DOI 10.31489/2022M1/13-24
UDC 519.63

A. Ashyralyev[1,2,3,*], M. Urun[4,5], I.C. Parmaksizoglu[6]

[1]*Bahcesehir University, Istanbul, Turkey;*
[2]*Peoples' Friendship University of Russia (RUDN University), Moscow, Russian Federation;*
[3]*Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan;*
[4]*Near East University, Nicosia, Turkey;*
[5]*Galatasaray University, Istanbul, Turkey;*
[6]*Istanbul Technical University, Turkey*
(E-mail: allaberen.ashyralyev@eng.bau.edu.tr, murun@gsu.edu.tr, parmaksizo@itu.edu.tr)

Mathematical modeling of the energy consumption problem

The importance of energy-saving and correct design is obvious for energy efficiency. Correct design means that before construction considerable things, such as orientation or isolation decisions, need to be made. This study gives a mathematical model of the nonstationary energy consumption calculation problems. The model is well-posedness in Holder spaces of the mixed one-dimensional parabolic problem with Robin boundary conditions. In this study, an effective numerical method is also developed for energy consumption calculation which is related to this mathematical model. The three case problems are taken to test this numerical method. The dynamic model results have been compared with the previous finite-difference or steady-state solutions. The study also aims to develop a mathematical model in which the result can be found at any time.

Keywords: mathematical modeling, heat diffusion equation, difference scheme, stability.

Introduction

An important part of energy consumption occurs in buildings. Energy Efficient Building Design (EEBD) is a design that reduces energy usage and pollution controlling the criteria. Architectural building design rules are functionality, stability, and aesthetics. Today, efficiency and healthiness also are added. An efficient design means not only doing things during operating but also doing correct design before the construction. There are numerous studies on EEBD all over the world (see [1–7]). A national software, that calculates the energy consumption of buildings according to the Turkish Standards Institute (TS EN 13790), exists in Turkey. Note that the problem is complicated because the energy consumption calculation depends on many variables, such as nonstationary external temperature and solar radiation, building materials, heat losses and gains and energy consumption change with time. Energy consumption numerical calculations take a lot of time because of the stability criterion. It is not easy to check hour by hour for the whole year. For these reasons, the mathematical model and theoretical solution are valuable. In this article, the mathematical model of a building's outer wall consisting of an opaque wall is obtained by taking as a boundary value problem for the annual energy consumption calculation. The heat conduction differential equation and the boundary equations of the one-dimensional nonstationary boundary value problem are given. This study also gives a one-dimensional nonstationary general solution for some energy consumption calculation problems. Finally, the dynamic model results were compared with the numerical results.

Theoretical background

In this section, we consider the theoretical background of the mathematical model of energy-saving problems. The well-posedness of differential and difference heat problems with third boundary conditions in Hölder spaces is established. Numerical results are provided.

*Corresponding author.
E-mail: allaberen.ashyralyev@eng.bau.edu.tr

Stability and coercive stability of differential problem

We study the initial-boundary value problem

$$\begin{cases} \frac{\partial u(t,x)}{\partial t} - \frac{\partial}{\partial x}\left(a\left(x\right)\frac{\partial u(t,x)}{\partial x}\right) + \delta u(t,x) = f\left(t,x\right),\ t\in\left(0,T\right),\ x\in\left(0,l\right), \\ u\left(0,x\right) = \varphi\left(x\right),\ x\in\left[0,l\right], \\ u\left(t,0\right)-\psi(t) = bu_{x}\left(t,0\right), -u\left(t,l\right)-\omega(t) = cu_{x}\left(t,l\right),\ t\in[0,T], \end{cases} \tag{1}$$

for the one-dimensional heat equation with Robin boundary conditions. Here $0 < a \leq a\left(x\right)$ and b, c, δ are positive constants. Under compatibility conditions problem, (1) has a unique solution $u(t,x)$ for smooth functions $a\left(x\right)$, $x\in(0,l)$, $\varphi(x)$, $x\in[0,l]$, $\psi(t)$, $\omega(t)$, $t\in[0,T]$, $f(t,x)$, $(t,x)\in(0,1)\times(0,l)$.

Assume that H be a Hilbert space and A be the self-adjoint positive-definite operator defined by the formula

$$Az = -\frac{d}{dx}\left(a(x)\frac{dz(x)}{dx}\right)+\delta z(x) \tag{2}$$

with domain

$$D(A)=\{z : z, z^{''}\in L_2(0,l),\ z(0)=bz^{'}(0),\ -z(l)=cz^{'}(l)\}.$$

Here and in the rest of this paper, $C_0^{\alpha}\left(\left[0,T\right],H\right)$ $(0<\alpha<1)$ stands for Banach spaces of all abstract continuous functions $\varphi(t)$ defined on $[0,T]$ with values in H satisfying a Hölder condition with weight t^{α} for which the following norm is finite

$$\|\varphi\|_{C_0^{\alpha}([0,T],H)} = \|\varphi\|_{C([0,T],H)} + \sup_{0\leq t<t+\tau\leq T}\frac{(t+\tau)^{\alpha}\|\varphi(t+\tau)-\varphi(t)\|_H}{\tau^{\alpha}}.$$

Here, $C\left(\left[0,T\right],H\right)$ stands for the Banach space of all abstract continuous functions $\varphi(t)$ defined on $[0,T]$ with values in H equipped with the norm

$$\|\varphi\|_{C([0,T],H)} = \max_{0\leq t\leq T}\|\varphi(t)\|_H .$$

Let the Sobolev space $W_2^2(0,l)$ be defined as the set of all functions $v(x)$ defined on $(0,l)$ such that both $v(x)$ and $v''(x)$ are locally integrable in $L_2(0,l)$, equipped with the norm

$$\|v\|_{W_2^2(0,l)} = \left(\int_0^{\ell}|v(x)|^2\,dx\right)^{1/2} + \left(\int_0^{\ell}|v''(x)|^2\,dx\right)^{1/2}.$$

Theorem 1. Assume that $f(t,x)$ and $\psi(t)$, $\omega(t)$ are continuous functions and satisfying a Hölder condition with weight t^{α}. Then the problem (1) has a unique solution $u\in C_0^{\alpha}\left(L_2(0,l)\right)$ and for the solution of problem (1) the following stability estimates

$$\|u\|_{C_0^{\alpha}([0,T],L_2(0,l))} \leq M\left(q,\delta\right)\left[\|\varphi\|_{L_2(0,l)}+\|f\|_{C_0^{\alpha}([0,T],L_2(0,l))}+\|\psi\|_{C_0^{\alpha}[0,T]}+\|\omega\|_{C_0^{\alpha}[0,T]}\right]$$

and coercive stability estimates

$$\|u_t\|_{C_0^{\alpha}([0,T],L_2(0,l))} + \|u\|_{C_0^{\alpha}\left([0,T],W_2^2(0,l)\right)} \leq M\left(q,\delta\right)\Big[\|\varphi\|_{W_2^2(0,l)} + \tfrac{1}{\alpha(1-\alpha)}\|f\|_{C_0^{\alpha}([0,T],L_2(0,l))}+\|\psi\|_{C_0^{\alpha}[0,T]}+\|\omega\|_{C_0^{\alpha}[0,T]}\Big]$$

are satisfied.

