МЕХАНИКА
MECHANICS

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Calculation of velocities and accelerations of points of a crank mechanism

In this article, presented is an analytical method of determining linear and angular velocities and accelerations of links of a crank mechanism on the basis of a grapho-analytical method (without the construction of displacement, velocity, and acceleration diagrams) with the use of angles, which form between vectors on velocity and acceleration diagrams, as well as between links on the mechanism diagram. Pointed out is the role of analytical methods (with the help of which the research of kinematics of mechanisms can be conducted with high degree of accuracy), which has especially increased in recent years due to the fact that by having analytical expressions that link the main kinematic and structural mechanism parameters with each other, it is possible to compile a calculation program for a counting machine at any moment, and with the help of such a machine all the necessary results can be obtained. Given are the results of the implementation of the proposed calculation algorithm on computer equipment, a comparative analysis of the obtained data and an estimation of the relative error of the calculation. A justification for the suitability of the developed algorithm of calculating velocities and accelerations of points of the mechanism is provided. The proposed calculation algorithm of kinematic parameters of a crank mechanisms allows to automate the process of calculating the velocities and accelerations, significantly reduces the amount of labor needed for the calculation, ensures a high degree of accuracy.

Keywords: crank mechanism, velocity, acceleration, analytical method.

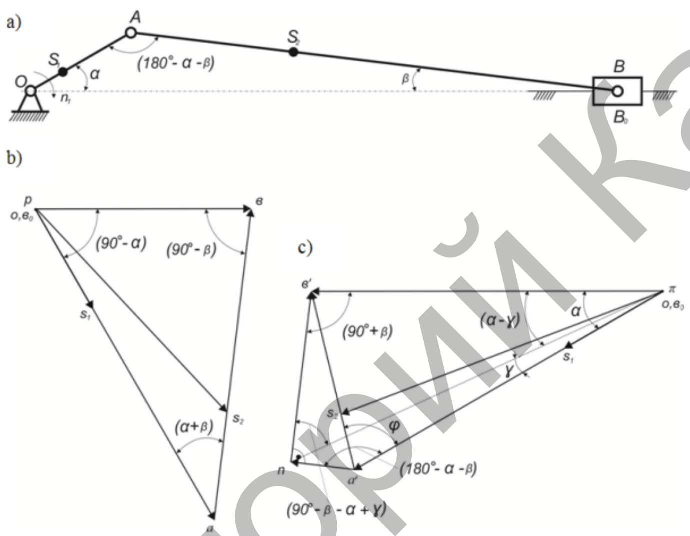
Introduction

During the process of research of the dynamics of mechanisms and machines, three problems of kinematics need to be solved: the position problem, the velocity problem, and the acceleration problem. Graphical, grapho-analytical and analytical methods exist to solve these problems. The graphical (charting method) and grapho-analytical methods (method of velocity and acceleration diagrams) of a kinematic analysis of mechanisms possess following flaws: low accuracy that is defined by the accuracy of graphical constructions, and great laboriousness. When using the graphical method, graphs of displacements, velocities and accelerations for each investigated point of the mechanism ought to be constructed. When the grapho-analytical method is utilized, several velocity and acceleration diagrams of the mechanism need to be constructed in order to define the time history of the velocity and acceleration of the points that we are interested in. To achieve this, vector equations for the velocities and accelerations of points of links, which make complex motions, are composed in advance. The solution of vector equations is executed graphically by constructing so-called velocity and acceleration diagrams, on which absolute velocities and accelerations are deposited, on a certain scale, from one point that called a pole [1].

There are no such flaws in the existing analytical methods. Given that, nevertheless, it is necessary to compose quite complex analytical dependences (formulas) and to be able to solve them with the use of computer equipment and technology, which is possible and accessible as of late. The role of analytical methods, with the help of which the research of kinematics of mechanisms can be conducted with high accuracy rate, has especially increased in recent years due to the fact that by having analytical expressions that link the main kinematic and structural mechanism parameters with each other, it is possible to compile a calculation program for a counting machine at any moment, and with the help of such a machine all the necessary results can be obtained.

