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RESONANT DISSIPATIVE MODEL OF THE FARADAY EFFECT IN A DIELECTRIC MULTILAYER NANO – STRUCTURES

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The theory of the rotation angle of the plane of polarization in dielectric media has been considered. In paper the dissipative model resonance Faraday effect in dielectric multilayer nanostructures is proposed. Found amplification condition of the Faraday effect in multilayer nano-structures. It is shown the possibility of resonance amplification of the Faraday rotation angle under the influence of an external magnetic field. The frequency dependence of the refractive indices for right and left polarized light was analyzed.

Keywords: Faraday effect, resonant amplification, refractive index, light field polarization

Introduction

The main problem of designing nano-dimensional structures for various applications is to increase the value of the angle of rotation of the plane of polarization and intensity of the light propagating through the multi-layer crystals. This is due to the fact that they play an important role for the development of magnetic devices of nanoelectronics. They are also important for the design of radically new magnetic memory elements.

In this paper we investigate the above mentioned problems for nano sized nonmagnetic multilayer dielectric crystals. This choice is due to the fact that only in a purely dielectric materials can be excited waveguide modes. At the same time, the metal - dielectric structures are coupled oscillations of the electrons and the localized electromagnetic field.

Given the presence of absorption in the non-magnetic insulators, we have shown that perhaps the resonance enhancement of the Faraday effect under the influence of an external magnetic field. To determine the transmission and reflection coefficients of multilayer systems, we used a method based on the summation of the Fresnel formulas.

1. Rotation of the plane of polarization in dielectric media

It is known that the permittivity of isotropic and homogeneous media, according to Maxwell's theory is given by [1]

$$\varepsilon = 1 + \alpha . \qquad (1)$$

It characterizes the polarizability of the medium and determines the resulting electric dipole moment of electrons per unit volume

$$\vec{p} = Nq\vec{r}_0 = \varepsilon_0 \alpha \vec{E}_0 . \qquad (2)$$

Where: N - the concentration of the electron, $q = -e$ - electron charge, $\vec{E}_0$ - the amplitude of the electric field of light, ε_0 - electrical constant (permittivity of free space), $\vec{r}_0$ - the amplitude of the displacement of the electron relative to its equilibrium position.

Consequently, for a finding of polarizability α dielectric permittivity ε and the amplitude of the electron oscillations $\vec{r}_0$ should be expressed in terms of the field amplitude $\vec{E}_0$. Strictly speaking, in this case one must consider itinerant electron oscillations, so-called polarization wave, along with the electromagnetic field [2]. But we shall confine ourselves to the case where the problem can be solved in the linear approximation.

In this case, the electric field of the light can be considered uniform, and assume that the quasi elastic restoring force and the drag force acting on the electron, too linear, i.e., proportional to the displacement $\vec{r}$ and velocity $\vec{\vartheta}$. Thus, a linear approximation equation of motion of the electrons can be written as follows

$$m\frac{d\vec{\vartheta}}{dt} = -a\vec{r} + b\vec{\vartheta} + q(\vec{E} + \text{[redacted]}, \vec{B}). \qquad (3)$$

Where: a - the effective coefficient of elasticity, b - effective coefficient of resistance, $\vec{E} = (E_x, E_y, 0)$ - the electric field strength of the electromagnetic wave, $\vec{B} = (0,0,B)$ - an induction of an applied magnetic field.

Further recording vector equation of motion (3) in the scalar form we obtain the following system of equations coupled oscillations

$$\left.\begin{aligned} m\frac{d\vartheta_x}{dt} &= -ax - b\vartheta_x + qE_x + qB\vartheta_y \\ m\frac{d\vartheta_y}{dt} &= -ay - b\vartheta_y + qE_y - qB\vartheta_x \end{aligned}\right\} \qquad (4)$$

The last set of equations of motion can be solved by taking advantage of its line, believing

$$\vec{E} = \vec{E}_0 e^{i\omega t}, \qquad (5)$$

and reducing it to a system of linear equations for the components $\vec{r}_0 = x_0\vec{i} + \vec{y}_0\vec{j}$. However, the use of conversion

$$\left.\begin{aligned} x &= \frac{1}{2}(r^+ + r^-), y = -\frac{i}{2}(r^+ - r^-) \\ E_x &= \frac{1}{2}(E^+ + E^-), E_y = -\frac{i}{2}(E^+ - E^-) \end{aligned}\right\}, \qquad (6)$$

It allows us to reduce the system of equations related to the vibrations of mind:

$$\left.\begin{aligned} m\ddot{r}^+ &= -ar^+ - b\dot{r}^+ + qE^+ - iqB\dot{r}^+, \\ m\ddot{r}^- &= -ar^- - b\dot{r}^- + qE^- + iqB\dot{r}^-. \end{aligned}\right\} \qquad (7)$$

i.e. to a system of independent variables r^+ and r^-.