Proof. Denote by

$$u\left(t,x\right) = w\left(t,x\right)+\left(1-\frac{x^2}{l^2+2lc}\right)\psi(t)-\frac{x^2}{l^2+2lc}\omega\left(t\right), \tag{3}$$

where $w\left(t,x\right)$ is the solution of the following initial-boundary value problem:

$$\begin{cases} w_t(t,x) - (a(x) w_x(t,x))_x + \delta w(t,x) \\ = f(t,x) + \psi_t(t)\left(1 - \frac{x^2}{l^2+2lc}\right) - \frac{x^2}{l^2+2lc}\omega_t(t) \\ +\delta\left(\psi(t)\left(1 - \frac{x^2}{l^2+2lc}\right) - \frac{x^2}{l^2+2lc}\omega(t)\right) + \frac{2}{l^2+2lc}\psi(t)(a(x)x)_x \\ +\frac{2}{l^2+2lc}\omega(t)(a(x)x)_x, \ t \in (0,T), \ x \in (0,l), \\ w(0,x) = \varphi(x) + \psi(0)\left(1 - \frac{x^2}{l^2+2lc}\right) - \frac{x^2}{l^2+2lc}\omega(0), \ x \in [-l,l], \\ w(t,0) = bw_x(t,0), \ -w(t,l) - cw_x(t,l) = 0, \ t \in [0,T]. \end{cases} \tag{4}$$

Applying (3), we get

$$\|u\|_{C_0^\alpha([0,T],L_2(0,l))} \le \|w\|_{C_0^\alpha([0,T],L_2(0,l))} + K_1(l)\left[\|\psi\|_{C_0^\alpha[0,T]} + \|\omega\|_{C_0^\alpha[0,T]}\right],$$

$$\|u\|_{C_0^\alpha\left([0,T],W_2^2(0,l)\right)} \le \|w\|_{C_0^\alpha([0,T],L_2(0,l)])} + K_2(l)\left[\|\psi\|_{C_0^\alpha[0,T]} + \|\omega\|_{C_0^\alpha[0,T]}\right].$$

Therefore, the following theorem will be complete the proof of Theorem 1.

Theorem 2. Under assumptions of Theorem 1, the problem (4) has a unique solution in $C([0,T], L_2(0,l))$ and the following stability estimate:

$$\|w\|_{C_0^\alpha([0,T],L_2(0,l))} \le M(q,\delta)\left[\|\varphi\|_{L_2[0,l]} + \|f\|_{C_0^\alpha([0,T],L_2[0,l])} + \|\psi\|_{C_0^\alpha[0,T]} + \|\omega\|_{C_0^\alpha[0,T]}\right]$$

and coercive stability estimate

$$\|w_t\|_{C_0^\alpha([0,T],L_2(0,l))} + \|w\|_{C_0^\alpha\left([0,T],W_2^2(0,l)\right)} \le M(q,\delta)\left[\|\varphi\|_{W_2^2(0,l)} + \frac{1}{\alpha(1-\alpha)}\|f\|_{C_0^\alpha([0,T],L_2(0,l))} \right.$$
$$\left. + \|\psi\|_{C_0^\alpha[0,T]} + \|\omega\|_{C_0^\alpha[0,T]}\right]$$

are satisfied.

Proof. Problem (4) can be written in the following abstract form

$$\begin{cases} w'(t) + Aw(t) = f(t) + \psi_t(t)q_1 + \psi_t(t)q_2 + \psi(t)q_3 + \omega(t)q_4, \ 0 < t < T, \\ w(0) = \varphi + \psi(0)q_1 + \omega(0)q_2 \end{cases} \tag{5}$$

in a Hilbert space $H = L_2(0,l)$ with the space operator $A = A^x$ defined by the formula (2). Here, $f(t) = f(t,x)$ is the given abstract function, $w(t) = w(t,x)$ is unknown function and

$$q_1 = q_1(x) = 1 - \frac{x^2}{l^2+2lc}, q_2 = q_2(x) = -\frac{x^2}{l^2+2lc}, q_3 = q_3(x) = \delta\left(1 - \frac{x^2}{l^2+2lc}\right) + \frac{2}{l^2+2lc}(a(x)x)_x,$$
$$q_4 = q_4(x) = -\delta\frac{x^2}{l^2+2lc} + \frac{2}{l^2+2lc}(a(x)x)_x$$

are known elements of $L_2(0,l)$. The proof of Theorem 2 is based on theorems on stability and coercive stability of the abstract problem (5) (see, [1,2]), the self-adjointness and positive definiteness of the space operator A^x defined by formula (2).

Stability and coercive stability of difference problem

Let $\alpha \in (0,1)$ is a given number and $C_\tau^\alpha(H) = C_0^\alpha([0,T]_\tau, H]), C_\tau(H) = C([0,T]_\tau, H)$ be Banach spaces of all H-valued mesh functions $w_\tau = \{w_k\}_{k=0}^N$ defined on

$$[0,T]_\tau = \{t_k = k\tau, 0 \le k \le N, N\tau = T\}$$

with the corresponding norms

$$\|w_\tau\|_{C_\tau(H)} = \max_{0 \le k \le N} \|w_k\|_H,$$

$$\|w_\tau\|_{C_\tau^\alpha(H)} = \sup_{1 \le k < k+n \le N} (N-n)^{-\alpha}(k)^\alpha \|w_{k+n} - w_k\|_H + \|w_\tau\|_{C_\tau(H)}.$$

