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## NUMERICAL STUDY OF GRADUAL CHANGE IN SHAFT DIAMETER AND RELIEF GROOVE RADIUS ON STRESS CONCENTRATION FACTOR

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**Abstract.** The purpose of this work is to investigate the effect of relief groove radius and the taper in gradual diameter change of shaft on stress concentration factors. Finite element analysis was used to conduct theoretical research in order to reduce the maximum value of stress in stepped shafts. In comparison based on a straight shaft without relief groove, the results show the decrease of taper angle has a significant effect in decrease the values of stress concentration factors. On the other hand, the increase of location of relief grooves from shaft shoulder tends to increase the stress concentration factors values, especially at high taper angle.

**Keywords:** stepped shafts, shaft shoulder fillet, relief groove, stress concentration factors (SCF), finite element analysis (FEA).

### 1. Introduction

Most machine parts have varying cross sections, and shafts often include steps or shoulders to accommodate bearings, gears, and other mechanical elements. These changes in cross – sectional geometry cause local stress concentration and, while it is not practical to eliminate such features, it is necessary to minimize their effects, given that the degree of stress concentration is a factor in the evaluation of fatigue strength and fatigue life.

The stress concentration factors (SCF) are based on the nominal stress at the minimum diameter or width can be defined as the following,

$$k_t = \frac{\sigma_{\max}}{\sigma_{\text{nom}}}$$

where  $\sigma_{\max}$  is the maximum stress at the root of relief groove obtained from numerical simulation, and  $\sigma_{\text{nom}}$  is the nominal stress calculated by  $\sigma_{\text{nom}} = 4F/(\pi d^2)$  for step shaft cross section.

Shoulders, grooves, keyways, and threads all contribute to increased localized high stresses in the parts. Stress concentration refers to the localization of high stress. The ratio of peak stress in the body to nominal stress is known as the stress concentration factor ( $K_t$ ). In experimental and analytical researches, Pertson and Roak presented a lot of data in the form of graphs to evaluated the effect of geometrical discontinuities on the stress concentration factor ( $k_t$ ) [1].

Amaral Dias et al., presented a method for numerically evaluating stress concentration factors in shafts when radial bearings are used under three pure load conditions: traction, bending, and torsion. Two designs of simple fillet bearing shoulder is used in studies. Ansys® software was used to carry out the study. A

validation is achieved by comparing data from the literature works. The results showed that, in some case, simple fillet present low stress concentration factor compared to the standard design, [2]. Finite element analysis by Solid Works program was used to study a rounded shaft with shoulder fillet. The stress concentration factors for various fillet radius were investigated under applying tensile load on the round shaft. The results showed that the error not exceed that 5% between theoretically stress compared to the maximum stress computed with Solid Works. In addition to, the error decreases with the increase of fillet radius [3].

The effect of stress concentration on fatigue life were numerically evaluated using finite element analysis by applying changes in geometric parameters related to the size and location of the relief grooves. The results showed that the stress concentration reduced by 22.3% compared with that without the relief grooves. In addition to, using of the relief grooves extends the fatigue life by a factor of more than 3.3 times compared with the case without relief grooves [4]. Park et al., showed that Stress relief grooves were more effective than side fillet cutting at reducing stress concentration. The stress concentrations under bending and torsion loading conditions were lowered by 27.3 % and 18.2 %, respectively, when the radius of the stress relief groove was increased up to 2.0 mm. According to this study, the stress concentration increased when the radius of the relief groove was small [5].

Masanobu et al., studied that the effect of stress relief groove design variables such as radius, tangential angle, and depth on fretting fatigue strength. The results showed that the stress relief groove increases the fretting fatigue strength under the loading conditions of a bending moment combined with fretting fatigue.[6]

Prajwal et al., investigated the stress concentration and fatigue life of stepped shaft subject to axial stress. Reduction of stress concentration in the range of 13% and 34% were achieved for  $d_o/d_i$  from 1.08 to 1.22 [7].

