

BOUNDARY CONDITIONS IN THE CALCULATION OF LAYERED ORTHOTROPIC PLATES BASED ON ONE MODIFIED REFINED BENDING THEORY

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The creation of new modern technology and the improvement of technological developments made it necessary to search and create new composites, in particular multilayer ones, with a wide range of operational properties that cannot be achieved using traditional materials. Moreover, the rapid development of scientific and technological progress requires the creation and implementation of new progressive materials and structures with predetermined properties. Orthotropy is one of these properties. Orthotropic materials are more difficult to analyze than isotropic materials because their properties depend on the direction.

The use of multilayer orthotropic composite materials in modern apparatuses and devices required taking into account their structural features, the physical properties of the materials used, as well as the creation of new methods for calculating the stress-strain state of such structures. The technical, physical and mechanical properties of structures made of multilayer inhomogeneous materials differ significantly in the thickness of their packages, therefore, the study of the features of the operation of structures, in particular plates, made of multilayer inhomogeneous materials in the thickness of their package using refined models is important when designing new innovative lightweight structures from multilayer materials.

We consider a rectangular layered plate with sides with orthotropic layers and a given thickness, consisting of an arbitrary number of orthotropic layers. We consider a plate in an orthogonal coordinate system $x_1, x_2, x_3 = z$. The axes x_1 and x_2 lie on the coordinate plane and their directions coincide with the orthotropy axes of the layers. The coordinate plane is positioned arbitrarily along the height of the plate cross-section.

The material layers are numbered from the bottom surface of the plate. The total number of layers in the package is denoted by n , then we take $k = 1, 2, \dots, n$, where k is the number of an arbitrary layer. All layers of the plate in aggregate in thickness form a package of layers.

In the general case, we assume that the package structure is formed by layers of different thicknesses and rigidities, the physical and mechanical characteristics of which are constant in their thickness. The number and order of the layers are arbitrary.

For an arbitrary k th layer of the plate, simplified hypotheses are adopted. These hypotheses satisfy the conditions for the joint operation of the layers without separation and displacement, the conditions on the plate surface, and determine the nonlinear law of variation of transverse shear stresses and normal stresses in the thickness of the plate [1]. The given hypotheses are obtained on the basis of the hypotheses proposed by professor A.Sh. Bozhenov [2], by neglecting a number of factors that insignificantly affect the stress-strain state of plates. It is assumed that normal displacements are equal to deflections.

Factors such as transverse shear in two directions and the pressure of the layers on each other, as well as the orthotropy of the layers are taken into account using a single shear function.

Equations for the bending multilayer orthotropic plates with an asymmetric structure in thickness are obtained from the Lagrange variational principle using the relations received on the basis of the accepted hypotheses. Then, by introducing the force functions, the system of equations and the boundary conditions are transformed into a mixed form. As a result, a system of three equations of the 12th order is obtained; this system describes the bending for a multilayer plate of an asymmetric structure in thickness with orthotropic layers. The system takes into account a transverse shear, a layer pressure, and normal strains. Three functions of the coordinate surface are

unknown; these functions are the function of force ϕ , the deflection function W , and the shear function χ [1, 3].

We note that the solution of the obtained systems of equations is possible when boundary conditions are satisfied on each contour with respect to the sought functions.

The boundary conditions for various cases of fixing the edges of the plate are obtained from the contour integral of the variational equation [2].

For edges $x_i = const$ we have

$$\begin{aligned} \phi_{,ll} \delta u_i = 0; \quad \phi_{,12} \delta u_l = 0; \quad M_{ii} \delta W_{,i} = 0; \quad (M_{ii,i} + 2M_{12,l}) \delta W = 0; \\ (Q_i'' - M'_{ii,i} - 2M'_{12,l}) \delta \chi = 0; \quad M'_{ii} \delta \chi_{,i} = 0; \quad (i = 1, 2; \quad l = 2, 1). \end{aligned} \quad (1)$$

The number of boundary conditions corresponds to the order of the system of equations.

From (1) we single out two groups of boundary conditions: the first group includes the first four, which in form correspond to the conditions of the classical theory of plate bending. They model the connections superimposed on the contour of the coordinate plane of a multilayer plate ($z = 0$) and determine the nature of its fixation, that is, they describe the contour anchoring of the coordinate plane of the plates. The remaining conditions are attributed to the second group, which models the connections that prevent mutual displacements of points on the end plane of the plate ($z \neq 0$) [2, 4]. The second group of equations models the type of deformation of the end surface of the plate and assumes the presence of various types of diaphragms at the end of the multilayer plate.

By combining the conditions of two groups of boundary conditions, any boundary conditions can be simulated. Here some common options for fixing the plate for the edges $x_i = const$ are obtained.

A movable hinged support with a rigid end diaphragm is determined by the conditions

$$\phi_{,12} = \phi_{,ll} = W = M_{ii} = \chi = \chi_{,i} = 0.$$

The relations for a movable hinged support with an end diaphragm, rigid in its plane and flexible from the plane, have the form

$$\phi_{,12} = \phi_{,ll} = W = M_{ii} = \chi = 0; \quad M'_{ii} = 0.$$

The conditions for a movable pinching with an end rigid diaphragm are written as

$$\phi_{,12} = \phi_{,ll} = W_{,i} = \chi_{,i} = \chi = 0.$$

A movable pinching with a diaphragm that is rigid in its plane and flexible from the plane is determined by the following relations

$$\phi_{,12} = \phi_{,ll} = W_{,i} = \chi = 0; \quad M'_{ii} = 0.$$

In the case of a free edge of a plate with a diaphragm that is flexible in its plane and rigid from the plane, we have the following conditions

$$\phi_{,12} = \phi_{,ll} = M_{ii} = 0; \quad M_{ii,i} + 2M_{12,l} = 0; \quad Q_i'' - M'_{ii,i} - 2M'_{12,l} = 0; \quad \chi_{,i} = 0.$$

Thus, the combination of conditions from the two groups makes it possible to write down the boundary conditions on the edges of the plate for any type of contour fastening.

In this paper, we also study some options for fixing end diaphragms when performing the Kirchhoff-Love hypotheses.

References

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