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About mixed problem for degenerate hyperbolic-parabolic equation

The hyperbolic-parabolic equation in the upper half plane with given boundary and initial conditions is considered. By the method of generalized Fourier transformation the existence and the uniqueness of the solution of the posed mixed problem are proved.

Key words: hyperbolic and parabolic equations, mixed problem, fourier method and transformation.

Introduction

By investigating the problems of aerodynamics, hydrodynamics, the membrane theory of shells, heat and mass exchange in the capillary porous environment and in the stratum environment it is very common that one have to solve the boundary value problems for partial differential equations which belong to different type in the different regions of its domain. Many works [1–19] have been addressed to the analysis of various boundary value problems for parabolic-hyperbolic equations of the second and the third order. In this work we solve the mixed problem for degenerate hyperbolic-parabolic equation of the second order in the upper half plane by applying for each parts of the domain the generalized Fourier transformations corresponding to the posed problem.

1. The statement of the problem

In the domain $\Omega = \Omega_1 \cup \Omega_2$, where $\Omega_1 = \{(x,t): -1 < x < 0, t > 0\}$, $\Omega_2 = \{(x,t): 0 < x < 1, t > 0\}$, we consider the mixed hyperbolic-parabolic type equation

$$0 = \begin{cases} t^m u_{tt} - u_{xx} + c_1(x)u, \ -1 < x < 0, \ t > 0, \ 0 < m < 1; \\ t^\alpha u_t - u_{xx} + c_2(x)u, \ \ 0 < x < 1, \ t > 0, \ 0 < \alpha < 1, \end{cases} \tag{1}$$

where $c_1(x) \in C[-1,0]$, $c_2(x) \in C[0,1]$.

We search the solution from the class of functions $u(x,t)$, satisfying the conditions

$$\begin{gathered} \forall t > 0 \ \lim_{x \to 0-} u(x,t) = \lim_{x \to 0+} u(x,t), \ -\infty < \int_{-1}^{0} |u(x,t)|^2 dx < +\infty, \ \forall t > 0; \\ \int_0^1 |u(x,t)|^2 dx < \infty, \ \forall t > 0 \ u(x) \in C_{x,t}^{(2,2)}(\Omega_1) \bigcup C(\bar{\Omega}_2) \bigcup C_{x,t}^{(2,1)}(\Omega_2) \bigcap L_2(\Omega). \end{gathered} \tag{2}$$

We consider the mixed problem for the equation (1) in the following statement.

Problem

Find the solution of the equation (1) satisfying the conditions (2), union conditions on the line $x = 0$

$$\lim_{x \to 0-} \left[u_x'(x,t) - h_2 u(x,t) \right] = \lim_{x \to 0+} \left[u_x'(x,t) - h_3 u(x,t) \right] = 0, \ t > 0, \tag{3}$$

boundary and initial conditions

$$u_x(-1,t) + h_1 u(-1,t) = 0, \ t > 0; \tag{4}$$

$$u_x'(1,t) + h_4 u(1,t) = 0, \ t > 0; \tag{5}$$

$$u(x,0) = \tau(x), \ -1 \le x \le 1; \tag{6}$$

$$\lim_{t \to 0+} u_t(x,t) = \upsilon(x), \ -1 \le x \le 0, \tag{7}$$

where $\tau(x) \in C^1[-1,0] \bigcap C[0,1] \bigcap C^2(-1,0) \bigcap C^1(0,1)$, $\upsilon(x) \in C[-1,0] \bigcap C^1(-1,0), h_1, h_2, h_3, h_4$ are positive numbers.

To solve the problem (1)–(7) we use the method based on the Fourier method of separation of variables and the method of generalized Fourier transformations. As known the method of separation of variables implies finding particular solutions of the equation (1) in the following form

$$u(x,t)=\theta_1(x)\omega_1(t),\ \text{if } -1<x<0$$

and

$$u(x,t)=\theta_2(x)\omega_2(t),\ \text{if } 0<x<1,$$

which in the domains Ω_1 and Ω_2 satisfy the homogenous boundary conditions (2) + (3) and (3) + (4) accordingly.

