

**REDUÇÃO DAS CONDIÇÕES DEFEITUOSAS DE OPERAÇÃO DA PLANTA DE ENERGIA EÓLICA AO RESERVAR OS MODOS DE OPERAÇÃO DE ENERGIA ALTERNATIVA****REDUCTION OF DEFECTIVE CONDITIONS OF THE WIND POWER PLANT OPERATION AT RESERVING THE OPERATION MODES OF ALTERNATIVE ENERGY****СНИЖЕНИЕ ДЕФЕКТНЫХ СОСТОЯНИЙ РАБОТЫ ВЕТРОВОЙ УСТАНОВКИ ПРИ РЕЗЕРВИРОВАНИИ РЕЖИМОВ ФУНКЦИОНИРОВАНИЯ АЛЬТЕРНАТИВНОЙ ЭНЕРГЕТИКИ**

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**RESUMO**

O ritmo atual da economia global requer um aumento significativo na produção de energia. Ao mesmo tempo, as reservas de combustíveis naturais tradicionais, como gás natural e petróleo, estão sendo gradualmente esgotadas. Portanto, a energia tradicional está sendo substituída por alternativa. O desenvolvimento da tecnologia de operação dos dispositivos que garantem o funcionamento de empresas de energia alternativa baseia-se no fato de que a confiabilidade desses dispositivos deve estar no nível das usinas de energia tradicionais. Cada dispositivo deve não apenas ser tolerante a falhas, mas também fácil de manter. Geralmente, são considerados dispositivos portáteis e independentes, capazes de gerar energia sem manutenção excessiva do equipamento pelo serviço técnico. O artigo objetivou resolver o problema do uso de energia eólica em uma área remota. O principal método de pesquisa foi a análise da teoria dos gráficos. A novidade da pesquisa é determinada pelo fato de oferecer a formação de um modelo que atenda aos padrões de funcionamento no nível dos maiores modelos industriais usados em energia alternativa como um dos objetos da produção pura de energia. Uma usina de geração de energia eólica é usada como uma instalação semelhante. Foi revelado que esse problema deve ser considerado apenas desde que essa instalação produza energia em uma quantidade não inferior a uma amostra semelhante de uma fonte de energia tradicional em consumo por consumidores industriais e padrão. A importância prática do estudo é determinada pelo fato de que cada um dos tipos de instalações projetados possa operar em quase todas as condições e ter alta tolerância a falhas.

**Palavras-chave:** *energia, consumo, padrão, desenvolvimento, fonte de energia alternativa.*

**ABSTRACT**

The current pace of the global economy requires a significant increase in energy production. At the same time, the reserves of traditional natural fuels, such as natural gas and oil, are gradually being depleted. Therefore, conventional energy is being replaced by an alternative. Developing the operation technology of the devices that ensure the functioning of alternative energy enterprises is based on the fact that the reliability of such devices should be at the level of traditional power plants. Each device must be not only fault-tolerant but also easy to maintain. It is often considered to be portable and self-contained devices capable of generating energy without excessive maintenance of the equipment by the technical service. The article aimed to solve the problem of using wind energy in a remote area. The leading research method was graph theory analysis. The

novelty of the research is determined by the fact that it offers the formation of a model that will meet the standards of functioning at the level of the most significant industrial models used in alternative energy as one of the objects of pure energy production. A wind power generation plant is used as a similar facility. It was revealed that this issue should be considered only provided that such a facility will produce energy in an amount of not less than a similar sample of a traditional energy source in consumption by both industrial and standard consumers. The practical significance of the study is determined by the fact that each of the designed types of facilities can operate in almost any conditions and have high fault tolerance.

**Keywords:** *energy, consumption, standard, development, alternative energy source.*

## АННОТАЦИЯ

Современные темпы развития мировой экономики требуют значительного увеличения производства энергии. Вместе с тем, запасы традиционных естественных видов топлива, таких как природный газ и нефть, постепенно истощаются. Поэтому на смену традиционной энергетике приходит альтернативная. Формирование технологии работы устройств, которые обеспечивают функционирование предприятий альтернативной энергетикой, основано на том, что надежность подобных устройств должна быть на уровне традиционных энергетических установок. Каждое устройство должно быть не только отказоустойчивым, но также и являться простым для обслуживания. В качестве подобного устройства зачастую рассматриваются портативные и автономные устройства, способные вырабатывать энергию без чрезмерного ухода за оборудованием со стороны технической службы. Целью статьи являлось решение задачи использования энергии ветра в отдаленном районе. Ведущим методом исследования был анализ с помощью теории графов. Новизна исследования определяется тем, что предложено формирование модели, которая будет отвечать стандартам функционирования на уровне крупнейших промышленных моделей, применяемых в альтернативной энергетике в качестве одного из объектов выработки энергии чистого типа. В качестве подобного объекта используется установка ветрового производства энергии. Было выявлено, что следует рассматривать данный вопрос только при условии того, что подобная установка будет производить энергию в количестве не меньшем чем подобный образец традиционного источника энергии в потреблении как промышленными, так и стандартными потребителями. Практическая значимость исследования определяется тем, что каждый из проектируемых видов установки может функционировать практически в любых условиях и обладать высокой отказоустойчивостью.

**Ключевые слова:** *энергия, потребление, стандарт, развитие, альтернативный источник энергии.*

## 1. INTRODUCTION:

The most common method of generating electricity for remote loads, both in areas and for special applications, is a diesel engine driving a generator set (Beinke *et al.*, 2017; Heredia *et al.*, 2018; Ballireddy and Modi, 2019). For small loads, a single diesel unit will be sufficient, but a more significant number of diesels will be needed for a larger group of consumers (Zheng *et al.*, 2015; Zhang *et al.*, 2018; Zhang *et al.*, 2019).

It is recommended to use diesel generators with a minimum load of not less than, usually, 40 percent to maintain high efficiency (efficiency), since fuel costs can be significant, which will be expensive, and to minimize machine wear (Kaiser and Snyder, 2012a; Bukar *et al.*, 2019; Rehman *et al.*, 2019). However, given that the load is continually changing and the need to match the load, in many areas, you can find diesels that have the appropriate capacity or are inefficiently used (Chakraborty, 2018). This quite often happens with diesels, for which the

operational demands are not fulfilled. However, diesel units guarantee reliability and long service life (Hau, 2013; Hong *et al.*, 2018; Chen *et al.*, 2017; Can, 2019).

Energy costs tend to be high, often many times higher than for higher-capacity networks (Grigorash *et al.*, 2014; Krasnobaev *et al.*, 2018; Obukhov *et al.*, 2018). Low efficiency is a result of the high cost of diesel fuel, including transportation costs, which are often the dominant factor, and operating and maintenance costs, as diesel systems are usually installed in remote areas. The main advantage of diesel systems is that they are highly reliable (Yang *et al.*, 2013; Sujith and Ramesh, 2017; Dai *et al.*, 2019).

The critical point is that despite the reliability of diesel-electric systems, the energy produced by them has a high cost. This ratio is unlikely to change in the future, with costs likely to rise (Miller and Keith, 2018; Finn and Sandeberg, 2019). The nature of the electrical load in remote areas is of crucial importance and

largely depends on the type of electric machine in the system (Chemekov and Kharchenko, 2013; Grigor'ev *et al.*, 2019; Grigoriev *et al.*, 2019).