Moreover, let $L_{2h}=L_2\left[0,l\right]_h$ and $W_{2h}^2=W_2^2(0,l)_h$ be normed spaces of all mesh functions $\gamma^h\left(x\right)=\left\{\gamma_n\right\}_{n=0}^M$ defined on

$$\left[0,l\right]_h=\left\{x_n=nh,0\leq n\leq M,Mh=l\right\}$$

equipped with norms

$$\left\|\gamma^h\right\|_{L_{2h}}=\left(\sum_{x\in[0,l]_h}\left|\gamma^h(x)\right|^2h\right)^{1/2}$$

and

$$\left\|\gamma^h\right\|_{W_{2h}^2}=\left\|\gamma^h\right\|_{L_{2h}}+\left(\sum_{x\in(0,l)_h}\left|\left(\gamma^h\right)_{x\overline{x},j}\right|^2h\right)^{1/2},$$

respectively. Furthermore, we introduce the difference operator A_h^x defined by the formula

$$A_h^xu^h(x)=\left\{-\frac{1}{h}\left(a_{n+1}\frac{u_{n+1}-u_n}{h}-a_n\frac{u_n-u_{n-1}}{h}\right)+\delta u_n\right\}_1^{M-1}, \quad (6)$$

acting in the space of mesh functions $u^h\left(x\right)=\left\{u_n\right\}_{n=0}^M$ defined on $\left[0,l\right]_h$ satisfying the conditions $\left(h+b\right)u_0-$ $-bu_1=0,-cu_{M-1}+\left(h+c\right)u_M=0$. For the numerical solution $\left\{u_k^h\left(x\right)\right\}_{k=0}^N$ of problem (1), we present DS of the first order of approximation

$$\begin{cases} \frac{u_n^k-u_n^{k-1}}{\tau}-\frac{1}{h}\left(a_{n+1}\frac{u_{n+1}^k-u_n^k}{h}-a_n\frac{u_n^k-u_{n-1}^k}{h}\right)+\delta u_n^k \\ =f_n^k,\ f_n^k=f\left(t_k,x_n\right),\ t_k\in k\tau,\ x_n=nh,\ k\in\overline{1,N},\ n\in\overline{1,M-1}, \\ u_n^0=\varphi_n,\ \varphi_n=\varphi\left(x_n\right),\ n\in\overline{0,M}, \\ \left(h+b\right)u_0^k-bu_1^k=h\psi_k,\ cu_{M-1}^k-\left(h+c\right)u_M^k=h\omega_k, \\ \psi_k=\psi(t_k),\ \omega_k=\omega(t_k),\ k\in\overline{0,N} \end{cases} \quad (7)$$

and of the second order of approximation

$$\begin{cases} \frac{u_n^k-u_n^{k-1}}{\tau}-\frac{1}{2h}\left(a_{n+1}\frac{u_{n+1}^k-u_n^k}{h}-a_n\frac{u_n^k-u_{n-1}^k}{h}\right)-\frac{1}{2h}\left(a_{n+1}\frac{u_{n+1}^{k-1}-u_n^{k-1}}{h}-a_n\frac{u_n^{k-1}-u_{n-1}^{k-1}}{h}\right) \\ +\delta\frac{u_n^k+u_n^{k-1}}{2}=f_n^k,\ f_n^k=f\left(t_k-\frac{\tau}{2},x_n\right),\ t_k\in k\tau,\ x_n=nh,\ k\in\overline{1,N},\ n\in\overline{1,M-1}, \\ u_n^0=\varphi_n,\ \varphi_n=\varphi\left(x_n\right),\ n\in\overline{0,M}, \\ \frac{u_0^k+u_0^{k-1}}{2}-\psi_k=b\left(\frac{u_1^k-u_0^k}{2h}+\frac{u_1^{k-1}-u_0^{k-1}}{2h}\right), \\ -\frac{u_M^k+u_M^{k-1}}{2}-\omega_k=c\left(\frac{u_M^k-u_{M-1}^k}{2h}+\frac{u_M^{k-1}-u_{M-1}^{k-1}}{2h}\right) \\ \psi_k=\psi(t_k),\ \omega_k=\omega(t_k),\ k\in\overline{0,N}. \end{cases} \quad (8)$$

Let us give the following results on the stability and coercive stability of DSs (7) and (8).

Theorem 3. For the solution of DSs (7) and (8) the stability estimates

$$\left\|\left\{u_k^h\right\}_{k=1}^N\right\|_{C_\tau^\alpha(L_{2h})}\leq M\left(q,\delta\right)\left[\left\|\varphi^h\right\|_{L_{2h}}\right.$$
$$\left.+\left\|\left\{f_k^h\right\}_{k=1}^N\right\|_{C_\tau^\alpha(L_{2h})}+\left\|\left\{\psi_k\right\}_1^N\right\|_{C_0^\alpha[0,T]_\tau}+\left\|\left\{\omega_k\right\}_1^N\right\|_{C_0^\alpha[0,T]_\tau}\right]$$

and coercive stability estimates

$$\left\|\left\{\frac{1}{\tau}\left(u_k^h-u_{k-1}^h\right)\right\}_{k=1}^N\right\|_{C_\tau^\alpha(L_{2h})}+\left\|\left\{\widetilde{u_k^h}\right\}_{k=1}^N\right\|_{C_\tau^\alpha\left(W_{2h}^2\right)}\leq M\left(q,\delta\right)\left[\left\|\varphi^h\right\|_{W_{2h}^2}\right.$$
$$\left.+\frac{1}{\alpha\left(1-\alpha\right)}\left\|\left\{f_k^h\right\}_{k=1}^N\right\|_{C_\tau^\alpha(L_{2h})}+\left\|\left\{\psi_k\right\}_1^N\right\|_{C_0^\alpha[0,T]_\tau}+\left\|\left\{\omega_k\right\}_1^N\right\|_{C_0^\alpha[0,T]_\tau}\right]$$

hold. Here,

$$\widetilde{u_k^h} = \begin{cases} u_k^h \quad , & \text{for first order DS,} \\ \frac{u_k^h + u_{k-1}^h}{2}, & \text{for Crank-Nicolson DS.} \end{cases}$$

