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ON THE WAVE TREATMENT OF A REST MASS

This work is devoted to research of methodological aspects of the quantum theory. Namely, the question about the main principles of a correlation between the common dynamical quantities of elementary particles with the corresponding wave parameters is investigated in the paper. It can be shown that the particle rest mass can be regarded as some structural wave parameter. In the present work the possibility of generalization this approach on systems in external field is investigated. Also the implementation of uncertainty principle to the model is verified.

Keywords: quantum theory, wave-particle duality, treatment of a rest mass, uncertainty principle, cylindrical wave functions

Introduction

Generally accepted the Standard Model is a huge achievement for theoretical physics. Almost monthly, its potential possibilities are supported with new experimental data that coincide with the theoretical predictions. However, with all its formal consistency there are essential problems with interpretations. For example, the interpretation of interaction process as change by bosons is looking very mechanically. Besides it the theory no suppose about the nature of elementary particles. Really, the Standard Model operates wave functions, Green functions, states et cetera and all this objects are associated with behavior of particles, but not with its interior structure. This situation, jointly with some contradictions with experimental data, stimulates research activity in the area of quantum foundations. It is possible that the root of the problems is in Copenhagen interpretation of wave function.

The article of Donald C. Chang “What is rest mass in the wave-particle duality? A proposed model” [1] shows that the approach to wave function as to the wave of some physical field for a free particle leads to possibility to treat the rest mass as a new quantum number. His model is based on three conceptual postulates:

Like the photon, a particle (such as an electron) is not a point-like object, instead, it behaves like a wave packet.

Like the photon, the matter wave of a particle is an excitation of a real physical field.

Different types of particles are different excitation modes of a unified field in the vacuum.

As it is shown in the article the model of D. Cheng can be generalized on including of an exterior electric field and the cylindrical solutions do not broke the uncertainty principle.

The theory for a free particle

Wave equation for electromagnetic field has the view

$$\Delta A_{\mu} = 0.$$

The solving of this equation can be performed as $A_{\mu} = (\varphi\psi, \vec{A}\psi)$, where φ and \vec{A} forms some four-vector regarding to Lorentz's transformations. So we get to the equation

$$\Delta\psi = 0.$$

In a cylindrical reference of frame we have

$$\frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} - \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \psi}{\partial r} \right) - \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \varphi^2} - \frac{\partial^2 \psi}{\partial z^2} = 0 \quad (1)$$

Let's perform the separation of variables in the next manner

$$\psi(r, \varphi, z, t) = \psi_L(z, t)\psi_T(r, \varphi),$$

where we denote with letter L – longitudinal component of ψ -function and with letter T – transversal one. So, besides (1) we will have the next system of two independent equations

$$\frac{1}{c^2} \frac{\partial^2 \psi_L}{\partial t^2} - \frac{\partial^2 \psi_L}{\partial z^2} = -l^2 \psi_L, \quad (2)$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \psi_T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \psi_T}{\partial \varphi^2} = -l^2 \psi_T,$$

where l^2 is a new parameter of variables separation.

It is obvious that the first equation can be interpret as the Klein-Gordon equation for free particles moving along z-axis, if we accept

$$l^2 = m^2 c^2 / \hbar^2.$$

The solution of this equation is a usual plane wave

$$\psi_L \propto \exp\left(\frac{i}{\hbar}(p_z z - Et)\right).$$

The second equation of the system (2) after separation of variables $\psi_T(r, \varphi) = \psi_r(r)\psi_\varphi(\varphi)$ leads us to the next expression for radial part

$$\frac{\partial^2 \psi_r}{\partial r^2} + \frac{1}{r} \frac{\partial \psi_r}{\partial r} - \left(l^2 + \frac{k^2}{r^2} \right) \psi_r = 0, \quad (3)$$

where $k = 0, \pm 1, \pm 2, \dots$. The angle part has the clear solution of the view

$$\psi_\varphi \propto e^{ik\varphi}.$$

It is easy to see that the solution of (3) has to have the view of cylindrical functions

$$\psi_r = C_1 J_n(lr) + C_2 N_n(lr),$$

where C are arbitrary constants and J_n , N_n are Bessel's functions. Therefore the complete solution of equation (1) has the form

$$\psi(r, \varphi, z) = (C_1 J_n(lr) + C_2 N_n(lr)) \exp(ik\varphi) \exp\left(\frac{i}{\hbar}(p_z z - Et)\right) \quad (4)$$

The detailed analysis of this solution contains in [1].

