$$J_{\nu,h}(t)=\sum_{k=0}^{\infty}\frac{(-1)^k t_h^{\nu+2k}}{k!\Gamma(\nu+k+1)2^{\nu+2k}}$$

which is the solution of the Bessel equation and is called the Bessel function of the first kind ν-th order.

The h-Bessel operator: In this article, we consider a discrete analogue of the Bessel operator, wherethe h-Bessel operator has in the following form:

$$(B_h y)(t)=t_h^{(-2\nu-1)}D_h\left[D_h y(t)\frac{1}{t_h^{(-2\nu-1)}}\right].$$

In addition, B_h is a linear operator, that is

$$B_h(\alpha y+\beta f)=\alpha B_h(y)+\beta B_h(f),\ \forall y,f\in L_{\nu,h}^2(a,b).$$

Theorem 4.(Orthogonality of eigenfunctions). Let (λ_1,y) and (λ_2,y) two pairs of eigenvaluesand eigenfunctions, and $\lambda_1\neq\lambda_2$. Then, for both regular and periodic problems, the correspondingeigenfunctions $y(t)$ and $f(t)$ are orthogonal with weight r (therefore $\langle y(t),f(t)\rangle=0$).

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References

1 Cheung P., Kac V. Quantum calculus // Edwards Brothers. Inc. Ann Arbor. MI. USA.- 2000. – P. 112.
2 Girejko E., Mozyrska, D. Overview of fractional *h*-difference operators // Advances in harmonic analysis and operator theory, Oper. Theory Adv. Appl., Birkhauser/Springer. Basel AG. Basel.- 2013. –Vol. 229. – P. 253–268.
3 Ferreira R.A.C., Torres D.F.M. Fractional *h*-difference equations arising from the calculus of variations // Appl. Anal. Discrete Math., - 2011.–Vol. 1 (5).–P. 110–121

ON THE NON-LOCAL PROBLEMS FOR A BARENBLATT - ZHELTOV - KOCHINATYPE TIME-FRACTIONAL EQUATIONS WITH HILFER DERIVATIVE

Ashurov R.R.[1], Fayziev Yu.E.[2], Tokhtaeva N.[3], Khushvaktov N.Kh.[4]
[1]*Institute of Mathematics, the Academy of Sciences of the Uzbekistan, Tashkent, Uzbekistan*
[2,3,4]*National University of Uzbekistan, Tashkent, Uzbekistan*
E-mail:[1]ashurovr@gmail.com,[2]fayziev.yusuf@mail.ru,
[3]nozimatoxtayeva3715@gmail.com,[4]nuriddinh@gmail.com

Let H be a separable Hilbert space and $A:H\to H$ be an arbitrary unbounded positive selfadjoint operator in$H$. Suppose that A has a complete in Hsystem of orthonormal eigenfunctions $\{v_k\}$ and a countable set of positive eigenvalues λ_k. It is convenient to assume that the eigenvalues do not decrease as their number increases, i.e. $0<\lambda_1\le\lambda_2\cdots\to+\infty$.

Let $\alpha\in(0,1)$, $\beta\in[0,1]$and a function $h(t)$ be defined on $[0,\infty)$.The the Riemann-Liouvillefractional integrals [1]of order γ function $h(t)$ has the form

$$J_{a+}^{g}h(t)=\frac{1}{G(b)}\int_a^t(t-t)^{g-1}h(t)dt\ .$$

The Hilfer derivative [2] defined as

$$D^{\alpha,\beta} f(t) = J^{\beta(1-\alpha)} \frac{d}{dt} J^{(1-\beta)(1-\alpha)} f(t).$$

Consider the following problem

$$\begin{cases} D_t^{\alpha,\beta} u(t) + A(D_t^{\alpha,\beta} u(t)) + Au(t) = f, & 0 < t \le T; \\ J_t^{(1-\beta)(1-\alpha)} u(t)\Big|_{t=\xi} = \lim_{t\to+0} J_t^{(1-\beta)(1-\alpha)} u(t) + \varphi, & 0 < \xi \le T, \end{cases} \quad \text{.....................(1)}$$

where $\varphi,\ f \in H$ and ξ is fixed point. These problems are also called *the forward problems*.

In this paper, we prove the existence and uniqueness of a solution to the forwardproblem (1).

Moreover, we study the inverse problem of finding the right side of the equation. For this, we need an additional condition and as an additional condition, we will get the following condition:

$$u(\tau) = \Psi, \quad 0 < \tau \le T, \tag{2}$$

where τ – a fixed point.In this case, the function*f*does not depend on *t*.

References

1. A.A. Kilbas, H.M. Srivastava, J.J. Trujillo, Theory and Applications of Fractional Differential Equations, Elsevier (2006).

2.R. Hilfer, Applications of Fractional Calculus in Physics. Singapore: World Scientific (2000).

3.R. Hilfer, Yu. Luchko, Z. Tomovski, Operational method for the solution of fractional differential equations with generalized Riemann-Liouville fractional derivatives, Fractional Calculus and Applied analysis, V. 12, No 3 (2009)

4. Ashurov R., Fayziev Yu. On the nonlocal problems in time for time-fractional subdiffusion equations. Fractal and Fractional. 2022. V. 6. No 41 https://doi.org/10.3390/fractalfract6010041.

SOLITARY AND PERIODIC WAVE SOLUTIONS OF THE LOADED NON-LINEAR KLEIN-GORDON EQUATION

Babajanov B.A. [1], Abdikarimov F.B. [2]

[1]*Urgench State University, Urgench, Uzbekistan*

[2]*Khorezm Mamun Academy, Khiva, Uzbekistan*

E-mail: a.murod@mail.ru; goodluck_0714@mail.ru

The Klein-Gordon (KG) equation is a valuable class of the PDEs, and it first appeared in the relativistic quantum mechanics and field theory, which is highly significant for the high energy physicist [1, 2, 3], and it is applied for the modeling of different phenomena, including the propagation of dislocations in crystals and the behavior of elementary particles.

This equation is expressed in the following basic form

$$u_{tt} - u_{xx} - u_{yy} = a(u). \tag{1}$$

The KG equation most probably first arose in a mathematical context with $a(u) = e^u$ in the theory of constant surfaces in Liouville's work. The KG equation with cubic non-linearity $a(u) = u^3 - u$ has been used in [4].

It should be noted that many methods have been developed to find special solutions of general forms of nonlinear KG equation (1) by several authors [5, 6].

In recent years, due to the intensive study of the problems of optimal management of the agroecosystem for instance, the problem of long-term forecasting and regulation of groundwater levels and soil moisture interest in loaded equations has increased significantly. Among the works devoted to loaded equations, one should especially note the works of A. Kneser [7], L. Lichtenstein [8], A. M. Nakhushev [9] and others. A full explanation of solutions of the non-linear loaded PDE and their applications can be found in the articles [10, 11, 12].