Stress concentration factor of stepped shaft subject to twisting moment was investigated, the shaft has circular-to-square cross-section. Diameter to diagonal ratio ( $D/d$ ) and radius of fillet to diagonal ratio ( $r/d$ ) were studied for different ratios. The results indicated that the maximum shear stress located at the root of square in the middle location [8]. The results of FEA based on ANSYS program to compute the stress concentration factor ( $K_t$ ) give a good result. The method can be considered good to produce accurate results for a wide range of geometry and loading conditions [9].

Stress concentration factor was studied by FEA analysis for keyway as discontinuity on shaft. The result shows the conformed agreement with theoretical calculations using Peterson equations. Torsion, bending and axial were applied as loading conditions. The results concluded that the increase of fillet radius of keyway tends to decrease the stress concentration factor (SCF) [10]. Different shapes and sizes of notched formed in cylindrical specimen subject to axial load, in order to reduce the effect of stress concentration. Studies were carried out by ANSYS software. The results showed notched shape, especially U shape notch, has a significant effect on stress concentration factor and fatigue life cycles [11].

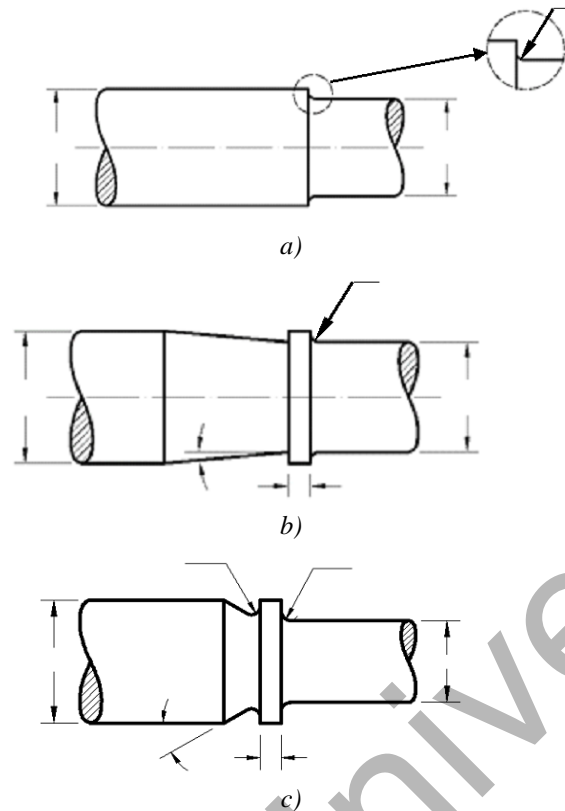
## 2. Finite element analysis

The finite element method is a computational procedure for obtaining solutions to a wide range of engineering problems with complicated geometries that involve stress analysis. All models are studied using the ANSYS workbench 2019 R1 software. In this work, a round shaft with a constant  $D/d$  ratio of 1.5 was used, the effect of relief groove geometry has been investigated using finite element analysis (FEA). The different geometric are shown in Figure 1 can be described as the following.

- 1) The angles of tapered shaft are 5, 10, 15 and 20°
- 2) Ratio of circular groove shaft diameter ( $r_g/d$ ) are 0.1, 0.15 and 0.2
- 3) Distance between relief groove and shaft shoulder respect to shaft diameter ( $L/d$ ) are 0.25, 0.5 and 0.75.

Steel material is used for simulations as an isotropic material with elastic modulus of 207 GPa, Poisson's ratio of 0.3. Figure 1 illustrates the geometric dimensions of the model used in the study.

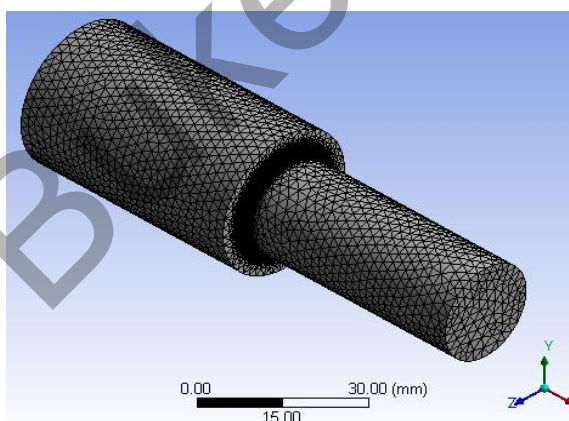
- 1) The greater and lower diameters are ( $D$ ) and ( $d$ ) simultaneously,
- 2) Distance between the shoulder fillet and the circular groove is ( $L$ ),
- 3) Applied normal force ( $F$ ) of 1 kN.