Thus, the functions $\vartheta_1(x)$ and $\vartheta_2(x)$ should be the solutions of the next Sturm-Liouville problems accordingly

$$\begin{cases} -\vartheta_1'' + c_1(x)\vartheta_1 = \lambda_1\vartheta_1,\ -1\le x\le 0; \\ \vartheta_1'(-1)+h_1\vartheta_1(-1)=0,\ \lim\limits_{x\to 0-}\left(\vartheta_1'(x)-h_2\vartheta_1(x)\right)=0 \end{cases} \tag{8}$$

and

$$\begin{cases} -\vartheta_2'' + c_2(x)\vartheta_2 = \lambda_2\vartheta_2, \qquad 0\le x\le 1; \\ \lim\limits_{x\to 0+}\left(\vartheta_2'(x)-h_2\vartheta_2(x)\right)=0,\ \vartheta_2'(1)+h_2\vartheta_2(1)=0 \end{cases} \tag{9}$$

and the functions $\omega_1(t)$ and $\omega_2(t)$ are the solutions of the corresponding ordinary differential equations

$$t^m\omega_1'' + \lambda_1\omega_1 = 0,\ t>0 \tag{10}$$

and

$$t^\alpha\omega_2' + \lambda_2\omega_2 = 0,\ t>0. \tag{11}$$

The problems (8) and (9) have been studied in [15]. The general solution of the equation (10) has the following form [20, 21]:

$$\omega_1(t)=c_1I_1(t,\lambda_1,m)+c_2I_2(t,\lambda_1,m), \tag{12}$$

where c_1 and c_2 are arbitrary constants,

$$I_1(t,\lambda_1,m)=t+\sum_{k=1}^{\infty}a_k(t,\lambda_1,m),\ \ I_2(t,\lambda_1,m)=1+\sum_{k=1}^{\infty}b_k(t,\lambda_1,m);$$

$$a_k(t,\lambda_1,m)=\frac{(-1)^k\lambda_1^k t^{2-m}k+1}{k!(2-m)^k\Pi_{n=1}^k((2-m)n+1)};$$

$$b_k(t,\lambda_1,m)=\frac{(-1)^k\lambda_1^k t^{2-m}k}{k!(2-m)^k\Pi_{n=1}^k((2-m)n-1)},\ 0<m<1$$

and the general solution of the equation (11) are presented by the following formula

$$\omega_2(t)=c_3\exp\left[-\frac{\lambda_2 t^{1-\alpha}}{1-\alpha}\right], \tag{13}$$

where c_3 is an arbitrary constant.

Under the assumption of continuity of $c_1(x)$ for $-1\le x\le 0$ and $c_2(x)$ for $0\le x\le 1$, and finiteness of the intervals [-1, 0] and [0, 1] the nonzero solutions of the problem (8) and the problem (9) can only exist for discreet values of $\lambda_1=\lambda_{1n}$ and $\lambda_2=\lambda_{2n}$, which are all real numbers and have as the accumulation points the infinity for points $\lambda_1=+\infty,\lambda_2=+\infty$ [1]. Also, as known eigenvalues λ_{1n} and λ_{2n} are paired with their corresponding eigenfunctions $\vartheta_{1n}(x)=\vartheta_1(x,\lambda_{1n})$ and $\vartheta_{2n}(x)=\vartheta_2(x,\lambda_{2n})$, and all eigenfunctions $\vartheta_1(x,\lambda_{1n})$ and $\vartheta_2(x,\lambda_{2n})$ are orthogonal for $[-1,0]$ and $[0,1]$ accordingly and they form a complete system in $L_2[-1,0]$ and $L_2[0,1]$ [1]. Due to the fact that $\forall x\in[-1,0],\ c_1(x)\ge 0,\ \forall x\in[0,1],\ c_2(x)\ge 0$ there are no negative eigenvalues in our case.

