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**THE THEORY OF DYNAMICAL STABILITY OF WAVE PACKETS**

V.V. Arkhipov

Karaganda State University, Universitetskaya St. 28, Karaganda, 100028, Kazakhstan, midav\_73@mail.ru

*In this article we show that a wave packet for massive particles can save pseudo stability, if it is composed with a discrete set of harmonics. This packet, after periodicity in a consequence of dispersion, is reconstructed at some smearing. Such behavior is denoted in this paper as dynamical stability and may be used as an illustration of a particle moving through physical vacuum, that is characterized by periodic annihilations with appropriate antiparticles and delivery of its momentum to the next identical particle.*

**Keywords:** wave packet, comprising waves, periodicity, annihilations, dynamical stability.

**Introduction**

The theory of the wave packet was proposed early in the 20th century to solve problems which are arising from the attempts to physical interpret de Broil waves. This model allowed to construct a local object which possessed had wave properties and moved with particle speed, in spite of super-light velocities of the compounds [1]. However, identification particles and wave packets had met a row of unsolvable problems. The basic counter-evidence is the De Broglie's waves dispersion for the particles with non zero rest mass though the wave packet is flowing apart during 10-11sec. As the second counter argument can be pointed that the wave packet does not preserve it's individuality when passing through some obstacles. In general, the Borne's probabilistic interpretation and further development of the quantum theory made the wave packet theory non relevant from scientific point of view. However, it became an integral part of the quantum mechanics course and it is used so far to demonstrate Geizenberg's uncertainty principle as well as for methodology purposes. Since we reckon it is relevant to show the first counter argument against wave packet theory is not crucial. Namely, we'll prove that it is possible to assemble a wave packet from a discrete set of harmonics, though it will have the ability to recreate itself with some periodicity after diffusion due its components dispersion. In the present paper such behavior is called dynamic stability. Being precise, a similar behavior of the wave packets was observed in experiments [2]. In the present work we study the general conditions to ensure dynamic stability of the free massive particle wave packets. The obtained results do not contradict with the probabilistic interpretation of the wave function. But the proposed model can be used to demonstrate an elementary particle movement through physical vacuum where the movement is a complex process accompanied by annihilations with antiparticles and initial momentum transfer to the identical particles.

**Conditions of periodicity**

Let some particle have an energy  $E_0$  and a momentum  $\vec{p}_0$ , that depend on its velocity by relativistic ratios:

$$E_0 = \frac{m_0 c^2}{\sqrt{1 - (v_0 / c)^2}}, \quad \vec{p}_0 = \frac{m_0 \vec{v}_0}{\sqrt{1 - (v_0 / c)^2}}, \quad (1)$$

where  $m_0$  – the rest mass of the particle. The connection with wave behavior is given by standard expressions:

$$E_0 = \omega_0 \hbar, \quad \vec{p}_0 = \vec{k}_0 \hbar, \quad (2)$$

where  $\omega_0$  is a cyclic frequency and  $\vec{k}_0$  is a wave vector. An expression for De Broglie's wave of free particle, that is moving along  $x$  - axis, can be written as

$$\psi_0 = A \exp i\{k_0 x - \omega_0 t\}. \quad (3)$$

The corresponding wave packet has to have a view of superposition waves closed by the wave number and the frequency to the basic mode  $\psi_0$ :

$$\psi(x, t) = \int_{k_0 - \Delta k}^{k_0 + \Delta k} A(k) \exp i\{kx - \omega(k)t\} dk, \quad (4)$$

where  $\omega(k)$  is defined by the fundamental equation of relativistic dynamic:

$$E^2 - p^2 c^2 = m_0^2 c^4. \quad (5)$$

So, we have an expression for  $\omega$  and  $\vec{k}$  connection:

$$(\omega \hbar)^2 - (\vec{k} \hbar)^2 c^2 = m_0^2 c^4. \quad (6)$$

The amplitudes  $A(k)$  in (4) have to be so small that the integral (4) at  $x=0$  and  $t=0$  is finite. For simplicity, hereafter the amplitudes will be considered equal for all components. Now, we will suppose the wave packet has a view like (4), but integration is replaced by summation:

$$\psi(x, t) = \sum_{m=-\Delta m}^{\Delta m} A \exp i\{k_m x - \omega_m t\}, \quad (7)$$

where  $\Delta m$  - some integer number defining quantity of components (harmonics) in the wave packet.