Turning on an exterior electromagnetic field

For including into consideration of electromagnetic field due to standard procedures it is sufficient to change usual derivatives along coordinates with the generalized ones

$$D_\mu = \partial_\mu + ieA_\mu, \quad (5)$$

where A_μ is a four-potential. So, the usual operator of D'Alembert has to be changed with the construction $g^{\mu\nu} D_\mu D_\nu$, which is named as a generalized D'Alembert operator here. In line with the previous scheme for a free particle we are choosing the equation for the wave function in the same form:

$$g^{\mu\nu} D_\mu D_\nu \psi = 0.$$

After the substitution (5) we have

$$g^{\mu\nu} \partial_\mu \partial_\nu \psi + ieg^{\mu\nu} (\partial_\mu A_\nu) \psi + ieg^{\mu\nu} A_\nu \partial_\mu \psi + g^{\mu\nu} ieA_\mu \partial_\nu \psi - g^{\mu\nu} e^2 A_\mu A_\nu \psi = 0. \quad (6)$$

For simplicity let's take into consideration the uniform electrostatic field, directed along z -axis:

$$A_\mu = (\varphi(z), 0, 0, 0).$$

Into a cylindrical frame of reference $\psi = \psi(r, \alpha, z, t)$ and besides (6) we have

$$\frac{1}{c^2} \partial_t^2 \psi - \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \psi}{\partial r} \right) - \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \alpha^2} - \frac{\partial^2 \psi}{\partial z^2} + 2i \frac{e}{c} \varphi \partial_t \psi - e^2 \varphi^2 \psi = 0$$

After separation of the variables $\psi = \Psi(z, t) \phi(r, \alpha)$ we will have the next set of independent equations:

$$\frac{1}{c} \frac{\partial^2 \Psi}{\partial t^2} - \frac{\partial^2 \Psi}{\partial z^2} + 2i \frac{e}{c} \varphi \frac{\partial \Psi}{\partial t} - e^2 \varphi^2 \Psi = -l^2 \Psi \quad (7)$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \phi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \alpha^2} = -l^2 \phi$$

The last equation doesn't contain any information about the potential, i.e. the angle part of the new wave remains the same. Taking account that this part in the Chang's model contains information about inner properties of the particle it's very important fact for the model.

It is not difficult to see that the first equation of the system (7) has the same view as for a plane wave spreading along z -axis, which is a solution of Klein-Gordon equation,

$$D_\mu D^\mu \psi = -\frac{m^2 c^2}{\hbar} \psi,$$

where

$$\frac{m^2 c^2}{\hbar^2} = l^2.$$

The result testifies relevance the model for existing of an exterior electrostatics field.

Uncertainty principle verification

Now we intend to check relevance of uncertainty principle for the cylindrical wave function of the view (4). For simplicity let's restrict ourselves by the partial case of $n=0$ and $C_2=0$, then

$$\psi_0 = C_1 J_0(r l) \exp \left[\frac{i}{\hbar} (p_z z - E t) \right]. \quad (8)$$

The norm condition $\int_{-\infty}^{\infty} \psi^* \psi dV = 1$ is the condition on the coefficient C_1 :

$$|C_1|^2 \int J_0^2(r l) r dr d\varphi dz = 1.$$

It's easy to see that this expression tends to infinity due to the dependence character (8) from the variable z . But the integral along the radial variable is divergent too. Really, due to the property of cylindrical functions [2]

$$\int x [z_p(\alpha x)]^2 dx = \frac{x^2}{2} \{ [z_p(\alpha x)]^2 - z_{p-1}(\alpha x) z_{p+1}(\alpha x) \},$$

we have

$$\int r [J_0(lr)]^2 dr = \frac{r^2}{2} \{ [J_0(lr)]^2 - J_{-1}(lr) J_1(lr) \} + const$$

where

$$J_{-n} = (-1)^n J_n \quad n = 1, 2, 3, \dots$$

Therefore, for the defined integral we will have the next expression:

$$\int_0^{\infty} r [J_0(lr)]^2 dr = \lim_{r \rightarrow \infty} \frac{r^2}{2} \{J_0^2(lr) + J_1^2(lr)\} - \lim_{r \rightarrow 0} \frac{r^2}{2} \{J_0^2(lr) + J_1^2(lr)\}.$$