Two factors are affecting the system: variability of electrical load; quality of the energy consumed, or "stability" of the network. In the first case, the total load tends to change more or less regularly throughout the day, often peaking during regular working hours and falling to a minimum early in the morning. In addition to the gradual change, there are also fluctuations of much shorter duration caused by the switching on or off powerful electrical equipment. The load can also vary significantly by day of the week or depending on the season of the year. From an economic point of view, it is necessary to know whether the annual change in wind energy potential corresponds to the change in load or there is a significant inconsistency. In the first case, the full potential of wind energy can be used (Tereshkin, 2018; Devederkin *et al.*, 2019). However, the downside of this will be the need to operate the diesel at low load. In case of incomplete compliance, there may be a situation of some advantage for individual systems, so diesel can be better loaded when the need for loads is high, although the downside may be the high cost of diesel fuel.

The second factor – "sustainability" applying – is slightly less obvious, but highly relevant. To properly feed the most potent load, the voltage and frequency of the current must be within the appropriate limits (Perzhabinsky and Karamov, 2017; Semenyutina *et al.*, 2019; Rodina, 2019). Fulfilling this condition is usually not tricky when only diesel generators are used. When other sources of energy, such as wind turbines, are added to the system, the task is more complicated. This is caused primarily by short fluctuations (of the order of seconds or minutes) in the wind speed, which causes corresponding changes in the generation of electricity from the wind energy (Nižetić *et al.*, 2017; Sabir *et al.*, 2019). The ability to account for these fluctuations is a requirement for any wind-diesel system (Essallah *et al.*, 2019; Ochoa and Martinez, 2019).

There are three areas of application of the autonomous power plant: for specialized applications in remote areas, e.g., communications, irrigation, for remote regions of industrialized countries and islands, local power plants in developing countries (Sebastián, 2017; Sivachandran, 2017).

Each has its unique requirements; for

example, the system reliability may be a more valuable property than the cost of energy for systems managed automatically (Deltenre and Runacres, 2019; Fernández-Guillamón *et al.*, 2019; Rabanal *et al.*, 2019). While consumers in industrialized areas expect high levels of energy quality, suitability, and competitiveness, ease of maintenance is a critical factor for the consumers in the countries that are in the process of development (Tsgoev, 2012; Dekkiche *et al.*, 2017; Rudenko *et al.*, 2019).

The article aimed to solve the problem of using wind energy in a remote area.

## 2. LITERATURE REVIEW:

To solve the problem of using wind energy in a remote area, it is necessary to compensate for the variability of the wind, which can be done with the help of a diesel generator. Such a system would take advantage of the free wind resource, saving a certain amount of fuel consumption, and would supply energy on-demand following the load of the consumer. Ideally, diesel could be used to provide a steady supply of energy in low wind, and WDPP could be used to save diesel when the wind power is sufficient (Wang and Bai, 2010).

The nature of wind energy has a significant impact on the overall efficiency, as well as on the organization of any wind-diesel system. The most significant aspect of this system is wind variability, which occurs over time, ranging from long – term changes (hourly to seasonal) to short-term turbulent fluctuations (seconds to minutes) (Morales Pedraza, 2015). Considering long-term changes, it would be desirable from an economic point of view that the wind speed in the long-term periods corresponds to the load.

While the correspondence between long-term changes in wind power and load can significantly affect the economy of the system, short fluctuations have the most significant impact on the design of the system. Most tasks with a simple combination of wind and diesel are reduced to adapting to this level of wind variability. Wind changes rapidly, leading to significant changes in the energy potential of the system (Tang *et al.*, 2017). This means that the amount of electricity that is affected by changes in a moderate time interval (such as an hour) can be much less than the average of the same range. Through the random nature of wind oscillations, reliable absolute minimum energy values can be guaranteed, although the probability of exceeding and most optimal levels

can be calculated (DeMeo and Steitz, 1990).

Since wind power is continually changing, diesel power also needs to be further modified to provide electricity to a relatively constant load. While the maximum of instantaneous wind energy is less than the load, the required level of energy can be expected from WDPP. WDPP will act as an unwanted load. Thus, in the scope of the diesel, the load will be less than the available wind energy. In addition to wind variability, the load itself can directly change over short time intervals. In much smaller systems, turning on or off any single device significantly affects their operation. When wind power exceeds the load capacity, the management of the system becomes even more complicated (Kaiser and Snyder, 2012b).

However, if the load is not always less than the energy supplied by the WDPP, the diesel should not be in the off state for a long time, because wind energy will sometimes fall to a level below the ability to provide the load. Trying to stop the diesel every time the wind power is above some level will result in having to turn the diesel on and off perhaps hundreds of times per hour. Such a strategy is not acceptable. This would lead to excessive deformation of the engine and starter motor, possibly resulting in their shorter service life (Tanasheva *et al.*, 2018). The pulse ratio could be aligned with some on and off strategy, but this would be at the expense of increased fuel consumption. In practice, the desirability of maintaining a minimum operating time also complicates the problem. It should also be noted that rapid load changes, especially in small networks, can have a similar and, at times, more severe effect than the effect of variable wind energy (Minin, 2012).

Among the autonomous power supply systems based on WDPP with asynchronous generator comprises, in addition to the facility, the control unit of generator start, battery pack with charge controller, inverter equipment, diesel-unit with a synchronous generator as a backup source, and a power matching output parameter of the system with the needs of the consumer. Depending on the production conditions, these component blocks can assemble block diagrams, working separately or in parallel to meet the needs of the consumer.

### 3. MATERIALS AND METHODS:

A simple method of integrating wind energy into a diesel system is to connect the WDPP to the grid in the same way as to a

massive power grid. Ideally, the operation of such a system is simple. When there is wind, the payload on the diesels is reduced and, if the wind force is strong enough, the diesel can be turned off altogether. When the desired significant fuel economy is achieved, the system should already contain additional components and control systems. It should be borne in mind that, in most cases, one of the key points is to reduce the overall cost of electricity production. Thus, the fuel economy should not be achieved at the expense of a significant increase in the value of the entire system (Conzalez-Rodriguez *et al.*, 2010).

It was analyzing the structural schemes of autonomous power supply systems based on the wind-diesel system and determine the dependence of the load intensity of the power supply channel of the consumer on the degree of its capacity. Let's consider the variants of structural schemes formed with such a set of components (Figure 1).

Such problems can be analyzed using graph theory. The algorithm of functioning of this system, as a set of dependencies, determines the necessary performance of a given process of providing energy to the consumer, we consider with the help of oriented graphs of these structures (Figure 2). The nodes of the graph are the structural components of the system. Ribs – energy flows between nodes, X1 – the flow of wind energy, X2 – the flow of traditional power (fuel), X3 – the flow of energy from batteries, Y1 – the flow of energy needs to the consumer.

The following mathematical methods were also used during the study: linear equations, Erlang equation, probability theory, theory of automatic control, Routh table, etc.

Transformation of energy flows in the system to meet the needs of the passing consumer: according to A (Equation 1), according to B (Equation 2), according to C (Equation 3). Provision of the consumer passes through two interconnected channels. The first is a wind power plant; the second is a diesel generator. In the absence of wind, the provision of the consumer is made at the expense of only the diesel generator. Channel selection, as well as the depth of correction, passes through the synchronization node 6 (Equation 4).

Providing the consumer passes through three interconnected channels: wind turbine, diesel generator, and stored energy in batteries. In the absence of wind, the consumer is only provided due to diesel-generator and

accumulator cells. In the case of discharge of batteries, and the lack of wind, the complete provision of electricity to the consumer is performed by a diesel generator. Channel selection, as well as correction depth, is carried out through the synchronization node 6. The matching coefficients of graph nodes k1, k2, k3, k4, k5, respectively refer to the wind power plant, diesel generator, capacitor bank, controller, storage unit, and inverter is less than 1.