Now, as usual, considering that

$$E^{\pm} = E_0^{\pm} e^{i\omega t}, \quad r^{\pm} = r_0^{\pm} e^{i\omega t}, \tag{8}$$

from (6) we find:

$$r_0^{\pm} = \frac{qE_0^{\pm}}{(a - m\omega^2 \mp qB\omega) + ib\omega}. \tag{9}$$

Finally, with the help of (2) of the expression (9) determine the polarizability of the dielectric medium:

$$\alpha^{\pm} = \frac{Ne^2}{\varepsilon_0 (a \pm eB\omega - m\omega^2) + ib\omega}. \tag{10}$$

Thus, due to the action of an external magnetic field with induction perpendicular to the dielectric medium appears toopolarizabiliti media, namely the difference polarizabilities right (α^{+}) and left (α^{-}) polarized light components. ($e = 1.6 \cdot 10^{-19} C$) - the value of the elementary charge, m - mass of the electron).

Concluding the section is the expression components of the electric field of an electromagnetic wave believing $E_0^{+} = E_0^{-} = E_0$ that $\vec{k} = (0,0,k)$. Then

$$\boldsymbol{E}^{\pm} = \boldsymbol{E}_0 \boldsymbol{e}^{i\omega t} \boldsymbol{e}^{-ikn^{\pm} z}, \; n^{\pm} = \sqrt{\varepsilon^{\pm}} \tag{11}$$

where $n^{\pm}$ - the refractive indices of the right and left polarized light component, to k - the wave vector of the light field.

Further, taking into account (11) and lower expression of the transformation (6) define E_x and E_y

$$\begin{Bmatrix} E_x \\ E_y \end{Bmatrix} = E_0 e^{i\omega t} e^{-\frac{i}{2}(k^{+}+k^{-})z} \cdot \begin{Bmatrix} \cos(\frac{k^{-} - k^{+}}{2} z), \\ \sin(\frac{k^{-} - k^{+}}{2} z). \end{Bmatrix} \tag{12}$$

It follows that rotates the plane of polarization of the electromagnetic wave- Faraday effect and the angle of rotation is determined by the difference between the refractive indices of the right (n^{+}) and left (n^{-}) polarized light components

$$\varphi(z) = \frac{\omega}{2c}(n^{-} - n^{+})z, \tag{13}$$

where: ω - the cyclic frequency of the light field, c - the speed of light in vacuum.

2. The increase in the Faraday rotation angle under the influence of an external magnetic field

Now we examine the angle of rotation of the plane of polarization in the dielectric layer thickness d:

$$\varphi = \frac{\omega}{2c}(n^{-} - n^{+})d\,. \tag{14}$$

The expressions for the refractive index difference and the right (n^{+}) polarized waves (n^{-}) manage to simplify the fact that for dielectric media polarizability much smaller one, i.e. $\alpha^{\pm} << 1$. Then according to the expression (1) and (11) we obtain

$$n^{-} - n^{+} \approx \frac{1}{2}(\alpha^{-} - \alpha^{+})\,. \tag{15}$$

Further, by introducing features

$$A^{\pm} = a \pm eB\omega - m\omega^{2} + ib\omega\,, \tag{16}$$

refractive index difference we give to the following form

$$n^{-} - n^{+} \approx \frac{Ne^{2}}{2\varepsilon_{0}} \cdot \frac{A_{1} - iA_{2}}{A_{1}^{2} + A_{2}^{2}} \cdot 2eB\omega\,. \tag{17}$$

Here A_1 and A_2 are the real and imaginary parts of the functions A^{+} and A^{-} are determined according to (16), (17) the following expressions

$$\left.\begin{aligned} A_{1} &= m^{2}\omega^{4} - \omega^{2}(2am + e^{2}B^{2} + b^{2}) + a^{2}, \\ A_{2} &= 2b\omega(a - m\omega^{2}). \end{aligned}\right\} \quad \begin{aligned} &(18.1) \\ &(18.2) \end{aligned}$$

As seen from (17) and (18), the difference in refractive indices $n^{-} - n^{+}$ is a complex function of the cyclic frequency of a light wave ω. Therefore, further restrict the analysis $A_2 = 0$. Then case of (18.2) it follows that

$$\omega_{1} = 0\,,\ \omega_{2} = \sqrt{\frac{a}{m}} \equiv \omega_{c}\,, \tag{19}$$

where ω_c - cyclic frequency of oscillations of electrons.

