The main result of the paper is stated in the theorem below.

Theorem 1. Let $u(x,t)$ and $\psi(x,\lambda_k,t)$ be solution of the problem (1)-(3). Then the spectrum of the problem (3) does not depend on t, and the spectral parameters $\xi_n=\xi_n(t)$, $\sigma_n=\sigma_n(t)$, $n\geq 1$ satisfy the analogue of the system of Dubrovin equations

$$\dot{\xi}_n=\left\{\frac{1}{2\xi_n}-\frac{1}{2}\sum_{j=1}^{\infty}\frac{1}{\xi_j}+\frac{1}{4}\sum_{k=0}^{\infty}\frac{1}{\lambda_k}+\sum_{k=0}^{\infty}\frac{\xi_n\alpha_k(t)s(\pi,\lambda_k,\mathrm{t})}{\xi_n-\lambda_k}\right\}h_n(\xi),$$

where

$$h_n(\xi)=-\frac{\sigma_n\xi_n\sqrt{\left(1-\frac{\xi_n}{\lambda_0}\right)\prod_{i=1}^{\infty}\left(1-\frac{\xi_n}{\lambda_{2i-1}}\right)\left(1-\frac{\xi_n}{\lambda_{2i}}\right)}}{\prod_{j\neq n,\,j=1}^{\infty}\left(1-\frac{\xi_n}{\xi_j}\right)}.$$

The sign $\sigma_n(t)=\pm 1$ changes at each collision of the point $\xi_n(t)$ with the boundaries of its gap $[\lambda_{2n-1},\lambda_{2n}]$. Moreover, the following initial conditions are fulfilled:

$$\xi_n(t)\big|_{t=0}=\xi_n^0,\quad \sigma_n(t)\big|_{t=0}=\sigma_n^0,\ n\geq 1,$$

where ξ_n^0, σ_n^0, $n\geq 1$ are the spectral parameters of the weighted Sturm-Liouville equations (3) corresponding to the coefficients $q_0(x)=u(x,0)-u_{xx}(x,0)<0$.

References

1. Camassa R. and Holm D. D. An integrable shallow water equation with peaked solitons// Phys. Rev. Lett. 1993. Vol.71, 1661_1664.
2. Constantin A. On the inverse spectral problem for the Camassa-Holm equation// J. Funct. Anal. 1998. Vol.155, 352_363.
3. Korotayev E. Inverse Spectral Problem for the Periodic Camassa-Holm Equation// Journal of Nonlinear Mathematical Physics. 2004. Vol.11, No 4. 499_507.
4. Gesztesy F. and Holden H. Real-valued algebro-geometric solutions of the Camassa-Holm hierarchy// Phil.Trans.R.Soc.A. 2008. Vol.366, 1025_1054.
5. Johnson R. The Camassa_Holm equation for water waves moving over a shear _ow// Fluid Dynam. Res. 2003. Vol.33, 97_111.
6. Alber M.S., Camassa R., Holm D.D. and Marsden J.E. The geometry of peaked solitons and billiard solutions of a class of integrable PDE's// Lett.Math.Phys, 1994. Vol.32, 137_151.
7. Huang Y., Yao Y., Zeng Y. On Camassa-Holm equation with self-consistent sources and its solutions// Communications in theoretical physics. 2010. Vol.53, No 3. 403-412.
8. Baltaeva I.I., Urazboev G.U. About the Camassa-Holm equation with a self-consistent source// Ufa mathematical journal, 2011. Vol.3, No 2. 10-18.

INTEGRATION OF THE FINITE COMPLEX TODA CHAIN WITH A SELF-CONSISTENT SOURCE

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The finite Toda lattice is a nonlinear Hamiltonian system which describes the motion of N particles moving in a straight line, with “exponential interactions”[1]. A huge number of papers has been devoted to the investigation of the Toda lattices and their various generalizations, from which we indicate here only [2, 3]. With regard to their applications we refer to works [4, 5].

We consider the following system of equations

$$\begin{cases} \dot{a}_n = a_n(b_n - b_{n+1}) + a_n \sum_{i=1}^{N}\left((g_n^i)^2 - (g_{n+1}^i)^2\right), \\ \dot{b}_n = 2(a_{n-1}^2 - a_n^2) - 2\sum_{i=1}^{N} g_n^i (a_n g_{n+1}^i - a_{n-1} g_{n-1}^i), \\ a_{n-1} g_{n-1}^k + b_n g_n^k + a_n g_{n+1}^k = \lambda_k g_n^k,\ k = 1, \dots, N,\ n = 0, 1, \dots, N-1 \end{cases} \quad (1)$$

with the boundary conditions

$$a_{-1} = a_{N-1} = 0. \quad (2)$$

The system (1),(2) is considered subject to the initial conditions

$$a_n(0) = a_n^0,\ b_n(0) = b_n^0, n = 0, 1, \dots, N-1, \quad (3)$$

where a_n^0, b_n^0 are given complex numbers such that $a_n^0 \neq 0$ $(n = 0, 1, \dots, N-2)$, $a_{N-1}^0 = 0$.

The main aim of this work is to derive representations for the solutions $a_n(t), b_n(t)$, $g_n^1(t), g_n^2(t), \dots, g_n^N(t)$, $n = 0, 1, \dots, N-1$ of the finite complex Toda lattice (1) with a self-consistent source by means of the inverse spectral problem for the complex Jacobi matrices. For this goal spectral data of the complex Jacobi matrices are introduced and an inverse spectral problem from the spectral data is solved. The time evolution of the spectral data for the Jacobi matrix associated with the solution of the Toda lattice is computed. Using the solution of the inverse spectral problem with respect to the time-dependent spectral data we reconstruct the time-dependent Jacobi matrix and hence the desired solution of the finite complex Toda lattice with self-consistent source.

References

1. Huseynov A., GuseinovG.Sh. Solution of the finite complex Toda lattice by the method of inverse spectral problem // App. Math. and Comp. 2010, v. 219, pp. 5550- 5563.
2. Toda M. Theory of Nonlinear Lattices. New York: Springer, 1981.
3. Flaschka H., The Toda lattice, I // Phys. Rev., 1974, B 9, pp. 1924-1925.
4. Moser J., Three integrable Hamiltonian systems connected with isospectral deformations // Adv. Math., 1975, vol. 16, pp. 197-220.
5. Muto V., Scott A.C., Christiansen P. L., Thermally generated solitons in a Toda lattice model of DNA // Physics Letters A, 1989, vol. 136, pp. 33-36.

A PROBLEM FOR LOADED DIFFERENTIAL EQUATIONS WITH PIECEWISE CONSTANT ARGUMENT OF GENERALIZED TYPE

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We consider the following linear three-point boundary-value problem for the system of loaded differential equations with piecewise constant argument of generalized type:

$$\frac{dx}{dt} = A_0(t)x + K(t)x(\gamma(t)) + \sum_{i=1}^{m+1} A_i(t)x(\theta_{i-1}) + f(t), \quad t \in (0, T), \quad (1)$$

$$Bx(0) + Dx(\theta_1) + Cx(T) = d, \quad d \in R^n, \quad x \in R^n, \quad (2)$$

where $(n \times n)$-matrices $A_j(t)$, $(j = \overline{0, m+1})$, $K(t)$are continuous on $[0, T]$,and n-vector-function $f(t)$ is piecewise continuous on $[0, T]$ with possible discontinuities of the first kind at the points