#### 4. RESULTS AND DISCUSSION:

The use of an asynchronous generator in WDPP requires constant correction of energy consumption because it is susceptible to its deficit. Therefore, there is a task of organizing feedback in an autonomous wind power system, which is to ensure the production of a control signal for the restructuring of composite systems to comply with the technological requirements of the consumer. Let's suppose that the processes in this system are linear (Equation 5). Where  $z(t)$  – system input value (consumer requirements);  $y(t)$  – input value of the system (power supply level);  $A, B, C$  – system component parameters.

Then, if at some point in time the state of the system is  $\bar{x}$ , and the output value will take the value  $\bar{y}$ , the system blocks must have the following parameters  $\bar{A}, \bar{B}_1, \bar{B}_2$  and  $\bar{C}$ , under which the following conditions would be fulfilled  $x(t) - \bar{y}(t) \rightarrow 0$  (Equation 6).

Since this system uses an asynchronous generator with a short-circuited rotor winding and a synchronous generator with permanent magnets of the diesel plant and excitation control is impossible, the feedback parameters are assumed to be single. That is, it is necessary to determine the state of the system under the following condition  $x(t) - \bar{x}(t) \rightarrow 0$  (Equation 7).

This requirement is met when  $\bar{B}_1 = B$  and in the real part (Equations 8-9). The stability of the system will be ensured when the parameters of the unit  $\bar{B}_2$  (synchronizer, Figure 2) will be controlled and have the property to be observed (at any time to provide the requirements of the consumer on the load power).

Analysis of schemes a and b shows their

limitations in providing energy needs of the consumer and dependence on the presence of a certain level of wind speed. Schemes C and D are almost little dependent on fluctuations in wind energy supply over time because they have an additional source of power supply (diesel generator) and can increase the fill factor of the load graph of the consumer almost to 1.

Relative throughput  $Q$  of the system for a single channel system is as follows (Equation 10). Where  $\lambda$  – the intensity of the flow of consumer demand (applications);  $\mu$  – the power of the flow of wind energy (maintenance). The organization of two-channel and three-channel systems will increase the degree of security of the consumer. According to the Erlang's equation for a two-channel system with failures (Equation 11) and a three-channel system with failures (Equation 12). At  $Q = 50\%$ ,  $\rho = 1$ , i. e. the intensity of the flow of service and requests is equalized. Compared to a dual channel system, the relative throughput of the system provided  $\rho = 1$  is less by 30%.

Figure 3 shows the dependence of (Equation 13) for single- two- and three-channel system. The latter has an advantage over the former since the application execution devices work with less intensity. The probability that the consumer is provided from one unit will be (Equation 14). From two blocks at once (Equation 15), from three blocks (Equation 16). That is, the probability of simultaneous operation of three sources at the same load level has a smaller value than two and one source (Figure 4).

The WDPP used in stand-alone operation consists of two sources of electrical energy: a diesel generator and a wind power plant. The power of the diesel generator can be changed during the process of the plant, depending on the needs of the consumer and on the arrival of wind. The power of the wind power plant is entirely uncontrolled since it depends on the wind speed. As a rule, a wind-diesel system is built according to the following scheme: a wind turbine rotor (with or without a speed controller), an electric generator, a diesel generator (with a fuel supply regulator), an electric energy storage (an electrochemical battery) and a dc-to-dc converter (one-or three-phase industrial frequency). The study of such systems requires considering that they often operate in the mode of commensurate power with the consumer with the pulsating nature of the wind speed change. That is, in the created channel of the energy flow, the above elements of the wind-diesel power plant (WDPP)

are most often in the transition process, which requires determining the boundary of the stability of the consumer's energy supply system. In general, the block diagram of an autonomous WDPP is as follows (Figure 5).

The conversion of wind energy in an autonomous wind-diesel power system follows the standard principle: the mechanical energy of the wind flow is converted by the WDPP rotor into electrical energy by an electric generator in the same way as the mechanical energy in an internal combustion engine when burning fuel. The electrical energy is then used by the consumer either directly or through a dc-to-dc converter. It should be noted that the nature of the load of the consumer can be active (heaters, lighting), capacitive (electrochemical batteries) or active-inductive (an electric drive of technological machines with constant or variable moment of resistance). That is, the functional scheme of an autonomous wind-diesel power plant will consist of five links (Figure 6).

Input  $f_1$ , characterizing the parameters of the wind flow (speed, duration of wind speed gradations, and period of calm) and the boundaries of the zone of useful wind energy. The link output signal  $W_1$  is the input signal of the link  $W_2$  and will be equal to (Equation 17). Where  $f(v)$  – wind speed distribution density. If the value  $f_2$  gives the opportunity of entering the zone of production load,  $f_3$  signal is formed, the value of which makes it possible to ensure the implementation of the consumer's production needs (link  $W_5$ ).

Input  $f_4$  characterizes the parameters and periods of fuel supply. Link output signal  $W_3$  is the input signal of the link  $W_4$  and will characterize the mechanical power that is supplied to the rotor of the generator. If the value  $f_5$  allows entering the zone of production load,  $f_6$  signal is formed, the value of which makes it possible to ensure the implementation of the consumer's production needs (link  $W_5$ ). Thus, we have a mixed connection of five links, each of which can have internal feedback, but the whole system lacks the main feedback; that is, the system cannot affect the input signal  $f_1$  (the wind flow parameters).

Let's analyze the operation of the above system in the channel of the wind flow and the diesel engine. For this purpose, each component of an autonomous wind-diesel power plant is considered separately, considering the influence of other components. The motion equation for the rotor of a wind power plant or a rotor of a diesel engine, considering the influence of the generator link, is as follows (Equation 18). Where  $M_p$  – mechanical torque on the rotor shaft of a wind farm or diesel engine rotor;  $M_G$  – electromagnetic torque on generator shaft;  $J$  – inertial torque of rotating rotor elements;  $\omega$  – angular speed of rotation of rotor and generator shafts.

The speed of rotation of the rotor shaft may vary due to changes in wind speed, changes the amount of fuel, and as a result of load change. If the rate of rotation of the rotor shaft, as a result of the action of wind or increased fuel supply increased by  $\Delta\omega$ , this will change the resulting torque to  $\Delta M$ , then (Equation 19). As  $M_p$  and  $M_G$  are nonlinear functional speed  $\omega$ , applying Taylor series expansion within  $t=0$  and limiting ourselves to the first two members, we have as follows (Equation 20). After the substitution of equation (19) to equation (20) and replacement of (Equations 21-22), we have as follows (Equation 23).

From the theory of automatic control, it is known that the system will be stable in the case when (Equation 24) i.e.  $\frac{dM_G}{d\omega} > \frac{dM_p}{d\omega}$ . This means

that the system will remain stable if the load increases. Moreover, the rate of its rise must exceed the rate of increase in the value of the wind speed or the rate of increase in the fuel supply. If this is not possible, it is necessary to limit the speed of rotation.

As the wind speed or fuel supply decreases, the equation (23) takes the following form (Equation 25) and, after the transformation have as follows (Equation 26). Then, to keep the system stable  $\left(\frac{dM_p}{d\omega} > \frac{dM_G}{d\omega}\right)$  it is necessary to

provide a condition for reducing the load, and the rate of its decline should be higher than the rate of change in the value of the wind speed or the value of the fuel supply.

The speed of the transient process is affected by the time constant of each system block. The analysis of the processes taking place in the blocks of the functional scheme of an

autonomous wind-diesel power plant (Figure 6) will lead to a differential equation of type (23) describing the first-order aperiodic link (Equation 27). Where  $T_1, T_2, T_3, T_4, T_5$ , – the timing of relevant links and  $\xi_1, \xi_2, \xi_3, \xi_4, \xi_5$ , – coefficients of self-alignment of the corresponding links.