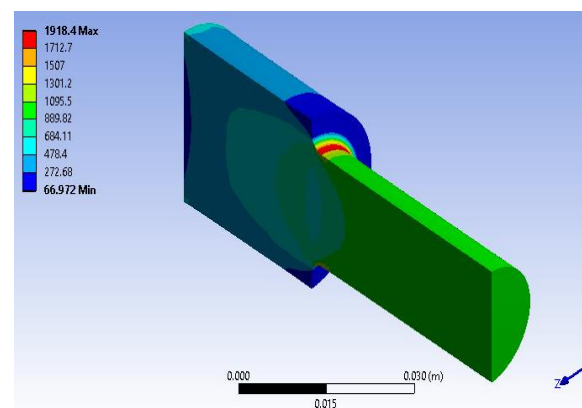
**Fig.1.** Geometric parameters of FEA models: a) Stepped shaft with shoulder fillet; b) Stepped shaft without relief groove; c) Stepped shaft with relief groove.

### 2.1 Mesh convergence

Different FEA models were studied with several different mesh sizes to determine the best mesh size to meet good results. Figures 2 and 3 show mesh refinement of fillet shoulder and equivalent stress distribution. 8 node elements with size of 0.25 mm along the fillet were applied to achieve high stress level. The relation between mesh densities on the stress concentration factor can be seen in Figure 4, the results showed that the stresses are increased by 17% when increasing the number of elements in the model.



**Fig.2.** Mesh refinement of shaft model with shoulder.



**Fig. 3.** Equivalent (von Mises) stress distribution of shaft model with shoulder fillet.

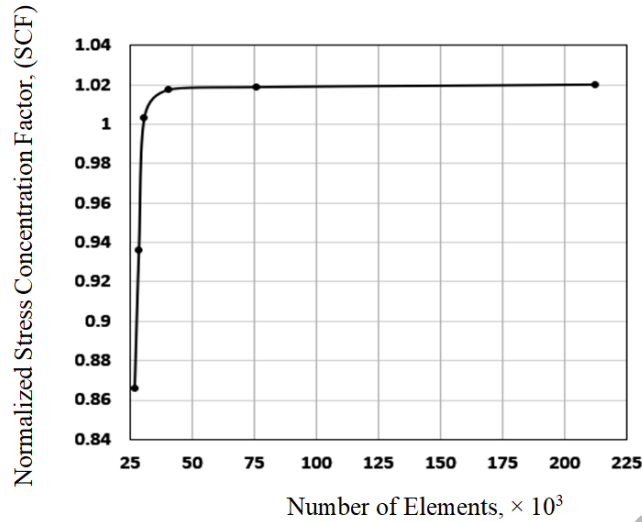


Fig.4. Stress concentration factor respect to mesh densities.

### 2.2 Verification of finite element model

ANSYS 2019 R1 is used to carry out FEA analysis. The validation of results obtained was compared to analytical data from Peterson,[1]. The shaft model has a circumferential groove. The material model is steel with a yield strength of 304 MPa, a Young's modulus of 207 GPa, and a Poisson ratio of 0.3 were used as input data in the FEA software.

Based on the results from the previous studies [1] and results from finite element analysis, Figure 5 illustrates the comparison between the stress concentration factor (SCF) for different ratios of (r/d) of stepped shaft (D/d) equal to 1.5. It may be noticed that a good agreement between two results in the range from 0.5 to 0.3.

