

# A GENERALIZED $(G'/G)$ - EXPANSION METHOD FOR THE LOADED MODIFIED BURGERS–KDV EQUATION

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This paper is dedicated to find the exact solutions of the equation of the loaded modified Burgers–KdV equation. It is shown to find the exact solutions via generalized  $(G'/G)$ - expansion method, that is one of the most effective way of finding exact solutions.

Consider the following loaded modified Burgers–KdV equation

$$u_t + pu^2u_x + qu_{xx} - ru_{xxx} + \gamma(t)u(0,t)u_x = 0, \quad (1)$$

where  $u(x,t)$  is an unknown function,  $x \in R$ ,  $t \geq 0$ ,  $\gamma(t)$  - is the given real continuous function.

## Description of the generalized $(G'/G)$ -expansion method

Let us be given a nonlinear partial differential equation in the form below

$$F(u, u_t, u_x, u_{tt}, u_{xx}, u_{xt}, \dots) = 0 \quad (2)$$

with two independent variables  $x$  and  $t$ .  $u = u(x,t)$  is a unknown function,  $F$  is a polynomial in  $q$  and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. Now we give the main steps of the generalized  $(G'/G)$ -expansion method [3]:

**Step 1.** We look for the  $u$  in the travelling form:

$$u(x,t) = u(\xi), \quad \xi = kx - \Omega(t), \quad (3)$$

where  $k$  is parameter and  $\Omega(t)$  is a continuous function dependent on  $t$ . We reduce equation (2) to the following nonlinear ordinary differential equation:

$$P(u, u', u'', u''', \dots) = 0, \quad (4)$$

where  $P$  is a polynomial of  $q(\xi)$  and its all derivatives  $u' = du(\xi)/d\xi$ ,  $u'' = d^2u(\xi)/d\xi^2$ .

**Step 2.** We assume that the solution of equation (4) has the form:

$$u(\xi) = \sum_{j=0}^m a_j \left( \frac{G'}{G} \right)^j, \quad (5)$$

where  $G = G(\xi)$  satisfies the following second order ordinary differential equation

$$G'' + \lambda G' + \mu G = 0, \quad (6)$$

where  $G' = dG(\xi)/d\xi$ ,  $G'' = d^2G(\xi)/d\xi^2$  and  $\lambda, \mu, a_j$  ( $j=1,2,\dots,m$ ) are constants that can be determined later, provided  $a_m \neq 0$ .

**Step 3.b** We determine the integer number  $m$  by balancing the nonlinear terms of the highest order and the partial product of the highest order of (4).

**Step 4.** Substitute (5) along with (6) into (4) and collect all terms with the same order of  $\left( \frac{G'(\xi)}{G(\xi)} \right)$ ,

the left-hand side of (4) is converted into a polynomial in  $\left( \frac{G'(\xi)}{G(\xi)} \right)$ . Then, set each coefficient of this

polynomial to zero to derive a set of over-determined partial differential equations for  $a_j$  ( $j=1,2,\dots,m$ ) and  $\xi$ .

**Step 5.** Substituting the values  $a_j$  ( $j=1,2,\dots,m$ ) and  $\xi$  as well as the solutions of equation (6) into (5) we have the exact solutions of equation (2).

## References

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## INTEGRATION OF THE LOADED SECOND-ORDER KORTEWEG-DE VRIES EQUATION WITH A FREE TERM INDEPENDENT OF THE SPATIAL VARIABLE

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Solutions in the class of periodic functions for KdV equation were studied in [1]–[3] in various formulations. In the works of [4] the KdV equation with free a term independent of the spatial variable, and in the work of [5], [6] the KdV equation with a loaded term was studied.

In this work, we study the loaded second-order KdV equation with a free term independent of the spatial variable, namely, we consider the following equation

$$q_t = \frac{1}{4}q_{xxxx} - 5q_x q_{xx} - \frac{5}{2}q q_{xxx} + \frac{15}{2}q^2 q_x + \gamma(t) \cdot q|_{x=0} \cdot q_x + f(t), \quad x \in R, t > 0 \quad (1)$$

with initial condition

$$q(x, t)|_{t=0} = q_0(x), \quad (2)$$

where  $\gamma(t) \in C[0, \infty)$  and  $f(t)$  is given real continuous function and  $q_0(x) \in C^5(R^1)$  is given real function. It is required to find a real function  $q(x, t)$ , that is  $\pi$ -periodic in a variable  $x$ :

$$q(x + \pi, t) \equiv q(x, t), \quad x \in R, t > 0 \quad (3)$$

and satisfies the smoothness conditions:

$$q(x, t) \in C_x^5(t > 0) \cap C_t^1(t > 0) \cap C(t \geq 0). \quad (4)$$

**Theorem.** Let  $q(x, t)$  be the solution of problem (1)-(4). Then the boundaries  $\lambda_n(t)$ ,  $n \geq 0$  of the spectrum of the following operator

$$L(\tau, t)y \equiv -y'' + q(x + \tau, t)y = \lambda y, \quad x \in R \quad (5)$$

satisfy the system of equations

$$\dot{\lambda}_n(t) = f(t), \quad n \geq 0, \quad (6)$$

and the spectral parameters  $\xi_n(\tau, t)$ ,  $n \geq 1$  satisfy the analogue of the system of equations of Dubrovin:

$$\frac{\partial \xi_n}{\partial t} = 2(-1)^n \sigma_n(\tau, t) \left[ -\frac{1}{2}q_{\tau\tau}(\tau, t) + \frac{3}{2}q^2(\tau, t) + 2\xi_n^2 q(\tau, t) + 4\xi_n^2 - \gamma(t)q(0, t) \right] h_n(\xi) + f(t)$$

$$h_n(\xi) = \sqrt{(\xi_n - \lambda_{2n-1})(\lambda_{2n} - \xi_n)} \times \sqrt{(\xi_n - \lambda_0) \prod_{\substack{k=1 \\ k \neq n}}^{\infty} \frac{(\lambda_{2k-1} - \xi_n)(\lambda_{2k} - \xi_n)}{(\xi_k - \xi_n)^2}}, \quad n \geq 1, \quad (7)$$

the sign  $\sigma_n(\tau, t)$  changes at each collision of the point  $\xi_n(\tau, t)$  with the boundaries of its gap  $[\lambda_{2n-1}, \lambda_{2n}]$ . Moreover, the following initial conditions are fulfilled:

$$\xi_n(\tau, t)|_{t=0} = \xi_n^0(\tau), \quad \sigma_n(\tau, t)|_{t=0} = \sigma_n^0(\tau), \quad n \geq 1, \quad (8)$$

where  $\xi_n^0(\tau)$ ,  $\sigma_n^0(\tau)$ ,  $n \geq 1$  are the spectral parameters of the Sturm-Liouville equation corresponding to the coefficients  $q_0(x + \tau)$ .

**Remark.** Using the trace formula