Proof. We will use

$$u_n^k = w_n^k + \left(1 - \frac{(nh)^2 - nh^2}{l^2 + 2lc + (c-l)\,h}\right)\psi_k - \frac{(nh)^2 - nh^2}{l^2 + 2lc + (c-l)\,h}\omega_k, \tag{9}$$

where $\left\{w_k^h(x)\right\}_{k=0}^N$ is the solution of the following DSs

$$\begin{cases} \frac{w_n^k - w_n^{k-1}}{\tau} - \frac{1}{h}\left(a_{n+1}\frac{w_{n+1}^k - w_n^k}{h} - a_n\frac{w_n^k - w_{n-1}^k}{h}\right) + \delta w_n^k \\ = f_n^k - \left(1 - \frac{(nh)^2 - nh^2}{l^2+2lc+(c-l)h}\right)\frac{\psi_k - \psi_{k-1}}{\tau} - \frac{(nh)^2 - nh^2}{l^2+2lc+(c-l)h}\frac{\omega_k - \omega_{k-1}}{\tau} \\ +\delta\left[\left(1 - \frac{(nh)^2 - nh^2}{l^2+2lc+(c-l)h}\right)\psi_k - \frac{(nh)^2 - nh^2}{l^2+2lc+(c-l)h}\omega_k\right] \\ +\frac{2}{l^2+2lc+(c-l)h}\left(na_{n+1} - (n-1)\,a_n\right)\left[\psi_k + \omega_k\right],\ k \in \overline{1,N},\ n \in \overline{1,M-1}, \\ w_n^0 = \varphi_n + \left(1 - \frac{(nh)^2 - nh^2}{l^2+2lc+(c-l)h}\right)\psi_0 - \frac{(nh)^2 - nh^2}{l^2+2lc+(c-l)h}\omega_0,\ n \in \overline{0,M}, \\ (h+b)\,w_0^k - bw_1^k = 0, cw_{M-1}^k - (h+c)\,w_M^k = 0,\ 9k \in \overline{0,N} \end{cases} \tag{10}$$

and

$$\begin{cases} \frac{w_n^k - w_n^{k-1}}{\tau} - \frac{1}{2h}\left(a_{n+1}\frac{w_{n+1}^k - w_n^k}{h} - a_n\frac{w_n^k - w_{n-1}^k}{h}\right) - \frac{1}{2h}\left(a_{n+1}\frac{w_{n+1}^{k-1} - w_n^{k-1}}{h} - a_n\frac{w_n^{k-1} - w_{n-1}^{k-1}}{h}\right) \\ +\delta\frac{w_n^k + w_n^{k-1}}{2} = f_n^k - \left(1 - \frac{(nh)^2 - nh^2}{l^2+2lc+(c-l)h}\right)\frac{\psi_k - \psi_{k-1}}{\tau} - \frac{(nh)^2 - nh^2}{l^2+2lc+(c-l)h}\frac{\omega_k - \omega_{k-1}}{\tau} \\ +\delta\left[\left(1 - \frac{(nh)^2 - nh^2}{l^2+2lc+(c-l)h}\right)\frac{\psi_k + \psi_{k-1}}{2}_k - \frac{(nh)^2 - nh^2}{l^2+2lc+(c-l)h}\frac{\omega_k + \omega_{k-1}}{2}\right] \\ +\frac{2}{l^2+2lc+(c-l)h}\left(na_{n+1} - (n-1)\,a_n\right)\left[\psi_k + \omega_k\right],\ k \in \overline{1,N},\ n \in \overline{1,M-1}, \\ w_n^0 = \varphi_n + \left(1 - \frac{(nh)^2 - nh^2}{l^2+2lc+(c-l)h}\right)\psi_0 - \frac{(nh)^2 - nh^2}{l^2+2lc+(c-l)h}\omega_0,\ n \in \overline{0,M}, \\ (h+b)\,w_0^k - bw_1^k = 0, cw_{M-1}^k - (h+c)\,w_M^k = 0,\ k \in \overline{0,N} \end{cases} \tag{11}$$

for (7) and (8), respectively. Applying (9), we obtain

$$\left\|\left\{u_k^h\right\}_{k=1}^N\right\|_{C_\tau^\alpha(L_{2h})} \leq \left\|\left\{w_k^h\right\}_{k=1}^N\right\|_{C_\tau^\alpha(L_{2h})} + K\left[\left\|\left\{\psi_k\right\}_{k=1}^N\right\|_{C_0^\alpha[0,T]_\tau} + \left\|\left\{\omega_k\right\}_{k=1}^N\right\|_{C_0^\alpha[0,T]_\tau}\right]$$

and

$$\left\|\left\{\widetilde{u_k^h}\right\}_{k=1}^N\right\|_{C_\tau^\alpha\left(W_{2h}^2\right)} \leq \left\|\left\{w_k^h\right\}_{k=1}^N\right\|_{C_\tau^\alpha\left(W_{2h}^2\right)} + K\left[\left\|\left\{\psi_k\right\}_{k=1}^N\right\|_{C_0^\alpha[0,T]_\tau} + \left\|\left\{\omega_k\right\}_{k=1}^N\right\|_{C_0^\alpha[0,T]_\tau}\right].$$

Therefore, the following theorem will be complete the proof of Theorem 3.

Theorem 4. For the solution of DSs (10) and (11) the stability estimates

$$\left\|\left\{w_k^h\right\}_{k=1}^N\right\|_{C_\tau^\alpha(L_{2h})} \leq K_3(a)\left[\left\|\varphi^h\right\|_{L_{2h}}\right.$$

$$\left. + \left\|\left\{f_k^h\right\}_{k=1}^N\right\|_{C_\tau^\alpha} + \left\|\left\{\psi_k\right\}_{k=1}^N\right\|_{C_0^\alpha[0,T]_\tau} + \left\|\left\{\omega_k\right\}_{k=1}^N\right\|_{C_0^\alpha[0,T]_\tau}\right]$$

and coercive stability estimates

$$\left\|\left\{\frac{1}{\tau}\left(w_k^h - w_{k-1}^h\right)\right\}_{k=1}^N\right\|_{C_\tau^\alpha(L_{2h})} \leq K_3(q)\left[\left\|\varphi^h\right\|_{W_{2h}^2}\right.$$

$$\left. + \frac{1}{\alpha(1-\alpha)}\left\|\left\{f_k^h\right\}_{k=1}^N\right\|_{C_\tau^\alpha} + \left\|\left\{\psi_k\right\}_{k=1}^N\right\|_{C_0^\alpha[0,T]_\tau} + \left\|\left\{\omega_k\right\}_{k=1}^N\right\|_{C_0^\alpha[0,T]_\tau}\right]$$

hold.