Therefore, what is proposed in this work is a more simple analytical method of a kinematic analysis of a crank mechanism on the basis of the grapho-analytical method without the construction of displacement, velocity, and acceleration diagrams. Moreover, a problem of an analytical determination of linear velocities and accelerations of points of links is set, as well as a problem of angular velocities and accelerations of links under following input data: number of crank revolutions n_1 (rev/min); crank length ℓ_A (m); length of connecting rod ℓ_{AB}(m); position of the center of gravity of crank ℓ_{S_1}(м); position of the center of gravity of connecting rod ℓ_{AS_2}(м); angle α, line segment $@0 = \pi 0'$ (mm).

The key to solve the given problem with this method is to use angles that form between vectors on velocity and acceleration diagrams, as well as between links on the mechanism diagram. All the necessary angles for the calculation of velocities and accelerations are depicted in Figure 1.



a – position diagram of mechanism; *b* – velocity diagram; *c* – acceleration diagram

Figure 1. Computational scheme of a crank mechanism

From ΔOAB we find angle β, i.e. the angle between connecting rod AB and column OB

$$\beta = \arcsin\left(\frac{\ell_{OA} \cdot \sin\alpha}{\ell_{AB}}\right). \tag{1}$$

Considering the construction principle of the velocity diagram, where $pb \| OB$, $ab \perp AB$, $pa \perp OA$, we find angles Δpab on the velocity diagram.

$$\angle pba = 90^\circ - \beta;$$

$$\angle apb = 90^\circ - \alpha,$$

$$\angle pab = 180^\circ - (90^\circ - \alpha) - (90^\circ - \beta) = 180^\circ - 90^\circ + \alpha - 90^\circ + \beta = \alpha + \beta.$$

From Δpab we derive an equation

$$\frac{pa}{\sin(90^\circ - \beta)} = \frac{pb}{\sin(\alpha + \beta)} = \frac{ab}{\sin(90^\circ - \alpha)}.$$

Lengths of line segments pb and ab are therefore

$$pb = \frac{pa \cdot \sin(\alpha + \beta)}{\sin(90^\circ - \beta)}; \tag{2}$$

$$ab = \frac{pa \cdot \sin(90^\circ - \alpha)}{\sin(90^\circ + \beta)}. \tag{3}$$

We will define the position of the centers of gravity on the velocity diagram by the principle of similarity

$$ps_1 = \frac{pa \cdot \ell_{OS_1}}{\ell_{OA}}; \qquad (4)$$

$$as_2 = \frac{ab \cdot \ell_{AS_2}}{\ell_{AB}}. \qquad (5)$$

From Δpas_2 we find the length of line segment ps_2

$$ps_2 = \sqrt{(pa)^2 + (as_2)^2 - 2 \cdot pa \cdot as_2 \cdot |\cos(\alpha + \beta)|}. \qquad (6)$$

Rotation frequency of the crank

$$\omega_1 = \frac{\pi \cdot n_1}{30}. \qquad (7)$$

Velocity of point A is determined by the formula

$$V_A = \omega_1 \cdot \ell_{OA}. \qquad (8)$$

Scale of velocity diagram

$$\mu_V = \frac{V_A}{pa}. \qquad (9)$$

The required absolute and relative velocities of points are calculated with the use of the scale of the velocity diagram

$$V_B = pb \cdot \mu_V;\ V_{S_1} = ps_1 \cdot \mu_V;\ V_{S_2} = ps_2 \cdot \mu_V;\ V_{BA} = ab \cdot \mu_V. \qquad (10)$$

The angular velocity of the connecting rod is determined by formula

$$\omega_2 = \frac{V_{BA}}{\ell_{AB}}. \qquad (11)$$

The acceleration of point A is determined by formula

$$a_A = \omega_1^2 \cdot \ell_{OA}. \qquad (12)$$

Scale of acceleration diagram

$$\mu_a = \frac{a_A}{\pi a'}. \qquad (13)$$

Normal acceleration is determined by formula

$$a_{BA}^n = \frac{V_{BA}^2}{\ell_{AB}}. \qquad (14)$$

Length of line segment $a'n$

$$a'n = \frac{a_{BA}^n}{\mu_a}. \qquad (15)$$

From $\Delta\pi a'n$ we find the length of line segment πn and the unknown angle γ

$$\pi n = \sqrt{(\pi a')^2 + (a'n)^2 - 2 \cdot \pi a' \cdot a'n \cdot \cos(180^\circ - \alpha - \beta)}; \qquad (16)$$

$$\gamma = \arccos\left(\frac{(\pi a')^2 + (\pi n)^2 - (a'n)^2}{2 \cdot \pi a' \cdot \pi n}\right). \qquad (17)$$

Considering the construction principle of the acceleration diagram, where $\pi b' \| OB$, $\pi a' \| OA$, $nb' \perp AB$, $a'n \| AB$, we find the angles of the triangles on the acceleration diagram.