The total transfer function of the entire system will be as follows (Equation 28). Substituting values of transfer functions of separate links, we have as follows (Equation 29). Opening the brackets and equating the denominator of the transfer function of the system to zero, we obtain the following characteristic equation (30).

The general view of the characteristic equation will be as follows (Equations 31-32). The following coefficients can be derived from the characteristic (Equations 33-38). The stability analysis is carried out using the algebraic Raus criterion. Let's make a Raus table in which the elements are determined from the following equations (39)-(40), where  $i \geq 3$  – line number,  $k$  – column number (Table 1).

Let's substitute the coefficients and determine the values of the table elements  $r_3, r_4, r_5, r_6$  by the given formulas in algebraic form. The expression to determine the coefficient  $r_3$  is equal to (Equation 41). The expression to determine the coefficient  $r_4$  is presented as follows (Equation 42). The expression to determine the coefficient  $r_5$  is (Equation 43).

Let's find all the  $c$  coefficients from the Raus table in algebraic form: coefficient  $c_{1,1}$  is equal to  $\alpha_0$  (Equation 44); coefficient  $c_{1,2}$  is equal to  $\alpha_1$  (Equation 45); coefficient  $c_{1,3}$  is equal to the difference between  $\alpha_2$  and  $r_3\alpha_3$  (Equation 46); coefficient  $c_{1,4}$  is equal to the difference between  $\alpha_3$  and  $r_4c_{2,3}$  (Equation 47); coefficient  $c_{1,5}$  is equal to the difference between  $c_{2,3}$  and  $r_5c_{2,4}$  (Equation 48); coefficient  $c_{1,6}$  is equal to  $\alpha_5$  (Equation 49); coefficient  $c_{2,1}$  is equal to  $\alpha_2$  (Equation 50); coefficient  $c_{2,2}$  is equal to  $\alpha_3$  (Equation 51); coefficient  $c_{2,3}$  is equal to the difference between  $\alpha_4$  and  $r_3\alpha_5$

(Equation 52); coefficient  $c_{2,4}$  is equal to the difference  $\alpha_5$  (Equation 53); coefficient  $c_{3,1}$  is equal to  $\alpha_4$  (Equation 54); coefficient  $c_{3,2}$  is equal to  $\alpha_5$  (Equation 55). For the stability of a linear stationary system it is necessary and sufficient that all the coefficients of the first column of the Raus table  $c_{1,1}, c_{1,2}, c_{1,3}, c_{1,4}, c_{1,5}, c_{1,6}$  were the same sign. If this is not done, the system is unstable.

Analysis of coefficient data in general form  $(c_{1,1}, c_{1,2}, c_{1,3}, c_{1,4}, c_{1,5}, c_{1,6})$  is difficult because their algebraic expression is too significant. Therefore, the stability study will be conducted for a specific wind-diesel power system, the parameters of which, as well as the time periods, need to be determined by simulation.

Since all-time steels have a positive sign, coefficient  $c_{1,1}$  is the product of all stable times of the system; then respectively the given coefficient will be positive. Sign of coefficients  $c_{1,3}, c_{1,4}, c_{1,5}$  is complicated to analyze because algebraic expressions are very cumbersome. Therefore, it is possible to determine the sign of these coefficients only by substituting the corresponding values of time constants and coefficients  $\xi$  for a specific system.

Let's analyze the coefficient (Equation 56). If an odd value (one, three, five) is the self-adjustment coefficient  $\xi$  will have a negative sign, it is the value  $c_{1,6}$  will be negative and therefore the system is not stable. Thus, for the stability of the system, it is necessary that all coefficients  $\xi$  had a positive sign, or even value  $\xi$  had a negative sign. Let's consider each case in more detail and analyze it (Equation 57).

Coefficient  $\xi_1 < 0$  characterizes the first link (WDPP rotor) and indicates a decrease in wind speed, and, accordingly, a reduction in the generated power in the system. Based on this, the system, of course, will lose stability if the load is not reduced. When reducing the load, coefficient  $\xi_5$  will take a negative value, and the system will remain stable. When substituting values, all coefficients  $c$  will have a positive sign that indicates the stability of the system (Equation 58).

Coefficient of self-regulation  $\xi_2 < 0$  characterizes the second link (WDPP generator)

and indicates a decrease in the generated power in the system. Consequently, the system will lose stability if the load is not reduced. When reducing the load, coefficient  $\xi_5$  will take a negative value, and the system will remain stable. When substituting values, all coefficients  $c$  will have a positive sign, which indicates the stability of the system (Equation 59)

$\xi_3 < 0$  characterizes the third link (diesel generator) and shows a decrease in the speed of fuel supply, and, accordingly, a reduction in the generated power in the system. The system will lose stability if the load is not reduced. When reducing the load, coefficient  $\xi_5$  will take a negative value, and the system will remain stable. When substituting values, all the coefficients will have a positive sign, which indicates the stability of the system (Equation 60).

$\xi_4 < 0$  characterizes the fourth link (diesel generator) and indicates a decrease in the generated power in the system. Based on this, the system, of course, will lose stability if the load is not reduced. When reducing the load, coefficient  $\xi_5$  will take a negative value, and the system will remain stable. When substituting values, all the coefficients will have a positive sign, which indicates the stability of the system (Equation 61).

$\xi_5 < 0$  characterizes the fifth link (load) and indicates a decrease in the value of the load. Consequently, the system will lose stability if one does not reduce the value of the generated power. When reducing the energy generation coefficient  $\xi_2$  or  $\xi_4$  will take a negative value, and the system will remain stable. When substituting values, all coefficients  $c$  will have a positive sign that indicates the stability of the system.

Let's analyze coefficient  $c_{1,2}$  (Equation 45). Since the time intervals  $T$  have a positive value, we can say that the coefficient  $c_{1,2}$  will be equal to the sum of all self-regulation coefficients multiplied by some value. If all coefficients (Equation 62) are above zero, the system will be stable. Let's consider the case when  $\xi_1 < 0$  or  $\xi_2 < 0$ . So that the system remains sustainable, it is necessary to ensure that the rate of increase of torque on a shaft of a diesel generator (flow of fuel) is greater than the rate of reduction of torque

on a shaft of the wind turbine (wind speed). I. e. it is necessary to compensate for the decrease in wind speed by increasing the fuel supply at a constant load or to reduce the load so that the system does not lose stability.

When  $\xi_3 < 0$  or  $\xi_4 < 0$  to ensure the stability of the system, it is necessary to compensate the decrease in torque of the diesel generator (fuel) by increasing the torque of wind turbine (wind speed) and the rate of increase must be greater than the rate of changing the torque on diesel generation or by reducing the load in accordance with the change in generation from diesel-generating. When  $\xi_5 < 0$  (load link) it is possible to compensate for the replaceable torque on the shaft of the diesel generator (fuel supply) or wind generator (wind speed).

Let's consider coefficient  $c_{1,3}$  (formula 46).

All  $T$  time intervals are positive values. To simplify the analysis, let's make some assumption: since coefficients  $\xi_1$  and  $\xi_2$  as well as  $\xi_3$  and  $\xi_4$  are part of the same systems, then combine them, i.e.  $\xi_1 = \xi_2 = \xi_{1,2}$  and  $\xi_3 = \xi_4 = \xi_{3,4}$  respectively. If time intervals and coefficients are not considered, equation 39 takes the following form (Equation 63).

In the case where all the self-regulation coefficients (Equation 64) are above zero, the stability of the system requires meeting the following condition (Equation 65). For other cases and analysis of the coefficients  $c_{1,4}$  and  $c_{1,5}$  in analytical form is difficult because of the bulkiness of expressions.