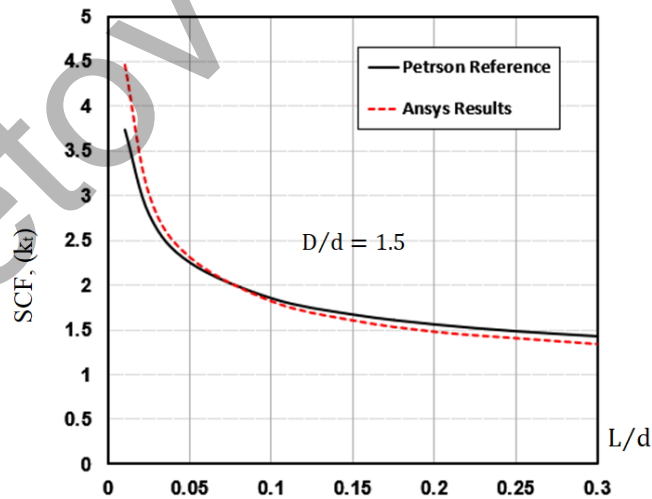


Fig.5. Validation of FEA results with the Analytical results given in reference, [1].

### 3. Results and Discussion

Figure 6 illustrates the effect of tapered shaft angle ( $\theta^\circ$ ) on stress concentration factor (SCF). The first curve shows the (SCF) of tapered shaft without relief groove, and the other is the percentage of these values relative to step shaft. The results show that the angle of tapered shaft has a significant effect, lowest values of (SFC) can be achieved at small angle value.

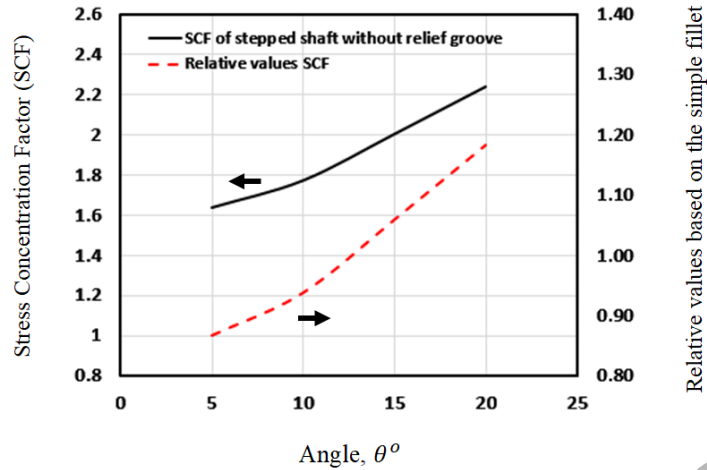


Fig.6. (SCF) of tapered shaft without relief groove.

As seen in Figure 7, The effect of groove location on stress concentration factor (SCF) is investigated. The increase in the distance ratio ( $L/d$ ) between the shaft shoulder and relief groove tends to increase the values of stress concentration factors (SCF) values.

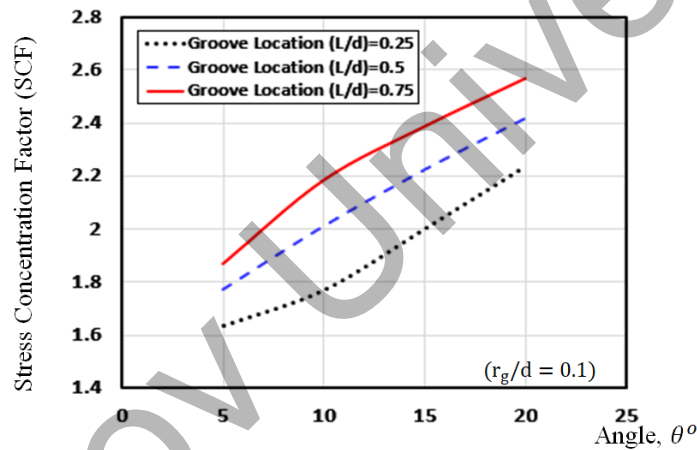


Fig.7. Effect of groove location of relief groove on (SCF).

The parameter of relief groove radius at different locations on the stress concentration factor (SCF) were studied. Figure 8, 9 and 10 illustrate the relationship between the stress concentration factor (SCF) versus the angle of tapered shaft at different ratio of relief groove radius ( $r_g/d$ ). With constant locations of relief groove ( $L/d$ ), It is clear that, the tapered shaft angle affects significantly the stress concentration factors (SCF). Furthermore, the effect of relief groove radius ( $r_g/d$ ) on the values of (SCF) can be clear observed at angles  $15^\circ$  and  $20^\circ$ .