From $\Delta\pi a'b'$ we find $a'\pi b' = \alpha$.

From $\Delta\pi s_2 b'$ we find $\angle b'\pi s_2 = \alpha - \gamma$.

From $\Delta\pi nb'$ we find $\angle\pi b'n = 90^\circ + \beta$.

From $\Delta\pi a'n$ we find $\angle\pi a'n = 180^\circ - \alpha - \beta$.

From $\Delta\pi nb'$ we will determine

$\angle\pi nb' = 180^\circ - (90^\circ + \beta) - (\alpha - \gamma) = 180^\circ - 90^\circ - \beta - \alpha + \gamma = 90^\circ - \beta - \alpha + \gamma$.

From $\Delta\pi n b'$ we will obtain the equation

$$\frac{\pi n}{\sin(90^\circ+\beta)}=\frac{\pi b'}{\sin(90^\circ-\beta-\alpha+\gamma)}.$$

Hence is the length of line segment $\pi b'$

$$\pi b'=\frac{\pi n\cdot\sin(90^\circ-\beta-\alpha+\gamma)}{\sin(90^\circ+\beta)}. \tag{18}$$

From $\Delta a'\pi b'$ we find the length of line segment $a'b'$

$$a'b'=\sqrt{(\pi a')^2+(\pi b')^2-2\cdot\pi a'\cdot\pi b'\cdot\cos\alpha}. \tag{19}$$

From $\Delta a'nb'$ we find the length of line segment nb'

$$nb'=\sqrt{(a'b')^2-(a'n)^2}. \tag{20}$$

We will determine the position of the centers of gravity on the acceleration diagram by the principle of similarity

$$\pi s_1=\frac{\pi a'\cdot\ell_{OS_1}}{\ell_{OA}}, \tag{21}$$

$$a's_2=\frac{a'b'\cdot\ell_{AS_2}}{\ell_{AB}}. \tag{22}$$

From $\Delta\pi a'b'$ we find the unknown angle φ

$$\varphi=\arccos\left(\frac{(\pi a')^2+(a'b')^2-(\pi b')^2}{2\cdot\pi a'\cdot a'b'}\right). \tag{23}$$

From $\Delta\pi a's_2$ we find the length of line segment πs_2

$$\pi s_2=\sqrt{(\pi a')^2+(a's_2)^2-2\cdot\pi a'\cdot a's_2\cdot\cos\varphi}. \tag{24}$$

The required absolute and relative accelerations of points are calculated with the use of the scale of the acceleration diagram

$$a_B=\pi b'\cdot\mu_a;\ a_{S_1}=\pi s_1\cdot\mu_a;\ a_{S_2}=\pi s_2\cdot\mu_a;\ a^{\tau}_{BA}=nb'\cdot\mu_a;\ a_{BA}=a'b'\cdot\mu_a. \tag{25}$$

Angular acceleration ε_1 of the crank, which executes uniform motion, equals to zero.

The angular acceleration of the connecting rod is determined by formula

$$\varepsilon_2=\frac{a^{\tau}_{BA}}{\ell_{AB}}. \tag{26}$$

To sum up, the algorithm of the proposed analytical method of determining the velocities and accelerations of points of a crank mechanism with the use of computer equipment and technology can be expressed by the following sequence of actions:

1 Accounting of the input data: number of crank revolutions n_1 (rev/min); crank length ℓ_{OA} (m); length of connecting rod ℓ_{AB}(m); position of the center of gravity of crank ℓ_{OS_1}(m); position of the center of gravity of connecting rod ℓ_{AS_2}(m); angle α , line segment $pa=\pi a'$ (mm).