## 5. CONCLUSIONS:

1. As a result, it was analyzed the structural schemes of an autonomous supply system based on a wind-diesel system, which showed that the relative capacity for a three-channel system is 30% less than for a two-channel system, i.e. a three-channel system allows increasing the degree of security of consumer requirements.

2. A mathematical model of the load modes of the wind-diesel system in the conditions of variable rotor speed of the wind turbine and load parameters based on the Raus criterion allows analyzing the nature of the dynamic processes.

3. The autonomous system must have a certain hierarchical subordination, that is, the electrical supply is carried out first from the units that convert wind energy, and only in the case of a decrease in the level of production or a decrease to critical values, a diesel plant is used. If we consider the operation of this system in terms of the queuing theory, it is necessary that the flow of customer requirements was stationary, ordinary, and had no consequences.

4. To study the stability of the system, it is necessary to operate with numerical values. The simulation model will calculate, if necessary, the time constants and coefficients of self-alignment of all the system parts and use them in the study of stability by the Raus criterion.

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$$\left\{ \begin{array}{l} X1 \rightarrow k1X1 \\ X1 > Y1, \\ Y1 \rightarrow 0 \\ X1 \rightarrow 0 \end{array} \right. \quad (\text{Eq. 1})$$

$$\left\{ \begin{array}{l} X1 + X2 \rightarrow k1X1k2X2 \\ X1 + X2 > Y1, \\ Y1 \rightarrow X2 \\ X1 = 0 \end{array} \right. \quad (\text{Eq. 2})$$

$$\left\{ \begin{array}{l} X1 + X3 \rightarrow k1k3k4k5X1 \\ X1 + X3 > Y1, \\ Y1 \rightarrow X3 \\ X1 = 0 \end{array} \right. \quad (\text{Eq. 3})$$

$$\left\{ \begin{array}{l} X1 + X2 \rightarrow k1k3k4k5X1 + k2X2 \\ X1 + X2 > Y1, \\ Y1 \rightarrow X2 + X \\ 1 = 0, \\ Y1 \rightarrow X2 \\ X1 + X3 = 0. \end{array} \right. \quad (\text{Eq. 4})$$

$$\left\{ \begin{array}{l} \frac{dx}{dt} = Ax + Bz, \\ y = Cx \end{array} \right. \quad (\text{Eq. 5})$$

$$\left\{ \begin{array}{l} \overline{\frac{dx}{dt}} = \overline{Ax} + \overline{B_1z} + \overline{B_2y}, \\ \overline{y} = \overline{Cx} \end{array} \right. \quad (\text{Eq. 6})$$

$$\frac{d[\bar{x}(t) - x(t)]}{dt} = \bar{A}x - Ax + (\bar{B}_1 - B)z + B_2y = \bar{A}x - Ax + \bar{B}_2Cx + (\bar{B}_1 - B)z \rightarrow 0 \quad (\text{Eq. 7})$$

$$\frac{d[\bar{x}(t) - x(t)]}{dt} < 0 \quad (\text{Eq. 8})$$

$$\frac{d[\bar{x}(t) - x(t)]}{dt} = (A - \bar{B}_2C)[\bar{x}(t) - x(t)] \quad (\text{Eq. 9})$$

$$Q = \frac{\mu}{\lambda + \mu} \quad (\text{Eq. 10})$$

$$Q = 1 - \frac{\rho^2}{2!} \frac{1}{\left(1 + \rho + \frac{\rho^2}{2}\right)} \quad (\text{Eq. 11})$$

$$Q = 1 - \frac{\rho^3}{3!} \frac{1}{1 + \rho + \frac{\rho^2}{2!} + \frac{\rho^3}{3!}} \quad (\text{Eq. 12})$$

$$\frac{\lambda}{\mu} = f(Q) \quad (\text{Eq. 13})$$

$$p_1 = \frac{\alpha}{1 + \alpha} \quad (\text{Eq. 14})$$

$$p_2 = \frac{\frac{1}{2!}\alpha^2}{1 + \alpha + \frac{1}{2!}\alpha^2} = \frac{\alpha^2}{1 + (1 + \alpha)^2} \quad (\text{Eq. 15})$$

$$p_3 = \frac{\frac{1}{3!}\alpha^3}{1 + \alpha + \frac{1}{2!}\alpha^2 + \frac{1}{3!}\alpha^3} = \frac{\frac{\alpha^3}{6}}{1 + \alpha + \frac{1}{2}\alpha^2 + \frac{\alpha^3}{6}} \quad (\text{Eq. 16})$$

$$f_2 = \int_0^{\infty} v^3 f(v) dv \quad (\text{Eq. 17})$$

$$j \frac{d\omega}{dt} = M_P - M_G \quad (\text{Eq. 18})$$

$$j \frac{d(\omega_0 + \Delta\omega)}{dt} = M_P - M_G + \Delta M \quad (\text{Eq. 19})$$

$$\begin{cases} M_P(\omega) = M_{P_0} + \left( \frac{dM_P}{d\omega} \right)_0 \Delta\omega + \Delta M \\ M_G(\omega) = M_{G_0} + \left( \frac{dM_G}{d\omega} \right)_0 \Delta\omega \end{cases} \quad (\text{Eq. 20})$$

$$\varphi = \frac{\Delta\omega}{\omega_0} \quad (\text{Eq. 21})$$

$$\mu = \frac{\Delta M}{M_0} \quad (\text{Eq. 22})$$

$$j \frac{\omega_0}{M_0} \frac{d\varphi}{dt} + \varphi \frac{\omega_0}{M_0} \left\{ \left( \frac{dM_G}{d\omega} \right)_0 - \left( \frac{dM_P}{d\omega} \right)_0 \right\} = \mu \quad (\text{Eq. 23})$$

$$\xi = \frac{\omega_0}{M_0} \left\{ \left( \frac{dM_G}{d\omega} \right)_0 - \left( \frac{dM_P}{d\omega} \right)_0 \right\} > 0 \quad (\text{Eq. 24})$$

$$j \frac{d(\omega_0 - \Delta\omega)}{dt} = M_P - M_G - \Delta M \quad (\text{Eq. 25})$$

$$j \frac{\omega_0}{M_0} \frac{d\varphi}{dt} + \varphi \frac{\omega_0}{M_0} \left\{ \left( \frac{dM_P}{d\omega} \right)_0 - \left( \frac{dM_G}{d\omega} \right)_0 \right\} = \mu \quad (\text{Eq. 26})$$

$$W_1 = \frac{k_1}{T_1 p + \xi_1}$$

$$W_2 = \frac{k_2}{T_2 p + \xi_2}$$

$$W_3 = \frac{k_3}{T_3 p + \xi_3} \quad (\text{Eq. 27})$$

$$W_4 = \frac{k_4}{T_4 p + \xi_4}$$

$$W_5 = \frac{k_5}{T_5 p + \xi_5}$$

$$W = (W_1 W_2 W_3 W_4) W_5 \quad (\text{Eq. 28})$$

$$W = \left( \frac{k_1}{T_1 p + \xi_1} \frac{k_2}{T_2 p + \xi_2} + \frac{k_3}{T_3 p + \xi_3} \frac{k_4}{T_4 p + \xi_4} \right) \frac{k_5}{T_5 p + \xi_5} \quad (\text{Eq. 29})$$

$$(T_1 p + \xi_1)(T_2 p + \xi_2)(T_3 p + \xi_3)(T_4 p + \xi_4)(T_5 p + \xi_5) = 0 \quad (\text{Eq. 30})$$

