THE BESSEL EQUATION ON h-CALCULUS

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Let $h>0$ and $\mathrm{T}_a := \{a, a+h, a+2h, ...\}$, $\forall a \in R$. [1-2].

*Definition1.*Let $f:\mathrm{T}_a \to R$. Then the h-derivative of the function $f = f(t)$ has the form and is defined as

$$D_h f(t) := \frac{f(\delta_h(t)) - f(t)}{h},\ t \in \mathrm{T}_a,$$

where $\delta_h(t) := t+h$.

The h-integral (or the h-difference sum) is given by

$$\int_a^x f(t) d_h t = \sum_{k=a/h}^{x/h-1} f(kh)h, \quad x \in T_a.$$

*Definition 2.*Let $t, \alpha \in R$. Then the h-fractional function $t_h^{(\alpha)}$ is defined as

$$t_h^{(\alpha)} = h^\alpha \frac{\Gamma\left(\frac{t}{h}+1\right)}{\Gamma\left(\frac{t}{h}+1-\alpha\right)},$$

where Γ is the gamma function of Euler, $\frac{t}{h} \notin \{-1,-2,-3,...\}$ and we use the convention that division at the pole gives zero. Notice that

$$\lim_{h\to 0} t_h^{(\alpha)} = t^\alpha.$$

Definition 3.(Foundamentaltheroem h-calculus) If $F(x)$ is an h-antidereivetive of $f(x)$ is continusat $x=0$, we get

$$\int_a^b f(x) d_h x = F(b) - F(a),$$

for $a, b \in T_a$.

The Bessel differential equation. We consider the h-difference equation in the following form:

$$t_h^{(2)} D_h^2 y(t-2h) + t_h^{(1)} D_h y(t-h) + t_h^{(2)} y(t-2h) - v^2 y(t) = 0 \qquad (1)$$

which is the equation (1) is called the h-Bessel equation of the indicator in v, where v is a real number.This equation has a special point $t=0$ (the coefficient at the highest derivative in (1) vanishes at $t=0$).

*Theorem 1.*Let $v \le 0$. Then there is a particular solution to equation (1), given by a uniformly convergent series

$$J_{\nu,h}(t)=\sum_{k=0}^{\infty}\frac{(-1)^k t_h^{\nu+2k}}{k!\Gamma(\nu+k+1)2^{\nu+2k}}$$

which is the solution of the Bessel equation and is called the Bessel function of the first kind ν-th order.

The h-Bessel operator: In this article, we consider a discrete analogue of the Bessel operator, wherethe h-Bessel operator has in the following form:

$$(B_h y)(t)=t_h^{(-2\nu-1)}D_h\left[D_h y(t)\frac{1}{t_h^{(-2\nu-1)}}\right].$$

In addition, B_h is a linear operator, that is

$$B_h(\alpha y+\beta f)=\alpha B_h(y)+\beta B_h(f),\ \forall y, f\in L^2_{\nu,h}(a,b).$$

Theorem 4.(Orthogonality of eigenfunctions). Let(λ_1, y) and (λ_2, y)two pairs of eigenvaluesand eigenfunctions, and$\lambda_1\neq\lambda_2$. Then, for both regular and periodic problems, the correspondingeigenfunctions $y(t)$ and $f(t)$are orthogonal with weight$r$ (therefore$\langle y(t), f(t)\rangle=0$).

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ON THE NON-LOCAL PROBLEMS FOR A BARENBLATT - ZHELTOV - KOCHINATYPE TIME-FRACTIONAL EQUATIONS WITH HILFER DERIVATIVE

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Let H be a separable Hilbert space and $A:H\to H$ be an arbitrary unbounded positive selfadjoint operator in*H*. Suppose that A has a complete in *H*system of orthonormal eigenfunctions $\{v_k\}$ and a countable set of positive eigenvalues λ_k. It is convenient to assume that the eigenvalues do not decrease as their number increases, i.e. $0<\lambda_1\leq\lambda_2\cdots\to+\infty$.

Let$\alpha\in(0,1)$, $\beta\in[0,1]$and a function $h(t)$be defined on $[0,\infty)$.The the Riemann-Liouvillefractional integrals [1]of order γ function $h(t)$has the form

$$J^{\gamma}_{a+}h(t)=\frac{1}{\Gamma(\beta)}\int_a^t(t-\tau)^{\gamma-1}h(\tau)d\tau .$$

The Hilfer derivative [2] defined as