2 Calculation of the value of the angle β by formula (1).

3 Calculation of the value of the length of line segment pb by formula (2).

4 Calculation of the value of the length of line segment ab by formula (3).

5 Calculation of the value of the length of line segment ps_1 by formula (4).

6 Calculation of the value of the length of line segment as_2 by formula (5).

7 Calculation of the value of the length of line segment ps_2 by formula (6).

8 Calculation of the value of the revolution frequency of crank ω_1 by formula (7).
9 Calculation of the value of the velocity of point A by formula (8).
10 Calculation of the value of the scale of velocity diagram μ_V by formula (9).
11 Calculation of the value of the absolute and relative velocities by formula (10).
12 Calculation of the value of the angular velocity of crank ω_2 by formula (11).
13 Calculation of the value of the acceleration of point A by formula (12).
14 Calculation of the value of the scale of acceleration diagram μ_a by formula (13).
15 Calculation of the value of normal acceleration a^n_{BA} by formula (14).
16 Calculation of the value of the length of line segment $a'n$ by formula (15).
17 Calculation of the value of the length of line segment πn by formula (16).
18 Calculation of the value of angle γ by formula (17).
19 Calculation of the value of the length of line segment $\pi b'$ by formula (18).
20 Calculation of the value of the length of line segment $a'b'$ by formula (19).
21 Calculation of the value of the length of line segment nb' by formula (20).
22 Calculation of the value of the length of line segment πs_1 by formula (21).
23 Calculation of the value of the length of line segment $a's_2$ by formula (22).
24 Calculation of the value of angle φ by formula (23).
25 Calculation of the value of the length of line segment πs_2 by formula (24).
26 Calculation of the value of absolute and relative by formula (25).
27 Calculation of the value of the angular velocity of connecting rod ε_2 by formula (26).

Currently, contemporary specialized professional programs are used for engineering calculations. However, in certain problems of mechanics, it is possible to achieve the assigned goal by using an old program written in the «Basic» language [2, 3].

The results of the implementation of the abovementioned algorithm on computer equipment in the domain of «Turbo Basic» (when $n_1 = 850$rev/min; $\ell_{OA} = 0,11$ m; $\ell_{AB} = 0,462$ m; $\ell_{OS_1} = 0,0363$ m; $\ell_{AS_2} = 0,15246$ m; $\alpha = 30°$; $pa = \pi a' = 50$ mm) are shown in Figure 2.

For the sake of a comparative analysis, the required magnitudes are determined by the grapho-analytical method with the use of the КОМПАС 3DV17 computer-aided design and put in Table.

Table

Results of the calculation of velocities and accelerations

Magnitude	Unit	Value		Relative error, δrel, %
		by the proposed method in the domain of «Turbo Basic»	by the grapho-analytical method with the help of КОМПАС 3D V17	
V	m/s	9,79129695892334	9,79	0,013
V_B	m/s	5,912344455718994	5,905164	0,12
V_{S_1}	m/s	3,231127977371216	3,2274	0,11
V_{S_2}	m/s	7,722815036773682	7,694904	0,36
V_B	m/s	8,540245056152344	8,530116	0,12
ω_2	s-1	18,48537826538086	18,46345454	0,12
a	m/s2	871,5409545898438	870,72	0,094
a_B	m/s2	860,3972778320312	860,445504	0,0056
a_{S_1}	m/s2	287,6085205078125	287,3376	0,094
a_{S_2}	m/s2	841,8800659179688	841,11552	0,091
a^n_{BA}	m/s2	157,8696594238281	157,49112	0,23
a^τ_{BA}	m/s2	419,677001953125	419,512896	0,036
a_{BA}	m/s2	448,3877868652344	448,246656	0,031
ε_2	s-2	908,3917846679688	908,0365714	0,039