Let's consider the problems (8) and (9) in the sense of the spectral functions $\rho_1(\lambda_1)$ and $\rho_2(\lambda_2)$. Therefore we need the solutions $\vartheta_1(x,\lambda_1)$ and $\vartheta_2(x,\lambda_2)$ of the corresponding Cauchy problems

$$-\vartheta_1''+c_1(x)\vartheta_1=\lambda_1\vartheta_1,\ \vartheta_1(-1,\lambda)=1,\ \vartheta_1'(-1,\lambda)=-h_1$$

and

$$-\vartheta_2''+c_2(x)\vartheta_2=\lambda_2\vartheta_2,\ \vartheta_2(0,\lambda)=1,\ \vartheta_2'(0,\lambda)=h_3.$$

The solution for the first problem has the next form [20, 21]:

$$\vartheta_1(x,\lambda_1)=h_1 I(x,\lambda_1)+J(x,\lambda_1)$$

where

$$I(x,\lambda_1)=x+1+\sum_{k=1}^{\infty}a_k(x,\lambda_1),\ J(x,\lambda_1)=1+\sum_{k=1}^{\infty}b_k(x,\lambda_1);$$

$$a_1(x,\lambda_1)=-\int_{-1}^{x}\int_{-1}^{y}(t+1)(c(t)-\lambda_1)dtdy;$$

$$a_k(x,\lambda_1)=-\int_{-1}^{x}\int_{-1}^{y}((c(t)-\lambda_1)a_{k-1}(t,\lambda_1)\mathrm{dtdy},\ (k=\overline{2,\infty});$$

$$b_1(x,\lambda_1)=-\int_{-1}^{x}\int_{-1}^{y}(c(t)-\lambda_1)dtdy;$$

$$b_k(x,\lambda_1)=-\int_{-1}^{x}\int_{-1}^{y}((c(t)-\lambda_1)b_{k-1}(t,\lambda_1)\mathrm{dtdy},\ (k=\overline{2,\infty})$$

and for the second problem the solution could be find as [20, 21]:

$$\vartheta_2(x,\lambda_2)=I(x)+h_2 J(x),$$

where

$$I(x)=x+\sum_{k=1}^{\infty}a_k(x),\ J(x)=1+\sum_{k=1}^{\infty}b_k(x);$$

$$a_1(x)=-\int_{0}^{x}\int_{0}^{y}t(c_2(t)-\lambda_2)dtdy,\ a_k(x)=-\int_{-1}^{x}\int_{-1}^{y}((c_2(t)-\lambda_2)a_{k-1}(t,\lambda_1)dtdy;$$

$$b_1(x)=-\int_{0}^{x}\int_{0}^{y}(c_2(t)-\lambda_2)dtdy,\ b_k(x)=-\int_{-1}^{x}\int_{-1}^{y}((c_2(t)-\lambda_2)b_{k-1}(t)\mathrm{dtdy}.$$

Let's introduce the generalized Fourier transformations for the functions $g_1(x)\in L_2\left[-1,0\right]$ and $g_2(x)\in L_2\left[0,1\right]$:

$$g_1(\lambda_1)=\int_{-1}^{0}g_1(x)\vartheta_1(x,\lambda_1)dx,\ {}_2(\lambda_2)=\int_{0}^{1}g_2(x)\vartheta_2(x,\lambda_2)dx.$$

For $\lambda_1=\lambda_{1n}$ and $\lambda_1=\lambda_{2n}$ according to the system of the eigenfunctions of the problems (8) and (9) we have the following Fourier coefficients for the functions $g_1(x)$ and $g_2(x)$

$$g_{1n}=\tilde{g}_1(\lambda_{1n})=\int_{-1}^{0}g_1(x)\vartheta_1(x,\lambda_{1n})dx,\ (n=\overline{1,\infty})$$

and

$$g_{2n}=\tilde{g}_2(\lambda_{2n})=\int_{0}^{1}g_2(x)\vartheta_2(x,\lambda_{2n})dx,\ (n=\overline{1,\infty}).$$

For the full orthogonal system of functions $\vartheta_1(x,\lambda_{1n})$ and $\vartheta_2(x,\lambda_{2n})$ in $L_2\left[-1,0\right]$ and $L_2\left[-1,0\right]$ the closure conditions, namely Parseval's identity, is fulfilled [1]

$$\int_{-1}^{0}g_1^2(x)dx=\sum_{n=1}^{\infty}\frac{\tilde{g}_1(\lambda_{1n})}{P\vartheta_1(x,\lambda_{1n})P^2}=\int_{-\infty}^{\infty}\tilde{g}_1^2(\lambda_1)d\rho_1(\lambda_1);$$

$$\int_{0}^{1}g_2^2(x)dx=\sum_{n=1}^{\infty}\frac{\tilde{g}_2(\lambda_{2n})}{P\vartheta_2(x,\lambda_{2n})P^2}=\int_{-\infty}^{\infty}\tilde{g}_2^2(\lambda_2)d\rho_2(\lambda_2),$$