In asymptotic approximations, as it is known, that can be written

$$J_0(x) \Big|_{x \rightarrow 0} \approx 1 - \frac{x^2}{n}, \quad J_n(x) \Big|_{x \rightarrow 0} \approx \frac{x^n}{2^n n!} \Rightarrow J_1(x) \Big|_{x \rightarrow 0} \approx \frac{x}{2}.$$

It means that

$$\lim_{r \rightarrow 0} \frac{r^2}{2} \{J_0^2(lr) + J_1^2(lr)\} = \lim_{r \rightarrow 0} \frac{r^2}{2} \left\{ \left(1 - \frac{(lr)^2}{n} \right) + \left(\frac{lr}{2} \right)^2 \right\} = 0.$$

And at the another limit $|x| \gg 1$ we have

$$J_p(x) \approx \sqrt{\frac{2}{\pi x}} \cos \left(x - \frac{p\pi}{2} - \frac{\pi}{n} \right).$$

So

$$J_0(x) \approx \sqrt{\frac{2}{\pi x}} \cos \left(x - \frac{\pi}{n} \right) \text{ and } J_1(x) \approx \sqrt{\frac{2}{\pi x}} \cos \left(x - \frac{3\pi}{n} \right) = \sqrt{\frac{2}{\pi x}} \sin \left(x - \frac{\pi}{n} \right).$$

Then

$$\lim_{r \rightarrow \infty} \frac{r^2}{2} \{J_0^2(lr) + J_1^2(lr)\} = \lim_{r \rightarrow \infty} \frac{r^2}{2} \left\{ \frac{2}{\pi dr} \cos^2 \left(lr - \frac{\pi}{n} \right) + \frac{2}{\pi lr} \sin^2 \left(lr - \frac{\pi}{n} \right) \right\} = \lim_{r \rightarrow \infty} \frac{r}{\pi l}.$$

Therefore,

$$\int_{-\infty}^{\infty} J_0^2(rl) r dr d\varphi = \lim_{r \rightarrow \infty} \frac{2r}{l} \quad (9)$$

i.e. the norm-factor goes to infinity.

Let's verify implementation Heisenberg's principle along the y -axis:

$$\langle \Delta p_y^2 \rangle \langle \Delta y^2 \rangle \geq \frac{\hbar^2}{4}.$$

So, for the quadratic mean deviation of the coordinate we have

$$\langle \Delta y^2 \rangle = \int \psi^* (y - \langle y \rangle)^2 \psi dV,$$

Due to the axial symmetry of system $\langle y \rangle = 0$ and with regard to $y = r \sin \varphi$ we have

$$\langle \Delta y^2 \rangle = \int \psi^* \psi r^2 \sin^2 \varphi dV.$$

After the substitution

$$\psi_0 = C_1 J_0(rl) e^{\frac{i}{\hbar}(\rho_z z - Et)}$$

we get to

$$\begin{aligned} \langle \Delta y^2 \rangle &= \int |C_1|^2 J_0^2(rl) r^2 \sin^2 \varphi r dr d\varphi dz = \int_{-\infty}^{\infty} |C_1|^2 dz \int_0^{\infty} r^3 J_0^2(rl) dr \int_0^{2\pi} \sin^2 \varphi d\varphi = \\ &= \frac{1}{2} \int_{-\infty}^{\infty} |C_1|^2 dz \int_0^{\infty} r^3 J_0^2(rl) dr. \end{aligned}$$

This expression, obviously, tends to infinity more rapidly than the integral (9).

Now, let's consider the deviation of momentum:

$$\langle \Delta p_y^2 \rangle = \int \psi^* \left(p_y - \langle p_y \rangle \right)^2 \psi dV.$$

Also, the middle value of the momentum equal to zero due to the cylindrical symmetry, $\langle p_y \rangle = 0$.
So we have

$$\langle \Delta p_y^2 \rangle = \int \psi^* p_y^2 \psi dV = -\hbar^2 \int \psi^* \frac{\partial^2 \psi}{\partial y^2} dV.$$