Let the particle be at the point  $x=0$  at the initial moment  $t=0$  and the packet have a maximum evidence there. After its smearing due to dispersion, let the next packet reconstruction realizes at a point  $x=l$ . This event has to have place at time  $t=\tau$ , when the particle has come to the point  $x=l$ . We will denote a phase of de Broglie's wave as

$$\varphi_0(x, t) = k_0 x - \omega_0 t. \quad (8)$$

Next condition has to be realized at the point  $x=l$  and  $t=\tau$ :

$$\varphi_0(l, \tau) = -2\pi n, \quad (9)$$

where  $n$  is any integer number. The sign "-" provides positivity of the number  $n$ , so the particle is moving slower than the wave humps. Thus

$$k_0 l - \omega_0 \tau = -2\pi n \quad (10)$$

is a condition on the space period of the reconstruction  $l$  and the "twinkling" time  $\tau$  of the wave packet. It is obviously that  $l$  and  $\tau$  can be intertwined with each other as

$$l = v_0 \tau \quad \text{or} \quad \tau = l / v_0. \quad (11)$$

It can switch from the wave parameters  $k_0$  and  $\omega_0$  to the corpuscular ones  $p_0$  and  $E_0$  in expression (10):

$$\frac{p_0}{\hbar} l - \frac{E_0}{\hbar} \frac{l}{v_0} = 2\pi n. \quad (12)$$

After replacements  $p_0$  and  $E_0$  due to (1) and some simple transformations we can get a definitive expression for possible values of the space reconstruction period:

$$l_n = \frac{2\pi \hbar v_0}{m_0 c^2 \sqrt{1 - (v_0 / c)^2}}. \quad (13)$$

The twinkling time  $\tau$  of the wave packet ( $\tau = l / v_0$ ) is

$$\tau_n = \frac{2\pi \hbar}{m_0 c^2 \sqrt{1 - (v_0 / c)^2}}. \quad (14)$$

We can rewrite  $l$  and  $\tau$  as dependent from the particle momentum:

$$l_n = \frac{2\pi \hbar p_0}{m_0^2 c^2}, \quad (15)$$

$$\tau_n = -\frac{2\pi \hbar \sqrt{p_0^2 + m_0^2 c^2}}{m_0^2 c^3}. \quad (16)$$

Let us consider a slow free electron moving at a speed 1 m/s. Numerical evaluation of the spatial period of the electron wave packet reconstruction  $l$  gets minimal ( $n = 1$ ) value  $10^{-20}$  m. This result implies distances passed by the wave packet in a degraded state are far less ones could be observed in experiments.

### A selection of the wave packet components

The components of the dynamical stable wave packet, directed along  $x$ -axis, have to have next standard view:

$$\psi(x, t) = A \exp i\{kx - \omega t\}, \quad (17)$$

where the wave number  $k$  and the cyclic frequency  $\omega$  can take only some discrete values. Appropriate to (17) the energy ( $E = \hbar \omega$ ) and the momentum ( $p = \hbar k$ ) have to satisfy the fundamental relation of relativistic dynamics (5). The wave (17) has to take on maximal values at the same points where the basic mode (3) does. Being more exactly, it has to be at points  $x = l \times [\text{integer number}]$ . But since all the points are peer, we can require realization of the maximum condition only at the point  $x = l$ , it has to take place at the moment  $\tau = l / v_0$ . So, we have a condition on the phase of the wave (17) analogously to (10):

$$kl - \omega\tau = -2\pi m, \quad (18)$$

where  $m$  is some integer number. Thus

$$\frac{p}{\hbar}l - \frac{E}{\hbar} \frac{l}{v_0} = -2\pi m. \quad (19)$$

After the next substitution

$$E = \sqrt{p^2 c^2 + m_0^2 c^4} \quad (20)$$

we have

$$\frac{l}{\hbar} \left( p - \frac{1}{v_0} \sqrt{p^2 c^2 + m_0^2 c^4} \right) = -2\pi m. \quad (21)$$

Now we can get a condition for  $l$ , similar to (13) or (15):

$$l = \frac{2\pi m \hbar}{\frac{1}{v_0} \sqrt{p^2 c^2 + m_0^2 c^4} - p}. \quad (22)$$

This expression have to be equated by  $l_n$  of the basic mode (15):

$$\frac{2\pi m \hbar}{\frac{1}{v_0} \sqrt{p^2 c^2 + m_0^2 c^4} - p} = \frac{2\pi \hbar p_0}{m_0^2 c^2}. \quad (23)$$

Taking into account, formulas for a velocity of the particle

$$v_0 = \frac{p_0 c^2}{E_0} = \frac{p_0}{\sqrt{p_0^2 + m_0^2 c^2}} \quad (24)$$

and get ridding of the radicals, we get a quadratic equation for possible values of the momentum  $p$ :

$$n^2 p^2 - 2nmpp_0 + (n^2 p_0^2 - (m^2 - n^2)m_0^2 c^2) = 0. \quad (25)$$

The discriminant

$$D = 4n^2 m^2 p_0^2 - 4n^2 (n^2 p_0^2 - (m^2 - n^2)m_0^2 c^2) \quad (26)$$

can be transformed to

$$D = 4n^2 (m^2 - n^2) \frac{E_0^2}{c^2}. \quad (27)$$

In that way we have derived an expression for the momentums of the harmonics, that can be in the dynamical stable wave packet:

$$p_m = \frac{m}{n} p_0 \pm \frac{\sqrt{m^2 - n^2}}{n} \frac{E_0}{c}. \quad (28)$$

If  $m = n$  we have  $p_m = p_0$ , i.e. the wave packet basic mode is reproduced. Its evident from the last formula that the next condition has to have place:

$$m \geq n. \quad (29)$$

The “minus” sign in (28) indicates possibility of constituent’s presence with negative phase velocity in the wave packet. It is easy to see it will have place under the condition:

$$mp_0 < \sqrt{m^2 - n^2} \frac{E_0}{c}, \quad (30)$$

or

$$mv_0 < \sqrt{m^2 - n^2} c. \quad (31)$$

If  $v_0 \ll c$  then the condition will be satisfied with a sizeable part of the wave packet constituents. If  $v_0 \rightarrow c$  then the condition will become impossible, i.e. the wave packet will hold only the constituents with positive phase velocities.

