$$D^{\alpha,\beta} f(t) = J^{\beta(1-\alpha)} \frac{d}{dt} J^{(1-\beta)(1-\alpha)} f(t).$$

Consider the following problem

$$\begin{cases} D_t^{\alpha,\beta} u(t) + A(D_t^{\alpha,\beta} u(t)) + Au(t) = f, & 0 < t \le T; \\ J_t^{(1-\beta)(1-\alpha)} u(t)\big|_{t=\xi} = \lim\limits_{t\to+0} J_t^{(1-\beta)(1-\alpha)} u(t) + \varphi, & 0 < \xi \le T, \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where $\varphi,\ f \in H$ and ξ is fixed point. These problems are also called *the forward problems*.

In this paper, we prove the existence and uniqueness of a solution to the forwardproblem (1).

Moreover, we study the inverse problem of finding the right side of the equation. For this, we need an additional condition and as an additional condition, we will get the following condition:

$$u(\tau) = \Psi, \quad 0 < \tau \le T, \tag{2}$$

where τ – a fixed point.In this case, the function*f*does not depend on *t*.

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SOLITARY AND PERIODIC WAVE SOLUTIONS OF THE LOADED NON-LINEAR KLEIN-GORDON EQUATION

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The Klein-Gordon (KG) equation is a valuable class of the PDEs, and it first appeared in the relativistic quantum mechanics and field theory, which is highly significant for the high energy physicist [1, 2, 3], and it is applied for the modeling of different phenomena, including the propagation of dislocations in crystals and the behavior of elementary particles.

This equation is expressed in the following basic form

$$u_{tt} - u_{xx} - u_{yy} = a(u). \tag{1}$$

The KG equation most probably first arose in a mathematical context with $a(u) = e^u$ in the theory of constant surfaces in Liouville’s work. The KG equation with cubic non-linearity $a(u) = u^3 - u$ has been used in [4].

It should be noted that many methods have been developed to find special solutions of general forms of nonlinear KG equation (1) by several authors [5, 6].

In recent years, due to the intensive study of the problems of optimal management of the agroecosystem for instance, the problem of long-term forecasting and regulation of groundwater levels and soil moisture interest in loaded equations has increased significantly. Among the works devoted to loaded equations, one should especially note the works of A. Kneser [7], L. Lichtenstein [8], A. M. Nakhushev [9] and others. A full explanation of solutions of the non-linear loaded PDE and their applications can be found in the articles [10, 11, 12].

In this paper, we consider the following non-linear loaded KG equation and non-linear loaded coupled KG equation with variable coefficients

$$u_{tt} - u_{xx} - u_{yy} + b(u) = 0, \quad (2)$$

$$\begin{cases} u_{xx} + u_{yy} - u_{tt} - u + 2u^3 + 2uv + \varphi(t)u(0,0,t)u = 0 \\ v_x + v_y - v_t - 4uu_t + \varphi(t)v(0,0,t)v_x = 0, \end{cases} \quad (3)$$

where $b(u) = \alpha u + \beta u^n + \gamma(t)u(0,0,t)u$, $u(x,y,t)$ and $v(x,y,t)$ are unknown functions, and $v(x,y,t)$ is a scalar field; $n = 2, 3$, $x \in R$, $y \in R$, $t \geq 0$, α and β are any constants, $\gamma(t)$ and $\varphi(t)$ are the given real continuous functions.

We construct exact travelling wave solutions of (2) and (3), that is the exact solutions of these equations including solitary wave solutions and periodic wave solutions are obtained by the functional variable method when these equations contains variable coefficients. All solutions of the loaded KG equation and the loaded coupled KG equation have been examined and three dimensional graphics of the obtained solutions have been drawn by using the Matlab software. The main advantage of the proposed functional variable method over other methods is that it provides more new exact traveling wave solutions along with additional free parameters when the equation contains variable coefficients.

The graphical representations of the soliton solutions and the periodic wave solutions by using distinct values of random parameter are demonstrated to betterunderstand their physical features. The exact solutions have its great importance to reveal the internal mechanism of the physical phenomena. Apart from the physical relevance, the close-form solutions of nonlinear evolution equations facilitate the numerical solvers to compare the accuracy of their results and help them in the stability analysis.

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