Consequently, in $\omega_1 = 0$ that under the influence of a constant electric field polarizabilities difference right (α^{+}) and left (α^{-}) polarized light waves disappear and the Faraday effect will not occur. However, the frequency of the light field is equal to the frequency of oscillations of electrons (17) that:

1) first, the difference in the imaginary parts of the complex refractive index of the right (n^{+}) and left (n^{-}) polarized light component is equal to zero;

2) in the second module of the Faraday rotation angle is determined by the difference of the real parts (n^{+}) and (n^{-}) will be expressed as follows:

$$|\varphi(d)| = \frac{Ne^2 d}{2\varepsilon_0 c} \cdot \frac{eB}{b^2 + e^2 B^2} \ . \qquad (20)$$

From the last expression it follows that the modulus of the Faraday rotation angle is at the maximum value of the induction of the external magnetic field (called critical) (see. Fig. 1):

$$B_{кр} = \frac{b}{e} \ . \qquad (21)$$

The maximum value of the modulus of the Faraday rotation angle in this case will be:

$$|\varphi_{макс}| = \frac{Ne^2 d}{4\varepsilon_0 bc} \ . \qquad (22)$$

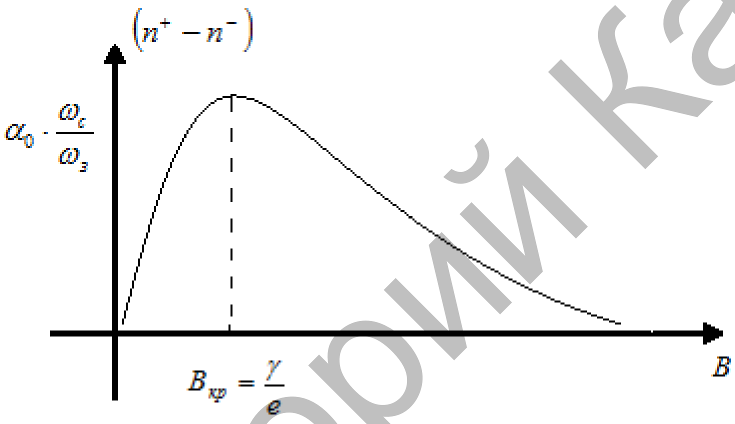


Fig.1. The dependence of the refractive index difference right - and left-polarized electromagnetic waves from the magnetic field

Further, given that the static polarizability of the dielectric medium is given by ($\alpha^{\pm}$ when $\omega \equiv 0$)

$$\alpha_0 = \frac{Ne^2}{\varepsilon_0 \alpha} \ , \qquad (23)$$

the preceding relation can be written as

$$|\varphi_{\max}(d)| \equiv \frac{\alpha_0 ad}{4bc} . \qquad (24)$$

Since usually the cyclic frequency of natural oscillations of the electrons ($\omega_c = \sqrt{\frac{a}{m}}$) substantially predominates over the cyclic frequency attenuation of light ($\omega_з = \frac{b}{m}$), the maximum difference between the right (α^+) and left (α^-) polarizations polarized components of the light field will be equal to

$$\max(\alpha^- - \alpha^+) = \alpha_0 \cdot \frac{\omega_c}{\omega_з}. \tag{25}$$

Thus, if the frequency of the light wave is equal to the frequency of the electron and the magnetic induction of the external field is equal B_{cr} (21), the maximum polarizability difference of left (α^-) and right (α^+) of polarized light components can be increased by two - three orders, as

$$\frac{\omega_з}{\omega_c} \approx 10^{-3} - 10^{-2} \text{ [3-4].}$$

Concluding the section, we present an expression of the maximum modulus of the Faraday rotation (22) to the following form

$$|\varphi_{\max}| = \frac{\alpha_0}{4c} \cdot \omega_c \cdot \frac{\omega_c}{\omega_з} d \equiv \frac{1}{4} \cdot \frac{2\pi d}{\lambda_0} \cdot \alpha_0 \frac{\omega_c}{\omega_з}, \tag{26}$$

convenient for applications. Here, λ_0 - wavelength of light in vacuum.

3. Amplification magneto-optical effects in multi-layer nano – structures

As shown in the previous section, the difference in the refractive indices, as well as the magnitude of the Faraday rotation angle can be substantially increased when $\omega = \omega_c$ action of a small external magnetic field $\sim 1.7 \cdot 10^{11} Cl/kg$. This is due to the fact that the specific charge of an electron is very large. At the same time the difference between the imaginary parts of the complex refractive index of the right (α^+) and left (α^-) polarized components of the light is zero. Otherwise, i.e. if it is $\mathrm{Im}(n^- - n^+) \neq 0$, it is well known, the amplitude of the light field is reduced during the passage through the medium in an exponential fashion. Thus, even when there is absorption in dielectric media magnetooptical effects can be enhanced by the above method. But there are also other ways of strengthening them, including on the basis of multi-layer nano-structures. In the latter case it used multiple transmission and reflection of light through the media periodically or multilayer crystals.

