ATOMIC AND ALGEBRAICALLY PRIME SETS OF FRAGMENTS

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To begin with, we introduce several key definitions related to the results presented in this abstract. We work in a countable first-order language with a fixed signature. Let T be a fixed Jonsson L-theory (1). By definition, a Jonsson theory T is inductive and thus $E_T \neq \emptyset$, where E_T denotes the class of existentially closed models of T. Let K be a subclass of E_T. We introduce the Jonsson spectrum of K, denoted $JSp(K)$. By the definition of the Jonsson spectrum, $JSp(K) = \{T \mid T \text{ is Jonsson theory}, K \subseteq Mod(T)\}$ (2).

On the class $JSp(K)$ we define the cosemanticness equivalence relation (denoted by $\bowtie$), which partitions $JSp(K)$ into elementary, pairwise disjoint classes of Jonsson theories such that theories within the same class have identical semantic models. The common semantic model for a class $[\Delta]$ will be denoted by $C_{[\Delta]}$. The resulting factor-set will be denoted by $JSp(K)_{/(\bowtie)}$. We fix some cosemanticness class $[\Delta] \in JSp(K)$.

Definition 1. (3) A cosemanticness class $[\Delta] \in JSp(K)_{/(\bowtie)}$ is said to be existentially complete if $[\Delta]$ consists only of theories that are complete with respect to existential sentences.

Similarly, we define an existentially-universally complete class $[\Delta] \in JSp(K)_{/(\bowtie)}$.

Definition 2. (4) A formula $\varphi(x_1, x_2, \ldots, x_n)$ is said to be complete for the set of formulas Γ (relative to T) if φ is consistent with T and for every formula $\psi(x_1, x_2, \ldots, x_n) \in \Gamma$ (with at most the same free variables as φ), we have either $T \models \forall x(\varphi \rightarrow \psi)$ or $T \models \forall x(\varphi \rightarrow \neg\psi)$.

Definition 3. (4) A structure A is called a Γ_1-atomic model of theory T if A is a model of T and for every n-tuple from A, there exists a formula from Γ_1 (complete for the set of Γ_2-formulas) that it satisfies.

Definition 4. (4) 1) The notation $(A, a_0, \ldots, a_{n-1}) \Rightarrow_\Gamma (B, b_0, \ldots, b_{n-1})$ means that for every formula $\varphi(x_1, \ldots, x_n) \in \Gamma$, if $A \models \varphi(a)$, then $B \models \varphi(b)$.

2) The notation $(A, a) \equiv_\Gamma (B, b)$ means that $(A, a) \Rightarrow_\Gamma (B, b)$ and $(B, b) \Rightarrow_\Gamma (A, a)$.

Definition 5. (4) 1) A structure A is called a Σ-nice algebraically prime model of a theory T if A is a countable model of T and for every model B of T, every $n \in \omega$, and every tuples $a_0, \ldots, a_{n-1} \in A$ and $b_0, \ldots, b_{n-1} \in B$, $(A, a_0, \ldots, a_{n-1}) \Rightarrow_\exists (B, b_0, \ldots, b_{n-1})$ implies that for every $a_n \in A$ there exists some $b_n \in B$ such that $(A, a_0, \ldots, a_n) \Rightarrow_\exists (B, b_0, \ldots, b_n)$.

2) A structure A is called a Σ^*-nice algebraically prime model of a theory T if A is a countable model of T, and for every model B of T, every $a_0, \ldots, a_{n-1} \in A$ and $b_0, \ldots, b_{n-1} \in B$ such that $(A, a_0, \ldots, a_{n-1}) \Rightarrow_\exists (B, b_0, \ldots, b_{n-1})$, then for every $a_n \in A$ there exists some $b_n \in B$ such that $(A, a_0, \ldots, a_n) \Rightarrow_\exists (B, b_0, \ldots, b_n)$.

Definition 6. (5) A set A is called (∇_1, ∇_2)-cl-atomic in a theory T if the following conditions are satisfied:

1) For every $a \in A$, there exists a formula $\varphi(x) \in \nabla_1$ such that for any formula $\psi(x) \in \nabla_2$, the formula $\varphi(x)$ is complete for $\psi(x)$, and $C \models \varphi(a)$ for some model C of T;

2) $cl(A) = M$, where $M \in E(T)$.

Theorem 1. Let $[\Delta]$ be complete for existential sentences (consists of $\exists$-complete theories), X be a countable cl-atomic Jonsson set. Then $(1) \Rightarrow (2)$ and $(2) \Leftrightarrow (3)$:

(1) X is (Σ, Σ)-atomic set;

(2) X is Σ^*-nice-algebraically prime set;

(3) X is Σ-nice-algebraically prime set.

Theorem 2. Let $[\Delta] \in JSp(K)$, a class $[\Delta]$ be universal and $\exists$-complete, $X \subseteq C_{[\Delta]}$ be a countable algebraically prime cl-atomic Jonsson set. Then there exists a Jonsson set $Y \subseteq C_{Fr(X)}$ such that $cl(Y) = M$, $M \in K$ and M is (Σ, ∇)-atomic model for $Fr(X)$.

All necessary definitions can be found from (1; 3; 5)

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ALGEBRAIC STRUCTURE OF THE JONSSON SPECTRUM

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We consider Jonsson theories of a countable first-order language L of the signature σ. By theory, we mean a consistent set of L-sentences. We write E_σ for the class of L-structures that are existentially closed in the class of all L-structures.

Let T_1 and T_2 be Jonsson L-theories, and let $M \in E_\sigma$ be a model of $T_1 \cup T_2$. Then $T_1 \cup T_2$ is a Jonsson L-theory.

Let K be a class of L-structures. A Jonsson spectrum (1) of K is the following set of theories:

$$JSp(K) = \{T \mid T \text{ is a Jonsson theory and } K \subseteq Mod(T)\}.$$

Let K_σ be a class of L-structures containing a structure $M \in E_\sigma$. Then $(JSp(K_\sigma), \cup)$ is a commutative monoid.

Two Jonsson theories are called cosemantic if their semantic models (2) coincide. Cosemanticness is an equivalence relation. Let K be any class of L-structures, and let $JSp(K)_{/\bowtie}$ be a factor-set of