where integrals in the right hand side are considered as Stieltjes integrals, and the spectral functions $\rho_1(\lambda_1)$ and $\rho_2(\lambda_2)$ are defined by [1]

$$\rho_1(\lambda_1)=\sum_{\lambda_{1n}<\lambda_1}\frac{1}{P\vartheta_1(x,\lambda_{1n})P^2}\varepsilon(\lambda_1-\lambda_{1n});$$

$$\rho_2(\lambda_1) = \sum_{\lambda_{2n}<\lambda_2} \frac{1}{P\vartheta_2(x,\lambda_{2n})P^2}\varepsilon(\lambda_2 - \lambda_{2n});$$

$$P\vartheta_1(x,\lambda_{1n})P^2 = \int_{-1}^{0} \vartheta_1^2(x,\lambda_{1n})dx,\ P\vartheta_2(x,\lambda_{2n})P^2 = \int_{0}^{1} \vartheta_2^2(x,\lambda_{2n})dx,$$

where $\varepsilon(\lambda_1 - \lambda_{1n})$ and $\varepsilon(\lambda_2 - \lambda_{2n})$ are Heaviside functions. The symbols $\sum_{\lambda_{1n}} < \lambda_1$ and $\sum_{\lambda_{2n}} < \lambda_2$ denote the sum over n, when $\lambda_{1n} < \lambda_1$ and $\lambda_{2n} < \lambda_2$ [1].

Thus, the spectral functions $\rho_1(\lambda_1)$ and $\rho_2(\lambda_2)$ hold the information about eigenvalues $\lambda_{1n},\lambda_{2n}$, and also about the normalizing coefficients $P\vartheta_1(x,\lambda_{1n})P$ and $P\vartheta_2(x,\lambda_{2n})P$ [16].

Notice, that for finite intervals $[-1,0]$ and $[0,1]$ there is also so called Weyls limited points case, where the only unique spectral functions $\rho_1(\lambda_1)$ and $\rho_2(\lambda_2)$ of the problems (8) and (9) exist [17].

Now, if we use the generalized Fourier transformations for the equations (1) in the domains Ω_1 and Ω_2

$$\tilde{u}_1(\lambda_1 t) = \int_{-1}^{0} u(x,t)\vartheta_1(x,\lambda_1)dx \tag{14}$$

and

$$\tilde{u}_2(\lambda_2 t) = \int_{0}^{1} u(x,t)\vartheta_2(x,\lambda_2)dx, \tag{15}$$

then, by finding the functions $\tilde{u}_1(\lambda_1,t)$ and $\tilde{u}_2(\lambda_2,t)$ from the equalities (1(1)) we obtain the equations (10) and (11), where $\omega_1(t,\lambda_1) \equiv \tilde{u}_1(\lambda_1,t), \omega_2(t,\lambda_2) \equiv \tilde{u}_2(\lambda_2,t),$ and initial conditions (6) and (7) turn these condition

$$\omega_1(0,\lambda_1) \equiv \tilde{u}_1(\lambda_1,0) = \tilde{\tau}_1(\lambda_1) = \int_{-1}^{0} \tau(x)\vartheta_1(x,\lambda_1)dx; \tag{16}$$

$$\lim_{t\to 0-0} \omega_1'(t,\lambda_1) \equiv \lim_{t\to 0+0} \tilde{u}_1(\lambda_1,t) = \upsilon(\lambda_1) = \int_{-1}^{0} \nu(x)\vartheta_1(x,\lambda_1)dx; \tag{17}$$

$$\omega_2(0,\lambda_2) \equiv \tilde{u}_2(\lambda_2,0) = \tilde{\tau}_2(\lambda_2) = \int_{0}^{1} \tau(x)\vartheta_2(x,\lambda_2)dx. \tag{18}$$

By using the initial conditions (16) and (17) we find c_1 and c_2, and for the functions $\omega_1(t,\lambda_1) = \tilde{u}_1(t,\lambda_{1n})$ we have

$$\omega_1(t,\lambda_{1n}) \equiv \tilde{u}_1(t,\lambda_{1n}) = \tilde{\nu}(\lambda_{1n})I_1(t,\lambda_{1n},m) + \tilde{\tau}(\lambda_{1n})I_2(t,\lambda_{1n},m). \tag{19}$$