$$\alpha_0 p^5 + \alpha_1 p^4 + \alpha_2 p^3 + \alpha_3 p^2 + \alpha_4 p + \alpha_5 = 0 \quad (\text{Eq. 31})$$

$$\begin{aligned} & (T_1 T_2 T_3 T_4 T_5) p^5 + (\xi_5 T_1 T_2 T_3 T_4 + \xi_4 T_1 T_2 T_3 T_5 + \xi_3 T_1 T_2 T_4 T_5 + \\ & + \xi_2 T_1 T_3 T_4 T_5 + \xi_1 T_2 T_3 T_4 T_5) p^4 + (\xi_4 \xi_5 T_1 T_2 T_3 + \xi_3 \xi_5 T_1 T_2 T_4 + \\ & + \xi_2 \xi_5 T_1 T_3 T_4 + \xi_1 \xi_5 T_2 T_3 T_4 + \xi_3 \xi_4 T_1 T_2 T_5 + \xi_2 \xi_4 T_1 T_3 T_5 + \\ & + \xi_1 \xi_4 T_2 T_3 T_5 + \xi_2 \xi_3 T_1 T_4 T_5 + \xi_1 \xi_3 T_2 T_4 T_5 + \xi_1 \xi_2 T_3 T_4 T_5) p^3 + \\ & + (\xi_3 \xi_4 \xi_5 T_1 T_2 + \xi_2 \xi_4 \xi_5 T_1 T_3 + \xi_1 \xi_4 \xi_5 T_2 T_3 + \xi_2 \xi_3 \xi_5 T_1 T_4 + \\ & + \xi_1 \xi_3 \xi_5 T_2 T_4 + \xi_1 \xi_2 \xi_5 T_3 T_4 + \xi_2 \xi_3 \xi_4 T_1 T_5 + \xi_1 \xi_3 \xi_4 T_2 T_5 + \\ & + \xi_1 \xi_2 \xi_4 T_3 T_5) p^2 + (\xi_2 \xi_3 \xi_4 \xi_5 T_1 + \xi_1 \xi_3 \xi_4 \xi_5 T_2 + \xi_1 \xi_2 \xi_4 \xi_5 T_3 + \\ & + \xi_1 \xi_2 \xi_3 \xi_5 T_4 + \xi_1 \xi_2 \xi_3 \xi_4 T_5) p + \xi_1 \xi_2 \xi_3 \xi_4 + \xi_5 = 0 \end{aligned} \quad (\text{Eq. 32})$$

$$\alpha_0 = T_1 T_2 T_3 T_4 T_5 \quad (\text{Eq. 33})$$

$$\alpha_1 = \xi_5 T_1 T_2 T_3 T_4 + \xi_4 T_1 T_2 T_3 T_5 + \xi_3 T_1 T_2 T_4 T_5 + \xi_2 T_1 T_3 T_4 T_5 + \xi_1 T_2 T_3 T_4 T_5 \quad (\text{Eq. 34})$$

$$\begin{aligned} \alpha_2 = & \xi_4 \xi_5 T_1 T_2 T_3 + \xi_3 \xi_5 T_1 T_2 T_4 + \xi_2 \xi_5 T_1 T_3 T_4 + \xi_1 \xi_5 T_2 T_3 T_4 + \xi_3 \xi_4 T_1 T_2 T_5 + \\ & + \xi_2 \xi_4 T_1 T_3 T_5 + \xi_1 \xi_4 T_2 T_3 T_5 + \xi_2 \xi_3 T_1 T_4 T_5 + \xi_1 \xi_3 T_2 T_4 T_5 + \xi_1 \xi_2 T_3 T_4 T_5 \end{aligned} \quad (\text{Eq. 35})$$

$$\begin{aligned} \alpha_3 = & \xi_3 \xi_4 \xi_5 T_1 T_2 + \xi_2 \xi_4 \xi_5 T_1 T_3 + \xi_1 \xi_4 \xi_5 T_2 T_3 + \xi_2 \xi_3 \xi_5 T_1 T_4 + \xi_1 \xi_3 \xi_5 T_2 T_4 + \\ & + \xi_1 \xi_2 \xi_5 T_3 T_4 + \xi_2 \xi_3 \xi_4 T_1 T_5 + \xi_1 \xi_3 \xi_4 T_2 T_5 + \xi_1 \xi_2 \xi_4 T_3 T_5 \end{aligned} \quad (\text{Eq. 36})$$

$$\alpha_4 = \xi_2 \xi_3 \xi_4 \xi_5 T_1 + \xi_1 \xi_3 \xi_4 \xi_5 T_2 + \xi_1 \xi_2 \xi_4 \xi_5 T_3 + \xi_1 \xi_2 \xi_3 \xi_5 T_4 + \xi_1 \xi_2 \xi_3 4 T_5 \quad (\text{Eq. 37})$$

$$\alpha_5 = \xi_1 \xi_2 \xi_3 \xi_4 \xi_5 \quad (\text{Eq. 38})$$

$$r_i = \frac{c_{1,i-2}}{c_{1,i-1}} \quad (\text{Eq. 39})$$

$$c_{k,l} = c_{k+1,i-2} - r_i c_{k+1,i-1} \quad (\text{Eq. 40})$$

$$r_3 = \frac{(T_1 T_2 T_3 T_4 T_5)}{(T_2 T_3 T_4 T_5 \xi_1 + T_1 T_3 T_4 T_5 \xi_2 + T_1 T_2 T_4 T_5 \xi_3 + T_1 T_2 T_3 T_5 \xi_4 + T_1 T_2 T_3 T_4 \xi_5)} \quad (\text{Eq. 41})$$

$$r_4 = (T_2T_3T_4T_5\xi_1 + T_1T_3T_4T_5\xi_2 + T_1T_2T_4T_5\xi_3 + T_1T_2T_3T_5\xi_4 + T_1T_2T_3T_4\xi_5) /$$

$$/ (T_3T_4T_5\xi_1\xi_2 + T_2T_4T_5\xi_1\xi_3 + T_1T_4T_5\xi_2\xi_3 + T_2T_3T_5\xi_1\xi_4 + T_1T_3T_5\xi_2\xi_4 +$$

$$+ T_1T_2T_5\xi_3\xi_4 + T_2T_3T_4\xi_1\xi_5 + T_1T_3T_4\xi_2\xi_5 + T_1T_2T_4\xi_3\xi_5 + T_1T_2T_3\xi_4\xi_5 -$$

$$- (T_1T_2T_3T_4T_5(T_4T_5\xi_1\xi_2\xi_3 + T_3T_5\xi_1\xi_2\xi_4 + T_2T_5\xi_1\xi_3\xi_4 + T_1T_5\xi_2\xi_3\xi_4 +$$

$$+ T_3T_4\xi_1\xi_2\xi_5 + T_2T_4\xi_1\xi_3\xi_5 + T_1T_4\xi_2\xi_3\xi_5 + T_2T_3\xi_1\xi_4\xi_5 + T_1T_3\xi_2\xi_4\xi_5 +$$

$$+ T_1T_2\xi_3\xi_4\xi_5)) / (T_2T_3T_4T_5\xi_1 + T_1T_3T_4T_5\xi_2 + T_1T_2T_4T_5\xi_3 + T_1T_2T_3T_5\xi_4 +$$

$$+ T_1T_2T_3T_4\xi_5)) \quad (\text{Eq. 42})$$

$$r_5 = (T_3T_4T_5\xi_1\xi_2 + T_2T_4T_5\xi_1\xi_3 + T_1T_4T_5\xi_2\xi_3 + T_2T_3T_5\xi_1\xi_4 + T_1T_3T_5\xi_2\xi_4 +$$