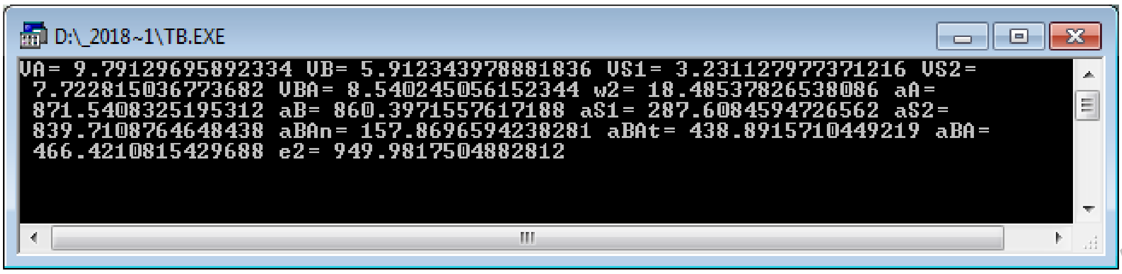


Figure 2. Results of the calculation in the domain of «Turbo Basic»

After comparing the obtained data, a conclusion can be made that the proposed calculation algorithm of kinematic parameters of a crank mechanism:

- allows to automate the process of calculating the velocities and accelerations;
- significantly reduces the amount of labor needed for the calculation;
- ensures a high degree of accuracy.

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Н.К. Бейсенов

Иінді-бұлғақты механизм нүктелерінің жылдамдықтарын және үдеулерін есептеу

Мақалада жылдамдықтар және үдеулер пландарындағы векторлардың арасында, оған қоса механизм планындағы звенолардың арасында құрылатын бұрыштарды пайдалана отырып, графоаналитикалық әдіс негізінде орындар, жылдамдықтар, үдеулер пландарын құрмай-ақ, иінді-бұлғақты механизмдегі нүктелердің сызықтық жылдамдықтары мен үдеулерін, звенолардың бұрыштық жылдамдықтары мен үдеулерін анықтаудың аналитикалық әдісі ұсынылған. Механизмдердің кинематикасын зерттеуде жоғары дәлдік дәрежесін қамтамасыз етуге мүмкіндік беретін аналитикалық әдістердің маңызы атап көрсетілген. Соңғы жылдары механизмнің кинематикалық және құрылымдық параметрлерін бір-бірімен байланыстыратын аналитикалық өрнектің көмегімен есептеу-шешу машинасы үшін есептеулер бағдарламасын жасап, барлық керекті нәтижелерді алу маңызы ерекше арты. Иінді-бұлғақты механизмнің кинематикалық параметрлерін анықтаудың ұсынылған алгоритмі жылдамдықтар мен үдеулерді есептеу үрдісін автоматтандыруға, есептеу уақытын едәуір қысқартуға, жоғары дәлдік дәрежесін қамтамасыз етуге мүмкіндік береді. Ұсынылған есептеудің алгоритмі компьютерлік техникада жүзеге асыру нәтижелерін, алынған мәліметтерді салыстыра талдап, есептеудің салыстырмалы ауытқуын бағалауға мүмкіндік береді. Механизмнің нүктелерінің жылдамдықтары мен үдеулерін есептеудің ұсынылған алгоритмінің жарамдылығы негізделген.

Кілт сөздер: иінді-бұлғақты механизм, жылдамдық, үдеу, аналитикалық әдіс.

Н.К. Бейсенов

Расчет скоростей и ускорений точек кривошипно-шатунного механизма

В статье представлен аналитический метод определения линейных скоростей и ускорений, угловых скоростей и ускорений точек звеньев кривошипно-шатунного механизма на основе графоаналитического метода без построения планов положений, скоростей, ускорений с использованием углов, образующихся между векторами на планах скоростей и ускорений, а также между звеньями на плане механизма. Отмечена роль аналитических методов, с помощью которых исследование кинематики механизмов может быть сделано с высокой степенью точности, особенно возросших в последние годы в связи с тем, что, имея аналитические выражения, связывающие между собой основные кинематические и структурные параметры механизма, можно всегда составить программу вычислений для счетно-решающей машины и с помощью машины получить все необходимые результаты. Приведены результаты реализаций предложенного алгоритма расчета на компьютерной технике, сравнительный анализ полученных данных и оценка относительной погрешности вычисления. Обоснована пригодность разработанного алгоритма расчета скоростей и ускорений точек механизма. Предложенный алгоритм вычисления кинематических параметров кривошипно-шатунного механизма позволяет автоматизировать процесс вычисления скоростей и ускорений, значительно сокращает трудоемкость расчета, обеспечивает высокую степень точности.

Ключевые слова: кривошипно-шатунный механизм, скорость, ускорение, аналитический метод.

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