Proof. Problems (10) and (11) can be written in the following abstract forms

$$\begin{cases} \frac{w_k^h - w_{k-1}^h}{\tau} + A^h w_k^h = f_k^h + q_1^h \frac{\psi_k - \psi_{k-1}}{\tau} + q_2^h \frac{\omega_k - \omega_{k-1}}{\tau} + q_3^h \psi_k + q_4^h \omega_k, \ 1 \le k \le N, \\ w_0^h = \varphi^h + q_1^h \psi_0 + q_2^h \omega_0 \end{cases} \quad (12)$$

and Crank-Nicolson

$$\begin{cases} \frac{w_k^h - w_{k-1}^h}{\tau} + A^h \frac{w_k^h + w_{k-1}^h}{2} = f_k^h + q_1^h \frac{\psi_k - \psi_{k-1}}{\tau} + q_2^h \frac{\omega_k - \omega_{k-1}}{\tau} + q_3^h \frac{\psi_k + \psi_{k-1}}{2} + q_4^h \frac{\omega_k + \omega_{k-1}}{2}, \ 1 \le k \le N, \\ w_0^h = \varphi^h + q_1^h \psi_0 + q_2^h \omega_0, \end{cases}$$

in a Hilbert space $H = L_{2h}$ with the space operator $A^h = A_h^x$ defined by the formula (6). Here, $f_k^h = f_k^h(x)$ is given abstract mesh function, $w_k^h = w_k^h(x)$ is unknown mesh function and $q_1^h = q_1^h(x) = \left(1 - \frac{(nh)^2 - nh^2}{l^2 + 2lc + (c-l)h}\right)$, $q_2^h = q_2^h(x) = -\frac{(nh)^2 - nh^2}{l^2 + 2lc + (c-l)h}$, $q_3^h = q_3^h(x) = \delta\left(1 - \frac{(nh)^2 - nh^2}{l^2 + 2lc + (c-l)h}\right)$ $+ \frac{2}{l^2 + 2lc + (c-l)h}\left(na_{n+1} - (n-1)a_n\right)$, $q_4^h = q_4^h(x) = -\delta \frac{(nh)^2 - nh^2}{l^2 + 2lc + (c-l)h} + \frac{2}{l^2 + 2lc + (c-l)h}\left(na_{n+1} - (n-1)a_n\right)$ are known elements of L_{2h}. The proof of Theorem 4 is based on theorems on stability and coercive stability of the abstract problem (12) (see [1, 2]), the self-adjointness and positive definiteness of the difference operator A_h^x defined by the formula (6).

Numerical results

Now, the numerical results for the solution of the initial boundary value problem

$$\begin{cases} u_t(t,x) - u_{xx}(t,x) = -\frac{3}{4}e^{-t}\cos\frac{x}{2}, \\ 0 < t < 1, \ 0 < x < \pi, \\ u(0,x) = \cos\frac{x}{2}, \ 0 \le x \le \pi, \\ u(t,0) - e^{-t} = u_x(t,0), \\ -u(t,\pi) - \frac{1}{2}e^{-t} = u_x(t,\pi), \ 0 \le t \le 1 \end{cases} \quad (13)$$

for the parabolic equation with Robin conditions are presented. The exact solution of this problem is

$$u(t,x) = e^{-t}\cos\frac{x}{2}.$$

For the approximate solution of problem (13), the set $[0,1]_\tau \times [0,\pi]_h$ of a family of grid points depending on the small parameters τ and h

$$[0,1]_\tau \times [0,\pi]_h$$
$$= \{(t_k, x_n) : t_k = k\tau, \ 0 \le k \le N, \ N\tau = 1, \ x_n = nh, \ 0 \le n \le M, \ Mh = \pi\}$$

is defined. For the numerical solution of problem (13), we present the first order of accuracy Rothe DS:

$$\begin{cases} \frac{u_n^k - u_n^{k-1}}{\tau} - \frac{u_{n+1}^k - 2u_n^k + u_{n-1}^k}{h^2} = f_n^k, f_n^k = -\frac{3}{4}e^{-t_k}\cos\frac{x_n}{2}, \\ 1 \le k \le N, \ 1 \le n \le M-1, \\ u_n^0 = \cos\frac{x_n}{2}, \ 0 \le n \le M, \\ u_0^k - e^{-t_k} = \frac{u_1^k - u_0^k}{h}, \\ u_M^k + \frac{1}{2}e^{-t_k} = -\frac{u_M^k - u_{M-1}^k}{h} = 0, \ 0 \le k \le N \end{cases} \quad (14)$$

and second order of accuracy Crank-Nicolson DS

$$\begin{cases} \frac{u_n^k - u_n^{k-1}}{\tau} - \frac{u_{n+1}^k - 2u_n^k + u_{n-1}^k}{2h^2} - \frac{u_{n+1}^{k-1} - 2u_n^{k-1} + u_{n-1}^{k-1}}{2h^2} = f_n^k, \\ f_n^k = -\frac{3}{4}e^{-t_k + \frac{\tau}{2}}\cos\frac{x_n}{2}, \ 1 \le k \le N, \ 1 \le n \le M-1, \\ u_n^0 = \cos\frac{x_n}{2}, \ 0 \le n \le M, \\ \frac{u_0^k + u_0^{k-1}}{2} - e^{-t_k + \frac{\tau}{2}} = \frac{u_1^k - u_0^k}{2h} + \frac{u_1^{k-1} - u_0^{k-1}}{2h}, \\ \frac{u_M^k + u_M^{k-1}}{2} + \frac{1}{2}e^{-t_k + \frac{\tau}{2}} = -\frac{u_M^k - u_{M-1}^k}{2h} - \frac{u_M^{k-1} - u_{M-1}^{k-1}}{2h}, \\ 1 \le k \le N. \end{cases} \quad (15)$$

For the computer implementation of DS (14), we can apply two approaches.