From (20) we can find a condition for discreet values of energy:

$$E_m = \frac{\sqrt{m^2 - n^2} p_0 c \pm m E_0}{n}. \quad (32)$$

Thus, the discreet harmonics of the dynamical stable wave packet reconstructed with period  $\tau$  has the next view:

$$\psi_m = A \exp \frac{i}{\hbar} \left( \frac{m}{n} p_0 x \pm \frac{\sqrt{m^2 - n^2} E_0}{n} x - \frac{\sqrt{m^2 - n^2}}{n} p_0 c t \mp \frac{m}{n} E_0 t \right). \quad (33)$$

### Uniqueness of the localization domain

In view of discontinuity of the wave packet constituents we have to investigate the question about uniqueness of its domain localization. In other words, if all harmonics give maximal positive contribution in the packet at the moment  $t = 0$  in the point  $x = 0$ , then is this point single? May be there are other points at  $t = 0$  where all harmonics are in phases  $2\pi s$ , where  $s$  is an integer number, somewhere in space. So, in this case we would get infinity number of wave packets, as the constituent monochromatic waves are infinity. This would be a wrong situation conflicted with experiment data. For checking the assumption noted above, we can investigate wave packet at the moment  $t = 0$ . The phases of the constituents (33) will have a view

$$\varphi_m = \frac{1}{\hbar} \left( \frac{m}{n} p_0 x \pm \frac{\sqrt{m^2 - n^2} E_0}{n} x \right). \quad (34)$$

A condition

$$\varphi_m = 2\pi s_m, \quad (35)$$

has to be satisfied at a point  $x \neq 0$ , where the wave packet has an identical view to the one at the point  $x = 0$ . The symbol  $s_m$  is any integer number and the index  $m$  notes that  $s_m$  can be changed against constituents. In that way we get a condition on the possible value of  $x \neq 0$ , where the reiteration of the wave packet takes place:

$$\frac{m}{n} p_0 x \pm \frac{\sqrt{m^2 - n^2}}{n} \frac{E_0}{c} x = 2\pi \hbar s_m. \quad (36)$$

It should be noted, that the point  $x$  where the wave packet reiterates has to be divisible to wave-length  $\lambda_0$  of the basic mode,

$$\lambda_0 = \frac{2\pi \hbar}{p_0}. \quad (37)$$

So, we can do the following change:

$$x = \lambda_0 w, \quad (38)$$

where  $w$  is an integer number. Thus we get from (36):

$$\frac{m}{n} w \pm \frac{\sqrt{m^2 - n^2}}{n} \frac{E_0}{c p_0} w = s_m. \quad (39)$$

Now we have to answer the question "Do such an integer number  $w$  and a value  $E_0 / c p_0$  exist which satisfy the ratio (39) for all values  $m$ ?"

The answer becomes obvious if to rewrite the last expression as

$$w = \frac{s_m n}{m \pm \sqrt{m^2 - n^2} \frac{E_0}{c p_0}}. \quad (40)$$

It is seen that if the values  $n$  and  $E_0 / c p_0$  are fixed then changes of the number  $m$  cannot be compensated by changes of the number  $s_m$ . In that way we have to conclude the wave packet is single and one nowhere reiterates in space.

### Concluding remarks

The obtained results take off the objection against the wave packet theory based on the dispersion of the comprising waves. This result does not prejudice the modern statistical interpretation of the wave function at least since the proposed hypothesis does not offer a solution to keep integrity of the particles when its passing through obstacles, for instance, potential barrier. However, this can significantly enrich it because the quantum states having the view of the wave packets are used in modern quantum mechanics everywhere, for example, in physical chemistry.

### References:

- A.A. Sokolov, Yu.M. Loskutov, I.M. Ternov. Quantum mechanics. - M.: - Prosvescheniye. - 1961. - 638 p.  
 Y.A. Smorodinsky, T.B. Romanovskaya. Louis de Broglie (1892 - 1987) (From hystory of Physycs)//UFN, **Vol. 166**, 1988. - pp.753-769.  
 V.V. Eryomin, S.I. Vetchinkin, I.M. Umanskii. Manifestations of wave packet fractional revivals in a Morse like anharmonic system // J. Chem. Phys. 1994. - **Vol.101**. - N12. - P.10730.10735.