By solving the problem (11) + (18), we find c_3 and obtain

$$\omega_2(t,\lambda_{2n}) \equiv \tilde{u}_2(t,\lambda_{2n}) = \tilde{\tau}_2(\lambda_{2n})\exp\left[-\frac{\lambda_{2n}t^{1-\alpha}}{1-\alpha}\right]. \tag{20}$$

Next, we apply the reverse generalized Fourier transformation to (14) and (15) and have the representation of the solution of the problem (1)–(7) by means of it's generalized Fourier transformation according to the system of eigenfunctions of the problems (8) and (9)

$$u(x,t) = \begin{cases} \int_{-\infty}^{\infty} \tilde{u}_1(\lambda_1,t)\vartheta_1(x,\lambda_1)d\rho_1(\lambda_1), & -1 \le x \le 0,\ t > 0; \\ \int_{-\infty}^{\infty} \tilde{u}_2(\lambda_2,t)\vartheta_2(x,\lambda_2)d\rho_2(\lambda_2), & 0 \le x \le 1,\ t > 0, \end{cases}$$

where $\tilde{u}_1(\lambda_1,y)$ and $\tilde{u}_2(\lambda_2,y)$ are defined by (19) and (20).

The latter is possible, because the conditions (2) fulfilled for function $u(x,t)$ make the generalized Fourier transformation applicable [22]. Uniqueness of the solution of the problem (1)–(7) follows from the uniqueness of the spectral functions $\rho_1(\lambda_1)$ and $\rho_2(\lambda_2)$. From the conditions (2) it follows

$$\int_{-\infty}^{\infty} \tilde{\nu}(\lambda_1)I_1(t,\lambda_1,m)\left[h_1 I(0,\lambda_1) + J(0,\lambda_1)\right]d\rho_1(\lambda_1) = \int_{-\infty}^{\infty} \tilde{\tau}_2(\lambda_2)\exp\left[-\frac{\lambda_2 t^{1-\alpha}}{1-\alpha}\right]d\rho_2(\lambda_2). \tag{21}$$

Thereby we proved the following theorem.

Theorem 1. If $c_1(x) \ge 0,$ for $x \in [-1,0],$ and $c_2(x) \ge 0$ for $x \in [0,1],$ and the equality (21) fulfills then the problem is unique solvable.

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Түрі өзгеретін гиперболалық-параболалық теңдеу үшін аралас есеп туралы

Мақалада жоғары жартыжолақта гиперболалық-параболлалық теңдеу үшін шекаралық және бастапқы шарттары бар есеп қарастырылды. Фурье жалпыланған түрлендіруін қолдана отырып, қойылған аралас есептің шешімі бар және жалғыз екендігі дәлелденген.

Т.Ж.Елдесбай, А.Б.Тунгатаров, М.У.Турсынбекова

О смешанной задаче для вырождающегося гиперболо-параболического уравнения

В статье в верхней полуполосе рассмотрено гиперболо-параболическое уравнение с заданными граничными и начальными условиями. С помощью метода обобщенного преобразования Фурье доказаны существование и единственность поставленной смешанной задачи.

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On the second boundary value problem for the equation of heat conduction in an unbounded plane angle

In the article, the second homogeneous boundary value problem is considered in an infinite angular domain. Solution of the problem is reduced to solving the singular Volterra integral equations of the second kind with kernel whose norm is equal to unity. By the method of Carleman-Vekua, solving the integral equation is reduced to solving the inhomogeneous equation of Abel. The theorem on the existence of a non-trivial solution of the second homogeneous boundary value problem in a non-cylindrical domain is proved. The solution of the given problem is obtained in an explicit form.

Key words: singular Volterra integral equation, Abel equation, non-cylindrical domain, non-trivial solution.

The need to study boundary value problems of heat conduction (diffusion) in the domain with moving boundaries is dictated by numerous practical applications in modeling the processes of electrocontact apparatuses in a related field of designing the plasma torches, the creation of new technologies, production of crystals, laser technology and other industries. Mathematical modeling these processes allows to carry out the optimal choice of parameters and operating modes of technological equipment and maximize economic