First, for obtaining the solution of difference scheme (14), we rewrite it as the initial value problem for the first order difference equation with respect to k and matrix coefficients

$$Au^{k+1} + Bu^{k} = If^{k},\ 1 \le k \le N,\ u^{0} = \varphi \tag{16}$$

where A, B are $(M+1)\times(M+1)$ square matrices and f^k is $(M+1)\times 1$ column matrix. Here,

$$A = \begin{bmatrix} 1+\frac{1}{h} & -\frac{1}{h} & 0 & \cdot & 0 & 0 & 0 \\ a & b & a & \cdot & 0 & 0 & 0 \\ 0 & a & b & \cdot & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & b & b & 0 \\ 0 & 0 & 0 & \cdot & a & b & a \\ 0 & 0 & 0 & \cdot & 0 & \frac{1}{h} & -1-\frac{1}{h} \end{bmatrix}_{(M+1)\times(M+1)},$$

$$B = \begin{bmatrix} 0 & 0 & 0 & \cdot & 0 & 0 & 0 \\ 0 & c & 0 & \cdot & 0 & 0 & 0 \\ 0 & 0 & c & \cdot & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & c & 0 & 0 \\ 0 & 0 & 0 & \cdot & 0 & c & 0 \\ 0 & 0 & 0 & \cdot & 0 & 0 & 0 \end{bmatrix}_{(M+1)\times(M+1)}$$

here and in future

$$a = -\frac{1}{h^2},\ b = \frac{1}{\tau} + \frac{2}{h^2},\ c = -\frac{1}{\tau}$$

and

$$f^k = \begin{bmatrix} f_0^k \\ f_1^k \\ \cdot \\ f_{M-1}^k \\ f_M^k \end{bmatrix}_{(M+1)\times 1} = \begin{bmatrix} e^{-t_k} \\ -0.75e^{-t_k}\cos\frac{x_1}{2} \\ \cdot \\ -0.75e^{-t_k}\cos\frac{x_{M-1}}{2} \\ \frac{1}{2}e^{-t_k} \end{bmatrix}_{(M+1)\times 1}.$$

From (16) it follows that

$$u^k = -inv(A)Bu^{k-1} + inv(A)If^k,\quad k = 1,\cdots,N,\quad u^0 = \varphi.$$

for DS (14) and

$$A = \begin{bmatrix} \frac{1}{2}+\frac{1}{2h} & -\frac{1}{2h} & 0 & \cdot & 0 & 0 & 0 \\ a & b & a & \cdot & 0 & 0 & 0 \\ 0 & a & b & \cdot & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & b & a & 0 \\ 0 & 0 & 0 & \cdot & a & b & a \\ 0 & 0 & 0 & \cdot & 0 & \frac{1}{2h} & -\frac{1}{2}-\frac{1}{2h} \end{bmatrix}_{(M+1)\times(M+1)},$$

$$B = \begin{bmatrix} \frac{1}{2}+\frac{1}{2h} & -\frac{1}{2h} & 0 & \cdot & 0 & 0 & 0 \\ a & c & a & \cdot & 0 & 0 & 0 \\ 0 & a & c & \cdot & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & c & a & 0 \\ 0 & 0 & 0 & \cdot & a & c & a \\ 0 & 0 & 0 & \cdot & 0 & \frac{1}{2h} & -\frac{1}{2}-\frac{1}{2h} \end{bmatrix}_{(M+1)\times(M+1)}$$

and

$$a = -\frac{1}{2h^2},\ b = \frac{1}{\tau} + \frac{1}{h^2},\ c = -\frac{1}{\tau} + \frac{1}{h^2},$$

and

$$f^k = \begin{bmatrix} f_0^k \\ f_1^k \\ \cdot \\ f_{M-1}^k \\ f_M^k \end{bmatrix}_{(M+1)\times 1} = \begin{bmatrix} e^{-t_k+\frac{\tau}{2}} \\ -0.75e^{-t_k+\frac{\tau}{2}} \cos\frac{x_1}{2} \\ \cdot \\ -0.75e^{-t_k+\frac{\tau}{2}} \cos\frac{x_{M-1}}{2} \\ \frac{1}{2}e^{-t_k+\frac{\tau}{2}} \end{bmatrix}_{(M+1)\times 1}$$

for DS (14). From (16) it follows that

$$u^k = -inv(A)Bu^{k-1} + inv(A)If^k, \quad k = 1, \cdots, N, \quad u^0 = \varphi.$$

Now, we will give the results of the numerical analysis. We recorded the numerical solutions u_n^k of these difference schemes at (t_k, x_n) for different N and M values. For their comparison, here and future errors are computed by

$$E_u = \max_{\substack{1\le k\le N \\ 0\le n\le M}} \left| u(t_k, x_n) - u_n^k \right|.$$

Table 1 demonstrates the error analysis between the exact solution and the solutions derived by the difference scheme. The error of Crank-Nicolson DS is $E_u = O(\tau^2 + h)$. It is constructed for $N = M = 20,\ 40$ and 80.

Table 1

Error analysis of first order Rothe DS (14)

Error	$N = M = 20$	$N = M = 40$	$N = M = 80$
E_u	0,0076	0,0038	0,0019

Table 2 illustrates the error analysis between the exact solution and the solutions derived by Crank-Nicolson. It is constructed for $N^2 = M = 100,\ 400$ and 1600.

Table 2

Error analysis of Crank-Nicolson DS (15)

Error	$N = 10$	$N = 20$	$N = 40$
E_u	0,0020	0,00046	0,00011

As it is seen in Tables 1 and 2, if N is multiplied by 2, the value of errors decreases approximately 1/2 for the DS (14) and 1/4 for the Crank-Nicolson DS (15). This shows that DS (15) has the second order of accuracy in time.

Mathematical modeling of the energy consumption problem

The importance of energy-saving and correct design is obvious for energy efficiency. Correct design means that before construction of something as orientation or isolation decisions needs to be made. The energy-saving means things to do during operation as automatic control. An important part of energy consumption occurs in buildings. For decision making, there are numerous studies on this subject all over the world. A national software calculates the energy consumption of buildings according to the TS EN 13790 standard.

The problem is complicated because the energy consumption calculation depends on many variables, such as the external temperature and the heat losses and gains, including the sun radiation change over time. Energy consumption numerical calculations given by the standard is time-consuming. Thus, the mathematical model and theoretical solution are valuable.

In this article, the annual energy consumption mathematical model of a house's room assumed heat loss and gain through the opaque outer wall. The heat conduction differential equation and boundary equations of the one-dimensional, nonstationary boundary value problem are obtained for the outer wall. This study aims at a dynamic model to compare the results of the numerical calculations ([7]). The study also aims to develop a mathematical model in which the result can be found at any time.

In this study, an effective numerical method is developed for energy consumption calculation. The three case problems are taken to test this method.