$$+ T_1T_2T_5\xi_3\xi_4 + T_2T_3T_4\xi_1\xi_5 + T_1T_3T_4\xi_2\xi_5 + T_1T_2T_4\xi_3\xi_5 + T_1T_2T_3\xi_4\xi_5 -$$

$$- (T_1T_2T_3T_4T_5(T_4T_5\xi_1\xi_2\xi_3 + T_3T_5\xi_1\xi_2\xi_4 + T_2T_5\xi_1\xi_3\xi_4 + T_1T_5\xi_2\xi_3\xi_4 +$$

$$+ T_3T_4\xi_1\xi_2\xi_5 + T_2T_4\xi_1\xi_3\xi_5 + T_1T_4\xi_2\xi_3\xi_5 + T_2T_3\xi_1\xi_4\xi_5 + T_1T_3\xi_2\xi_4\xi_5 +$$

$$+ T_1T_2\xi_3\xi_4\xi_5)) / (T_2T_3T_4T_5\xi_1 + T_1T_3T_4T_5\xi_2 + T_1T_2T_4T_5\xi_3 + T_1T_2T_3T_5\xi_4 +$$

$$+ T_1T_2T_3T_4\xi_5)) / (T_4T_5\xi_1\xi_2\xi_3 + T_3T_5\xi_1\xi_2\xi_4 + T_2T_5\xi_1\xi_3\xi_4 + T_1T_5\xi_2\xi_3\xi_4 +$$

$$+ T_3T_4\xi_1\xi_2\xi_5 + T_2T_4\xi_1\xi_3\xi_5 + T_1T_4\xi_2\xi_3\xi_5 + T_2T_3\xi_1\xi_4\xi_5 + T_1T_3\xi_2\xi_4\xi_5 +$$

$$+ T_1T_2\xi_3\xi_4\xi_5)) / (T_2T_3T_4T_5\xi_1 + T_1T_3T_4T_5\xi_2 + T_1T_2T_4T_5\xi_3 + T_1T_2T_3T_5\xi_4 +$$

$$+ T_1T_2T_3T_4\xi_5)) / (T_4T_5\xi_1\xi_2\xi_3 + T_3T_5\xi_1\xi_2\xi_4 + T_2T_5\xi_1\xi_3\xi_4 + T_1T_5\xi_2\xi_3\xi_4 +$$

$$+ T_3T_4\xi_1\xi_2\xi_5 + T_2T_4\xi_1\xi_3\xi_5 + T_1T_4\xi_2\xi_3\xi_5 + T_2T_3\xi_1\xi_4\xi_5 + T_1T_3\xi_2\xi_4\xi_5 +$$

$$+ T_1T_2\xi_3\xi_4\xi_5 - ((T_2T_3T_4T_5\xi_1 + T_1T_3T_4T_5\xi_2 + T_1T_2T_4T_5\xi_3 + T_1T_2T_3T_5\xi_4 +$$

$$+ T_1T_2T_3T_4\xi_5)) (T_5\xi_1\xi_2\xi_3\xi_4 + T_4\xi_1\xi_2\xi_3\xi_5 + T_3\xi_1\xi_2\xi_4\xi_5 + T_2\xi_1\xi_3\xi_4\xi_5 +$$

$$+ T_1\xi_2\xi_3\xi_4\xi_5 - (T_1T_2T_3T_4T_5\xi_1\xi_2\xi_3\xi_4\xi_5)) / (T_2T_3T_4T_5\xi_1 + T_1T_3T_4T_5\xi_2 +$$

$$+ T_1T_2T_4T_5\xi_3 + T_1T_2T_3T_5\xi_4 + T_1T_2T_3T_4\xi_5)) / (T_3T_4T_5\xi_1\xi_2 + T_2T_4T_5\xi_1\xi_3 +$$

$$+ T_1T_4T_5\xi_2\xi_3 + T_2T_3T_5\xi_1\xi_4 + T_1T_3T_5\xi_2\xi_4 + T_1T_2T_5\xi_3\xi_4 + T_2T_3T_4\xi_1\xi_5 +$$

$$+ T_1T_3T_4\xi_2\xi_5 + T_1T_2T_4\xi_3\xi_5 + T_1T_2T_3\xi_4\xi_5 - (T_1T_2T_3T_4T_5(T_4T_5\xi_1\xi_2\xi_3 +$$

$$+ T_3T_5\xi_1\xi_2\xi_4 + T_2T_5\xi_1\xi_3\xi_4 + T_3T_4\xi_1\xi_2\xi_5 + T_2T_4\xi_1\xi_3\xi_5 +$$

$$+ T_1T_4\xi_2\xi_3\xi_5 + T_2T_3\xi_1\xi_4\xi_5 + T_1T_3\xi_2\xi_4\xi_5 + T_1T_2\xi_3\xi_4\xi_5)) / T_2T_3T_4T_5T_1 +$$

$$+ T_1T_3T_4T_5\xi_2 + T_1T_2T_4T_5\xi_3 + T_1T_2T_3T_5\xi_4 + T_1T_2T_3T_4\xi_5)) \quad (\text{Eq. 43})$$

$$c_{1,1} = T_1T_2T_3T_4T_5 \quad (\text{Eq. 44})$$

$$c_{1,2} = \xi_5T_1T_2T_3T_4 + \xi_4T_1T_2T_3T_5 + \xi_3T_1T_2T_4T_5 + \xi_2T_1T_3T_4T_5 + \xi_1T_2T_3T_4T_5 \quad (\text{Eq. 45})$$

$$\begin{aligned}
c_{1,3} = & T_3 T_4 T_5 \xi_1 \xi_2 + T_2 T_4 T_5 \xi_1 \xi_3 + T_1 T_4 T_5 \xi_2 \xi_3 + T_2 T_3 T_5 \xi_1 \xi_4 + T_1 T_3 T_5 \xi_2 \xi_4 + \\
& + T_1 T_2 T_5 T_3 T_4 + T_2 T_3 T_4 \xi_1 \xi_5 + T_1 T_3 T_4 \xi_2 \xi_5 + T_1 T_2 T_4 \xi_3 \xi_5 + T_1 T_2 T_3 \xi_4 \xi_5 - \\
& - (T_1 T_2 T_3 T_4 T_5 - (T_4 T_5 \xi_1 \xi_2 \xi_3 + T_2 T_3 T_4 \xi_1 \xi_5 + T_1 T_3 T_4 \xi_2 \xi_5 + T_1 T_2 T_4 \xi_3 \xi_5 + \\
& + T_1 T_2 T_3 \xi_4 \xi_5 - (T_1 T_2 T_3 T_4 T_5 - (T_4 T_5 \xi_1 \xi_2 \xi_3 + T_3 T_5 \xi_1 \xi_2 \xi_4 + T_2 T_5 \xi_1 \xi_3 \xi_4 + \\
& + T_1 T_5 \xi_2 \xi_3 \xi_4 + T_3 T_4 \xi_1 \xi_2 \xi_5 + T_2 T_4 \xi_1 \xi_3 \xi_5 + T_1 T_4 \xi_2 \xi_3 \xi_5 + T_2 T_3 \xi_1 \xi_4 \xi_5 + \\
& + T_1 T_3 \xi_2 \xi_4 \xi_5 + T_1 T_2 \xi_3 \xi_4 \xi_5))) / (T_2 T_3 T_4 T_5 \xi_1 + T_1 T_3 T_4 T_5 \xi_2 + T_1 T_2 T_4 T_5 \xi_3 + \\
& + T_1 T_2 T_3 T_5 \xi_4 + T_1 T_2 T_3 T_4 \xi_5)
\end{aligned}
\tag{Eq. 46}$$