Let's consider separately the second derivative:

$$\begin{aligned} \frac{\partial^2 J_0(rl)}{\partial y^2} &= \frac{\partial}{\partial y} \left(\frac{dJ_0(rl)}{d(rl)} \frac{\partial(rl)}{\partial y} \right) = \frac{\partial}{\partial y} \left(\frac{\partial J_0(rl)}{\partial(rl)} l \frac{\partial \sqrt{x^2 + y^2}}{\partial y} \right) = \\ &= \frac{\partial}{\partial y} \left(\frac{\partial J_0(rl)}{\partial(rl)} \frac{1}{2} \frac{2y}{\sqrt{x^2 + y^2}} \right) = \frac{\partial}{\partial y} \left(\frac{\partial J_0(lr)}{\partial r} \right) \frac{y}{r} + \frac{\partial J_0(lr)}{\partial r} \frac{\partial}{\partial y} \left(\frac{y}{r} \right) = \\ &= \frac{\partial^2 J_0(lr)}{\partial r^2} \frac{1}{2} \frac{2y}{\sqrt{x^2 + y^2}} \frac{y}{r} + \frac{\partial J_0(lr)}{\partial r} \frac{\partial}{\partial y} \frac{\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2}} \frac{r - y^2}{r^2} = \\ &= \frac{\partial^2 J_0(lr)}{\partial r^2} \frac{y^2}{r^2} + \frac{\partial J_0(lr)}{\partial r} \frac{r^2 - y^2}{r^3} = \frac{\partial^2 J_0(lr)}{\partial r^2} \frac{y^2}{r^2} + \frac{\partial J_0(lr)}{\partial r} \frac{x^2}{r^3}. \end{aligned}$$

Due to the links between Cartesian and cylindrical frames ($x = r \cos \varphi$, $y = r \sin \varphi$) we have

$$\frac{\partial^2 J_0(lr)}{\partial y^2} = \frac{\partial^2 J_0(lr)}{\partial r^2} \cos^2 \varphi + \frac{\partial J_0(lr)}{\partial r} \frac{\sin^2 \varphi}{r^2}.$$

So as

$$\frac{dJ_0(z)}{dz} = J_{-1}(z) = -J_1(z),$$

we have

$$\frac{\partial J_0(z)}{\partial r} = -lJ_1(lr).$$

And due to the known expressions

$$\frac{d}{dz} z^n J_n(z) = z^n J_{n-1}(z),$$

it follows to the form

$$\frac{\partial J_1(lr)}{\partial r} = l \frac{\partial}{\partial lr} \left(\frac{J_1(lr)}{lr} lr \right) = l \frac{\partial}{\partial lr} \frac{(J_1(lr)lr) - J_1(lr)lr}{(lr)^2} = lJ_0(lr) - \frac{1}{r} J_1(lr)$$

and

$$\frac{\partial^2 J_0(rl)}{\partial r^2} = -l \frac{\partial}{\partial r} J_1(lr) = \frac{l}{r} J_1(lr) - l^2 J_0(lr).$$

Therefore for second derivative along y - axis we will have

$$\frac{\partial^2 J_0(rl)}{\partial y^2} = \left\{ \frac{l}{r} J_1(lr) - l^2 J_0(lr) \right\} \cos^2 \varphi - \frac{l}{r} J_1(lr) \sin^2 \varphi = \frac{l}{r} J_1(lr) \cos 2\varphi - l^2 J_0(lr) \cos^2 \varphi.$$

And finally, for middle square deviation along the momentum we have the expression:

$$\langle \Delta P_y^2 \rangle = -\hbar^2 \int |C_1|^2 J_0(lr) \frac{\partial^2 J_0(lr)}{\partial y^2} r dr d\varphi dz =$$

$$= -\hbar^2 \int_{-\infty}^{\infty} |C_1|^2 dz \int \left\{ J_0(lr) \frac{l}{r} J_1(lr) \cos 2\varphi - l^2 J_0(lr) \cos^2 \varphi \right\} d\varphi.$$

After integration along φ we will have

$$\langle \Delta P_y^2 \rangle = \hbar^2 l^2 \frac{1}{2} \int |C_1|^2 dz J_0^2(lr) r dr.$$

Taking account the above result for the norm procedure

$$\int_{-\infty}^{\infty} \psi_0^* \psi_0 dV = |C_1|^2 \int J_0^2(lr) r dr d\varphi dz = 2\pi \int_{-\infty}^{\infty} |C_1|^2 dz \int_0^{\infty} J_0^2(lr) r dr = 1,$$

we can write finally

$$\langle \Delta p_y^2 \rangle = \frac{\hbar^2 l^2}{4\pi}.$$

This non-zero result means that the Heisenberg' principle is not broken in the investigated model. Really, since the middle deviation along the coordinate tends to infinity the product of

$\langle \Delta p_y^2 \rangle \langle \Delta y^2 \rangle \geq \frac{\hbar^2}{4}$ also tend to infinity.

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