Case 1. Outer wall with different convection boundary problems; outer wall of a building which is initially 20^oC, is suddenly subjected to the convection boundary condition from the outer surface with air at 0^oC and the convection coefficient $25\,W/m^2K$ while inner temperature 20^oC and inner convection resistance $0.13\,m^2K/W$ are constant. Time-dependent temperature distribution and how long it will take to reach steady-state conditions are needed to be determined. Thermo-physical properties of the wall; $\rho = 2000\,kg/m^3$, $k = 1\,W/mK$, $c = 1000\,J/kgK$.

$$\begin{cases} u_t(t,x) - 5.10^{-7}u_{xx}(t,x) = 0, \ 0 < t < 3600, \ 0 < x < 0.2, \\ u\left(0,x\right) = 0, \ 0 \le x \le 0.2, \\ u_x\left(t,0\right) = 25u\left(t,0\right), \ 0 \le t \le 3600, \\ u_x\left(t,0.2\right) = 140 - 7u\left(t,0.2\right), \ 0 \le t \le 3600. \end{cases}$$

Case 2. Time-dependent on outer temperature problem; outer wall of a building which is initially 20^oC, is suddenly subjected to the convection boundary condition from the outer surface with time-dependent air temperature with the convection coefficient $25\,W/m^2K$ while inner temperature 20^oC and inner convection resistance $0.13\,m^2K/W$ are constant. Time-dependent temperature distribution and energy consumption are needed to be determined. Thermo-physical properties of the wall; $\rho = 2000\ kg/m^3$, $k = 1\,W/mK$, $c = 1000\,J/kgK$.

$$\begin{cases} u_t(t,x) - 5.10^{-7}u_{xx}(t,x) = 0, \ 0 < t < 3600, \ 0 < x < 0.2, \\ u\left(0,x\right) = 20, \ 0 \le x \le 0.2, \\ 25\left(u\left(t,0\right) - 20|\sin(\pi t/86400)|\right) = u_x\left(t,0\right), \ 0 \le t \le 3600, \\ -1.438u\left(t,0.2\right) + 28.76 = u_x\left(t,0.2\right), \ 0 \le t \le 3600. \end{cases}$$

Case 3. Time-dependent on outer temperature and solar radiation problems; An outer wall of a building which is initially 20^oC, is suddenly subjected to the convection boundary condition from the outer surface with time-dependent air temperature with the convection coefficient $25\,W/m^2K$ while inner temperature 20^oC and inner convection resistance $0.13\,m^2K/W$ are constant and time-dependent (constant) solar energy gain. Time-dependent temperature distribution and energy consumption are needed to be determined. Thermo-physical properties of the wall; $\rho = 2000\,kg/m^3$, $k = 1\,W/mK$, $c = 1000\,J/kgK$.

$$\begin{cases} u_t(t,x) - 5.10^{-7}u_{xx}(t,x) = f(t), \ 0 < t < 86400, \ 0 < x < 0.2, \\ u\left(0,x\right) = 20, \ 0 \le x \le 0.2, \\ 25\left(u\left(t,0\right) - 20\sin^2(\pi t/86400)\right) = u_x\left(t,0\right), \ 0 \le t \le 3600, \\ 7[20 - u\left(t,0.2\right)] = u_x\left(t,0.2\right), \ 0 \le t \le 3600, \end{cases}$$

$$f(t) = \begin{cases} 0, & t \le 21600, \\ 5.10^{-4}\sin^2(\pi t/43200), & 21600 < t < 64800, \\ 0, & 64800 \le t \le 86400. \end{cases}$$

Results

The results are compared with the previous finite-difference or steady-state solutions [7].

Case 1. One layer residence outer wall composed of one material initially is at the homogenously 20^oC. Then suddenly outside air temperature falls 0^oC and stays stable. Wall is 20 cm thick. Wall material properties are wall conduction coefficient $1\,W/mK$ and specific heat $1000\,J/kgK$, density $2000\,kg/m^3$. Heat convection coefficients inner and outer temperatures are 7.69 and $25\,W/m^2K$ respectively. This method's time-dependent results for the wall inner temperature distribution are given in Table 3.

Table 3

Temperature distribution for Case 1

Time(h)		Temperature				$q(W/m^2K)$
	Outside temp.	Outer surface	Mid point	Inner surfce	Inner Temp.	Heat Loss
0	20	20	20	20	20	0
6	0	5.48	14.33	17.53	20	17
12	0	3.50	10.92	15.36	20	35
24	0	2.43	8.26	13.52	20	49
48	0	2.11	7.38	12.55	20	57

The limit of this time-dependent solution for $t \to \infty$ is the steady-state solution, which is shown in Table 4. Steady-state temperature distribution goes to the linear line. Integrating heat loss over time we can get energy consumption rate approximation $2000\,Wh/m^2$.

Table 4

Steady state temperature distribution for Case 1

Time(h)		Temperature				$q(W/m^2K)$
	Outside temp.	Outer surface	Mid point	Inner surfce	Inner Temp.	Heat Loss
0	0	2.16	7.57	12.97	20	54

If we compare Table 3 results with Table 4, steady-state solutions are reasonable.

Case 2. Similar wall with Case 1, subjected this time with variable outer temperature according to $u_{outside}(t) = 20\sin^2(t/24)$ function. The temperature distribution of this wall is found by this method in Table 5 and compared with finite difference solution, Table 6.

Table 5

Temperature distribution for Case 2 variable outside temperature with sin function

Time(h)		Temperature				$q(W/m^2K)$
	Outside temp.	Outer surface	Mid point	Inner surfce	Inner Temp.	Heat Loss
0	20	20	20	20	20	0
6	7	13.93	15.49	17.08	20	22
12	20	18.88	17.05	16.91	20	28
24	0	5.12	13.42	15.87	20	32
48	0	4.91	12.25	14.58	20	41

Table 5 temperatures are over Table 3 temperatures as expected. Energy consumption rate is approximately $1200\,Wh/m^2$.

Table 6

The finite difference temperature distribution for Case 2

Time(h)		Temperature				$q(W/m^2K)$
	Outside temp.	Outer surface	Mid point	Inner surfce	Inner Temp.	Heat Loss
0	20	20	20	20	20	0
6	7	6.05	12.60	16.56	20	26
12	20	17.91	14.80	16.42	20	38
24	0	3.81	11.50	15.89	20	32
48	0	3.77	11.37	15.80	20	32

If the heat losses are integrated over a time period, heat energy consumption can be found.

The finite-difference numerical results of the article [7] for Case 2 are illustrated in Table 6. If we compare this study result of Table 5 with Table 6, then time-dependent solutions are reasonable.

Case 3. Similar wall with Case 1, subjected this time with variable outer temperature according to $u_{outside}(t) = 20\sin^2(t/24)$ function and variable sun radiation with a periodic sin function $6 < t < 18$, $q'' = 20\sin^2(t/24)$ function. The temperature distribution of this wall is found by this method in Table 7 and compared with finite difference solution in Table 8.