Next to consider the specificity of light propagation through the dielectric layer with permittivity ε_2, and bounded by the planes $z = 0$ that $z = d$ separates the dielectric constants ε_1 and ε_3 the environment. Let this layer normally falls to the surface of the $z < 0$ electromagnetic wave field. We want to determine when executed, any conditions reflectance dielectric layer is minimal. It is clear that only in this case, the transmittance is maximized, and may increase the magneto-optical effects in multilayer structures.

Note that the reflectance of the dielectric layer can be found in two ways:

1) By using the continuity conditions for the electric field and its derivative at the boundaries of adjacent layers, i.e. $z = 0$ and $z = d$ when. In this case, it is necessary to solve the system of algebraic equations, which determines the amplitude of the reflected field in the first medium, the amplitude of the field in the intermediate layer - traveling in opposite directions and the amplitude of the field in the third medium. When a similar problem is solved for multilayer structures with finite - dimensional layers (for example, four or five layer structure), this system is much more complicated, and therefore applied a numerical method for solving the so-called matrix method propagation [3-5].

2) There is a second method, which is based on the use of the Fresnel formulas for finding and reflection coefficients of transmission and reflection of the adjacent layers.

In this case, the resulting reflectance is found as the sum of the amplitudes of multiple reflections:

$$R = \alpha_{12} + \beta_{12}\alpha_{23}\beta_{21}\sum_{S=1}^{\infty} \left(\alpha_{21}\alpha_{23}\exp(2ik_2 d_2)\right)^{S-1} . \tag{27}$$

Here

$$\alpha_{ik} = \frac{n_i - n_k}{n_i + n_k}, \ \beta_{ik} = \frac{2n_i}{n_i + n_k}, \ k_2 = \frac{\omega}{c} n_2 , \tag{28}$$

describe the partial reflection and transmission coefficients of adjacent layers, k_2 - the wave vector of light in the second, ie, in the intermediate layer.

Next, using the formula for an infinitely decreasing geometric progression (since $|\alpha_{12}|, |\alpha_{23}| < 1$), and using (27) - (28), we obtain for the reflection coefficient of the three-layer structure similar to the following expression [6-7]

$$R = \frac{\alpha_{12} + \alpha_{23} e^{-2ik_2 d_2}}{1 + \alpha_{12}\alpha_{23} e^{-2ik_2 d_2}} . \tag{29}$$

Now using (29) we can find the reflectance of the multilayer structure of the reflection coefficients multiplying successively the following layers:

$$R = \prod_{l=0}^{S} \frac{\alpha_{l,l+1} + \alpha_{l+1,l+2} e^{-2ik_{l+1} d_{l+1}}}{1 + \alpha_{l+1,l+2} e^{-2ik_{l+1} d_{l+1}}} . \tag{30}$$

Where: S - number of the layer from which light is incident on the subsequent layers. Typically, the dielectric multilayers is located vacuum, or air.

Consequently, the reflection coefficient of a three-layer structure will be zero if the following two conditions:

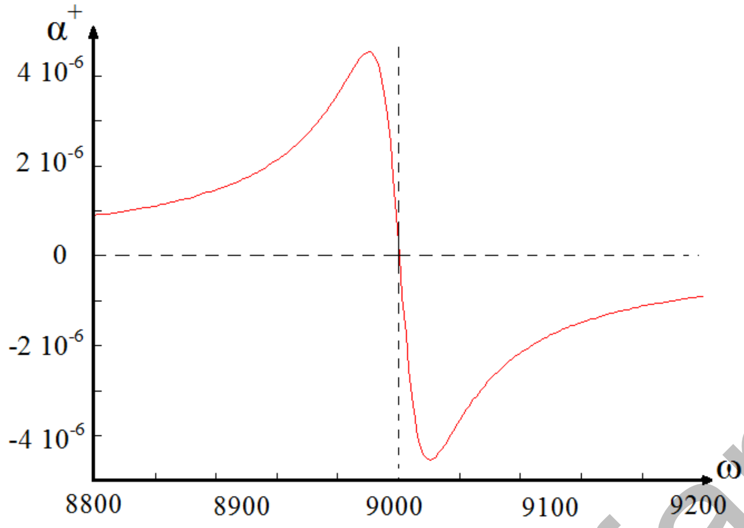
$$|\alpha_{l,l+1}| = |\alpha_{l+1,l+2}|, \tag{31.1}$$

$$\varphi_{l+1,l+2} - \varphi_{l,l+1} - 2k_{l+1} d_{l+1} = \pi \cdot (j+1) . \tag{31.2}$$