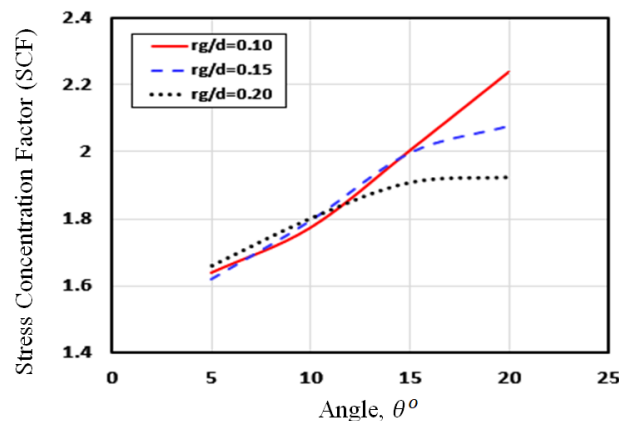


Fig.8. Effect of taper angle on (SCF),  $L/d = 0.25$

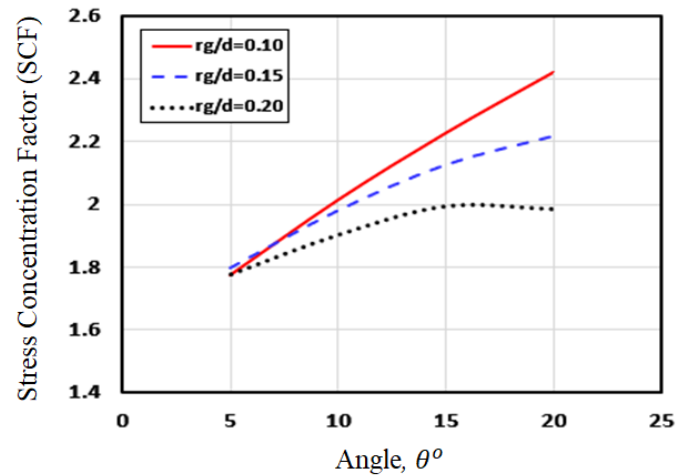


Fig.9. Effect of taper angle on (SCF), L/d= 0.5

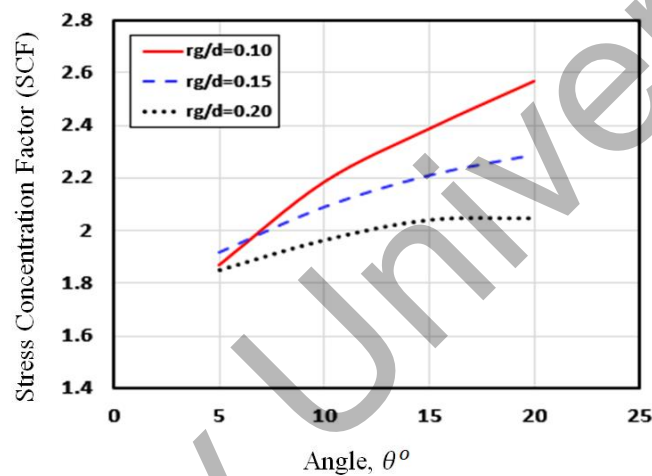


Fig.10. Effect of taper angle on (SCF), L/d= 0.75

#### 4. Conclusions

In this study, a numerical model using ANSYS software is developed to study the effect of geometric parameter such as radius of relief groove ( $r_g$ ) and angle of tapered shaft ( $\theta^\circ$ ) on stress concentration factor (SCF). Based on the results, the following conclusions can be drawn:

- 1) ANSYS software can be considered as a suitable FEA tool to study the stress concentration factor (SCF).
- 2) The angle of tapered shaft ( $\theta^\circ$ ) has a significant effect on the values of stress concentration factor (SCF).
- 3) The increase of relief groove radius ( $r_g$ ) tends to increase the values stress concentration factor (SCF).
- 4) The stress concentration factor (SCF) of tapered stepped shaft is lower than that of a stepped shaft without a taper by 5-10% at small values of taper angles.

#### Conflict of interest statement

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

#### CRedit author statement

**Sinossi M.I.:** Conceptualization, Software; **Sadak T.W.:** Writing - Review & Editing, Methodology.

The final manuscript was read and approved by all authors.

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