Table 7

Temperature distribution for Case 3 variable outside temperature and sun radiation with sin function

Time(h)		Temperature				$q(W/m^2K)$
	Outside temp.	Outer surface	Mid point	Inner surfce	Inner Temp.	Heat Loss
0	20	20	20	20	20	0
6	7	11.65	16.02	18.20	20	13
12	20	19.00	18.11	18.80	20	13
24	0	5.43	11.53	15.65	20	24
48	0	5.34.	11.27	15.44	20	24

Table 7 temperatures exceed Table 3 and Table 5 temperatures as expected. The finite-difference numerical result of the article [7] for Case 3 is pointed out in Table 8. If we compare this study results of Table 7 with Table 8, then time-dependent solutions are reasonable.

Table 8

The finite-difference temperature distribution solution for Case 3 variable outside temperature and sun radiation with sin function for a window and an opaque wall [7]

Time(h)		Temperature				$q(W/m^2K)$
	Outside temp.	Outer surface	Mid point	Inner surfce	Inner Temp.	Heat Loss
0	20	20	20	20	20	0
6	7	6	12.43	16.24	20	32
12	20	18.10	15.65	16.92	20	24
24	0	3.85	11.58	15.74	20	36
48	0	4.13	10.95	13.26	20	36

Conclusions

The energy consumption problem is complicated because the energy consumption calculation depends on many variables, such as the external temperature and the heat losses and gains, including the sun radiation change over time. Energy consumption numerical calculations given by the standard take a lot of time. In the present paper, we have examined the model of the nonstationary energy consumption calculation problems. The theoretical background of this model has been provided. The well-posedness of the mixed problem for parabolic equations with Robin conditions has been studied. The first and second-order accuracy single-step absolute stable difference schemes have been constructed. Well-posedness in Hölder spaces on time of these differential and difference parabolic problems has been established. Finally, these difference schemes have been applied for the energy consumption problems for the heat equations. The developed results are justified.

Acknowledgments

The authors are grateful to Charyyar Ashyralyyev for his support and to reviewers for the useful comments and relevant references.

References

1 Ashyralyev, A., & Sobolevskii, P.E. (1994). *Well-Posedness of Parabolic Difference Equations, Operator Theory Advances, and Applications.* Birkhauser Verlag, Basel, Boston, Berlin.

2 Ashyralyev, A., & Sobolevskii, P.E. (2004). *New Difference Schemes for Partial Differential Equations, Operator Theory Advances, and Applications.* Birkhauser Verlag, Basel, Boston, Berlin.

3 TS EN ISO 13790:2008. (2013). Energy performance of buildings – Calculation of energy use for space heating and cooling.

4 Incropera, F.P., Dewitt, D.P., Bergman, T.L., & Lavine, A.S. (2007). *Fundamentals of Heat and Mass Transfer.* United States, America.

5 Kokogiannakis, G., Strachan, P., & Clarke, J. (2008). Comparison of the simplified methods of the ISO 13790 standard and detailed modeling programs in a regulatory context. *Journal of Building Performance Simulation, 1*(4), 209–219.

6 Michalak, P. (2014). The simple hourly method of EN ISO 13790 standard in Matlab/Simulink: A comparative study for the climatic conditions of Poland. *Energy 75*, 568–578.

7 Koc, I., Sayar, E., & Parmaksizoglu, I.C. (2017). Thermal Modelling of the Building Energy Consumption. *National Thermal System Engineering Congress.*

А. Ашыралыев[1,2,3], М. Урун[4,5], И.Д. Пармаксизоглу[6]

[1] *Бахчешехир университеті, Стамбул, Түркия;*
[2] *Ресей халықтар достығы университеті, Мәскеу, Ресей;*
[3] *Математика және математикалық модельдеу институты, Алматы, Қазақстан;*
[4] *Таяу Шығыс университеті, Никосия, Түркия;*
[5] *Галатасарай университеті, Стамбул, Түркия;*
[6] *Стамбул техникалық университеті, Стамбул, Түркия;*

Энергияны тұтыну мәселелерін математикалық модельдеу

Энергияны үнемдеу мен дұрыс жобалау энергия тиімділігі үшін маңызды. Дұрыс дизайн дегеніміз — құрылысқа дейін бағдарлау немесе оқшаулау жұмыстарын жасау керек. Бұл зерттеуде бейстационарлық энергияны тұтынуды есептеу есептерінің математикалық моделі ұсынылған, яғни Гельдер кеңістігіндегі Робен шарттары бар аралас бір өлшемді параболалық есептің корректілігі. Авторлар осы математикалық модельге байланысты энергияны тұтынуды есептеудің тиімді сандық әдісін жасаған. Бұл сандық әдісті тексеру үшін үш есеп алынды. Динамикалық модельдің нәтижелері алдыңғы айырымдық немесе стационарлық шешімдермен салыстырылды. Сонымен қатар, зерттеу нәтижені кез келген уақытта табуға болатын математикалық модельді жасауға бағытталған.

Кілт сөздер: математикалық модельдеу, жылуөткізгіштік теңдеуі, айырымдық схемасы, тұрақтылық.

А. Ашыралыев[1,2,3], М. Урун[4,5], И.Д. Пармаксизоглу[6]

[1] *Университет Бахчешехир, Стамбул, Турция;*
[2] *Российский университет дружбы народов, Москва, Россия;*
[3] *Институт математики и математического моделирования, Алматы, Казахстан;*
[4] *Ближневосточный университет, Никосия, Турция;*
[5] *Галатасарайский университет, Стамбул, Турция;*
[6] *Стамбульский технический университет, Стамбул, Турция*

Математическое моделирование проблемы энергопотребления

Важность энергосбережения и правильного проектирования очевидна для энергоэффективности. Правильный дизайн означает, что перед строительством нужно сделать что-то вроде решения по ориентации или изоляции. В данном исследовании предложена математическая модель задач расчета нестационарного энергопотребления, которая представляет собой корректность в пространствах Гёльдера смешанной одномерной параболической задачи с условиями Робена. Авторами разработан эффективный численный метод расчета энергопотребления, связанный с данной математической моделью. Для проверки этого численного метода взяты три задачи. Результаты динамической модели сравнивались с предыдущими конечно-разностными или стационарными решениями. Кроме того, исследование направлено на разработку математической модели, в которой результат может быть найден в любое время.

Ключевые слова: математическое моделирование, уравнение теплопроводности, разностная схема, устойчивость.