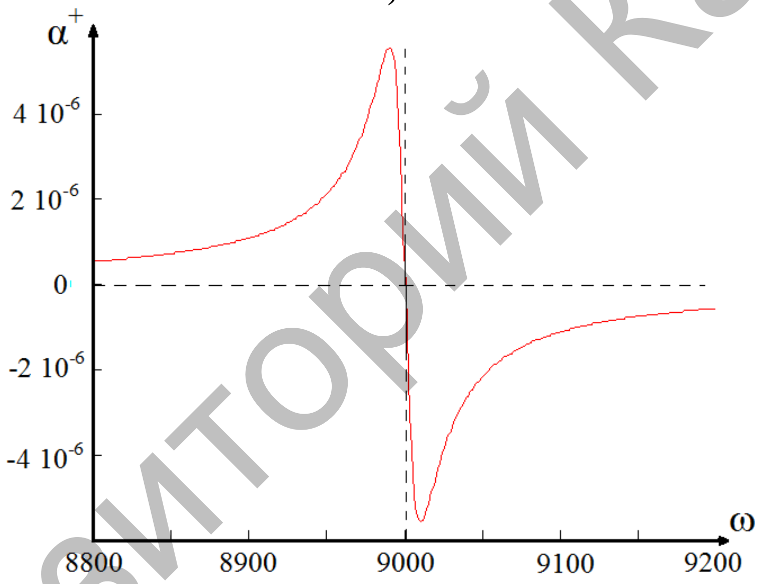
Where: j - an arbitrary integer, $\varphi_{l,l+1}$ - the argument is a complex partial reflectance. It is easy to see that the conditions (31.1 - 31.2) lead with l = 1 to the following relations:

$$\left.\begin{aligned} n_2^2 &= n_1 n_3 + \frac{n_3 \chi_1^2 - n_1 \chi_3^2}{n_1 - n_3}, \\ tg(2k_2 d) &= \frac{n_2 (n_1 - n_3)}{n_1 \chi_3 + n_3 \chi_1}. \end{aligned}\right\} \tag{32}$$

Where: n_1, n_2 and n_3 - the refractive indices of the dielectric medium, χ_1 and χ_3 - uptakes outer layers, the intermediate layer is considered transparent, i.e. $\chi_2 = 0$. Obviously, the latter made only to simplify the final formulas of the system (32).



a)



b)

Fig.2. Dispersion curves: a) right polarized (+) and b) left-polarized (-) light waves.

Thus, if the partial reflection coefficients of adjacent layers at an arbitrary *l* satisfies (31.1) and (31.2) in the light passes without being reflected multistructures, i.e. completely. Naturally, the last conclusion holds for the right and left polarized components of the light field.

Concluding the section, we give expression the refractive index of the right (+) or left (-) polarized light component in the frequency range near the resonance frequency, i.e., if

$$\varpi \mp \frac{eB}{2m} - \omega_c \to 0$$

that follows from (17) in this approximation:

$$n^{\pm} \equiv 1 + \frac{{\omega_{эл}}^2}{4 \cdot (\omega_c \pm \frac{eB}{2m})} \cdot \frac{\omega_c \pm \frac{eB}{2m} - \omega}{(\omega_c \pm \frac{eB}{2m} - \omega)^2 + (\frac{b}{2m})^2}. \qquad (33)$$

Here: $\omega_{el} = \sqrt{\frac{Ne^2}{m\varepsilon_0}}$ - electron plasma frequency.

Thus, in the frequency range near resonance can enhance the magneto-optical effects, including increasing the Faraday rotation angle, selecting a cyclic frequency ω of the light so that the refractive index difference right (n^+) and left (n^-) polarized light waves was greatest (Fig. 2).

Conclusion

In paper studied the Faraday effect in the multilayer dielectric nanostructures and obtained the following results:

1. In the purely dielectric media, magneto-optical effects may occur under the influence of external magnetic field.
2. In the light frequency equal to the frequency of oscillations of electrons can maximize the Faraday rotation angle at the critical magnetic field.
3. The conditions for amplification of these phenomena in multilayer nano-structures near the resonant frequencies of the refractive indices of the right and left polarized components of the light field.
4. These results can be used to develop new systems of magnetic nanostructures.

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