} \\ M^{10} & M^{11} \end{pmatrix},$$

acting in the space  $C^{1,\mu}(\Gamma) \times C^\mu(\bar{D})$ , all elements except  $M^{01}$  are compact. Therefore, by the general theory of Fredholm operators [5] the operators  $N = \text{diag}(N^0, 1)$  and  $N + M$  are Fredholm equivalent and their indices

coincide. Recalling that  $\dim P_{2l-2} = l(2l-1)$  and  $\xi \in \mathbb{R}^l$ , taking into account (20) and the corresponding properties of Fredholm operators we conclude that the next theorem is proved.

*Theorem.* Suppose that condition

$$\det[C(t)B] \neq 0, \quad t \in \Gamma$$

is satisfied. Then the problem (1), (2) is Fredholm in the class  $C^{2l,\mu}(\overline{D})$ , and its index  $\alpha$  is calculated by the formula

$$\alpha = -\frac{1}{\pi}[\arg \det(CB)]|_{\Gamma} + 2l^2,$$

where the increment of a continuous branch of the argument on the contour  $\Gamma$  is taken in the counterclockwise direction.

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#### References

- 1 Koshanov, B.D., & Soldatov, A.P. (2016). *Differential Equations*, 52, 12, 1594–1609.
- 2 Soldatov, A.P. (2004). *Journal Mathematics sciences*, 17, 1–111.
- 3 Abapolova, E.A., & Soldatov, A.P. (2010). *Scientific statements BSU*, 76, 5, 6–20.
- 4 Muskhelishvili, I.N. (1968). *Singular integral equations*. Moscow: Nauka.
- 5 Palais, R.S. (1965). *Seminar on the Atiyah-Singer Index theorem*, Princeton: Princeton Univ. Press.
- 6 Kalmenov, T.Sh., Koshanov, B.D., & Nemchenko, M.Y. (2008). *Complex variables and Elliptic equations*, 53, 2, 177–183.
- 7 Kalmenov, T.Sh., Koshanov, B.D., & Nemchenko, M.Y. (2008). *Doklady Mathematics*, 78, 1, 528–530.
- 8 Koshanov, B.D., & Koshanova, M.D. (2015). AIP Conference Proceedings, 1676, 20020. *dx.doi*. Retrieved from <http://dx.doi.org/10.1063/1.4930446>.
- 9 Kanguzhin, B.E., & Koshanov, B.D. (2010). *Ufa Mathematical Journal*, 2, 41–52.
- 10 Koshanov, B.D. (2014). *AIP Conference Proceedings*, 1611, 119–127.
- 11 Koshanov, B.D., & Otelbaev, M.O. (2016). AIP Conference Proceedings, 1759, 020005, *dx.doi*.. Retrieved from <http://dx.doi.org/10.1063/1.4959619>.
- 12 Koshanov, B.D., & Smatova, G.D. (2016). *AIP Conference Proceedings*, 1759, 020050. *dx.doi*. Retrieved from <http://dx.doi.org/10.1063/1.4959664>.
- 13 Koshanov, B.D., & Smatova, G.D. (2017). *AIP Conference Proceedings*, 1880, 020050. *dx.doi*. Retrieved from <http://dx.doi.org/10.1063/1.5000653>.
- 14 Sadybekov, M.A., Torebek, B.T., & Turmetov, B.K. (2014). *AIP Conference Proceedings*, 1611, 255–260.

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### Жазықтықта жоғары дәрежелі эллиптикалық теңдеулер үшін шеттік есептердің шешімділігі туралы

Мақалада тұрақты (тек жоғары дәрежелері) нақты коэффициентті  $2l$ -дәрежелі, шекарада  $(k_j - 1)$ -дәрежелі нормал туындылары берілген шеттік есептер қарастырылған,  $j = 1, \dots, l$ ,  $1 \leq k_1 < \dots < k_l \leq 2l - 1$ . Бұл есеп  $k_j = j$  болған кезде — Дирихле есебі, ал  $k_j = j + 1$  кезде Нейман есебі болады. Авторлар осы есептің фредгольмді шешімділігінің шартын тауып, индексін есептеген.

*Кілт сөздер:* эллиптикалық теңдеулер, шеттік есептер, Дирихле есебі, Нейман есебі, шеттік есептердің шешімділігі.

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## О разрешимости краевых задач для эллиптического уравнения высокого порядка на плоскости

В статье для эллиптического уравнения  $2l$ -го порядка с постоянными (и только старшими) вещественными коэффициентами рассмотрена краевая задача, заключающаяся в задании нормальных производных  $(k_j - 1)$ -го порядка,  $j = 1, \dots, l$ , где  $1 \leq k_1 < \dots < k_l \leq 2l - 1$ . При  $k_j = j$  она переходит в задачу Дирихле, а при  $k_j = j + 1$  – в задачу Неймана. Авторами найдено условие фредгольмовой разрешимости этой задачи и вычислен индекс.

*Ключевые слова:* эллиптическое уравнение, краевые задачи, задача Дирихле, задача Неймана, разрешимость краевых задач.

Репозиторий Карту