$$\begin{aligned}
c_{1,4} = & T_4 T_5 \xi_1 \xi_2 \xi_3 + T_3 T_5 \xi_1 \xi_2 \xi_4 + T_2 T_5 \xi_1 \xi_3 \xi_4 + T_1 T_5 \xi_2 \xi_3 \xi_4 + T_3 T_4 \xi_1 \xi_2 \xi_5 + T_2 T_4 \xi_1 \xi_3 \xi_5 + \\
& + T_1 T_4 \xi_2 \xi_3 \xi_5 + T_2 T_3 \xi_1 \xi_4 \xi_5 + T_1 T_3 \xi_2 \xi_4 \xi_5 + T_1 T_2 \xi_3 \xi_4 \xi_5 - ((T_2 T_3 T_4 T_5 \xi_1 + T_1 T_3 T_4 T_5 \xi_2 + \\
& + T_1 T_2 T_4 T_5 \xi_3 + T_1 T_2 T_3 T_5 \xi_4 + T_1 T_2 T_3 T_4 \xi_5)(T_5 \xi_1 \xi_2 \xi_3 \xi_4 + T_4 \xi_1 \xi_2 \xi_3 \xi_5 + T_3 \xi_1 \xi_2 \xi_4 \xi_5 + \\
& T_2 \xi_1 \xi_2 \xi_4 \xi_5 + T_1 \xi_2 \xi_3 \xi_4 \xi_5 - (T_1 T_2 T_3 T_4 T_5 \xi_1 \xi_2 \xi_3 \xi_4 \xi_5)) / (T_2 T_3 T_4 T_5 \xi_1 + T_1 T_3 T_4 T_5 \xi_2 + \\
& + T_1 T_2 T_4 T_5 \xi_3 + T_1 T_2 T_3 T_5 \xi_4 + T_1 T_2 T_3 T_4 \xi_5))) / (T_3 T_4 \xi_5 \xi_1 \xi_2 + T_2 T_4 \xi_5 \xi_1 \xi_3 + T_1 T_4 \xi_5 \xi_2 \xi_3 + \\
& + T_2 T_3 \xi_5 \xi_1 \xi_4 + T_1 T_3 \xi_5 \xi_2 \xi_4 + T_1 T_2 \xi_5 \xi_3 \xi_4 + T_2 T_3 \xi_4 \xi_1 \xi_5 + T_1 T_3 \xi_4 \xi_2 \xi_5 + T_1 T_2 \xi_4 \xi_3 \xi_5 + \\
& + T_1 T_2 \xi_3 \xi_4 \xi_5 - (T_1 T_2 T_3 T_4 T_5 (T_4 T_5 \xi_1 \xi_2 \xi_3 + T_3 T_5 \xi_1 \xi_2 \xi_4 + T_2 T_5 \xi_1 \xi_3 \xi_4 + T_1 T_5 \xi_2 \xi_3 \xi_4 + \\
& + T_3 T_4 \xi_1 \xi_2 \xi_5 + T_2 T_4 \xi_1 \xi_3 \xi_5 + T_1 T_4 \xi_2 \xi_3 \xi_5 + T_2 T_3 \xi_1 \xi_4 \xi_5 + T_1 T_3 \xi_2 \xi_4 \xi_5 + T_1 T_2 \xi_3 \xi_4 \xi_5))) / \\
& (T_2 T_3 T_4 T_5 \xi_1 + T_1 T_3 T_4 T_5 \xi_2 + T_1 T_2 T_4 T_5 \xi_3 + T_1 T_2 T_3 T_5 \xi_4 + T_1 T_2 T_3 T_4 \xi_5)
\end{aligned}
\tag{Eq. 47}$$

$$\begin{aligned}
c_{1,5} = & T_5 \xi_1 \xi_2 \xi_3 \xi_4 + T_4 \xi_1 \xi_2 \xi_3 \xi_5 + T_3 \xi_1 \xi_2 \xi_4 \xi_5 + T_2 \xi_1 \xi_3 \xi_4 \xi_5 + T_1 \xi_2 \xi_3 \xi_4 \xi_5 - \\
& - (TTT T T_5 \xi_1 \xi_2 \xi_3 \xi_4 \xi_5) / (T_2 T_3 T_4 T_5 \xi_1 + T_1 T_3 T_4 T_5 \xi_2 + T_1 T_2 T_4 T_5 \xi_3 + \\
& + T_1 T_2 T_3 T_5 \xi_4 + T_1 T_2 T_3 T_4 \xi_5) - (\xi_1 \xi_2 \xi_3 \xi_4 \xi_5 (T_3 T_4 T_5 \xi_1 \xi_2 + T_2 T_4 T_5 \xi_1 \xi_3 + \\
& + T_1 T_4 T_5 \xi_2 \xi_3 + T_2 T_3 T_5 \xi_1 \xi_4 + T_1 T_3 T_5 \xi_2 \xi_4 + T_1 T_2 T_5 \xi_3 \xi_4 + T_2 T_3 T_4 \xi_1 \xi_5 + \\
& + T_1 T_3 T_4 \xi_2 \xi_5 + T_1 T_2 T_4 \xi_3 \xi_5 + T_1 T_2 T_3 \xi_4 \xi_5 - (T_1 T_2 T_3 T_4 T_5 (T_4 T_5 \xi_1 \xi_2 \xi_3 + \\
& + T_3 T_5 \xi_1 \xi_2 \xi_4 + T_2 T_5 \xi_1 \xi_3 \xi_4 + T_1 T_5 \xi_2 \xi_3 \xi_4 + T_3 T_4 \xi_1 \xi_2 \xi_5 + T_2 T_4 \xi_1 \xi_3 \xi_5 + \\
& + T_1 T_4 \xi_2 \xi_3 \xi_5 + T_2 T_3 \xi_1 \xi_4 \xi_5 + T_1 T_3 \xi_2 \xi_4 \xi_5 + T_1 T_2 \xi_3 \xi_4 \xi_5)) / (T_2 T_3 T_4 T_5 \xi_1 + \\
& + T_1 T_3 T_4 T_5 \xi_2 + T_1 T_2 T_4 T_5 \xi_3 + T_1 T_2 T_3 T_5 \xi_4 + T_1 T_2 T_3 T_4 \xi_5))) / (T_4 T_5 \xi_1 \xi_2 \xi_3 + \\
& + T_3 T_5 \xi_1 \xi_2 \xi_4 + T_2 T_5 \xi_1 \xi_3 \xi_4 + T_1 T_5 \xi_2 \xi_3 \xi_4 + T_3 T_4 \xi_1 \xi_2 \xi_5 + T_2 T_4 \xi_1 \xi_3 \xi_5 + \\
& + T_1 T_4 \xi_2 \xi_3 \xi_5 + T_2 T_3 \xi_1 \xi_4 \xi_5 + T_1 T_3 \xi_2 \xi_4 \xi_5 + T_1 T_2 \xi_3 \xi_4 \xi_5 - ((T_2 T_3 T_4 T_5 \xi_1 + \\
& + T_1 T_3 T_4 T_5 \xi_2 + T_1 T_2 T_4 T_5 \xi_3 + T_1 T_2 T_3 T_5 \xi_4 + T_1 T_2 T_3 T_4 \xi_5) (T_5 \xi_1 \xi_2 \xi_3 \xi_4 + \\
& + T_4 \xi_1 \xi_2 \xi_3 \xi_5 + T_3 \xi_1 \xi_2 \xi_4 \xi_5 + T_2 \xi_1 \xi_2 \xi_4 \xi_5 + T_1 \xi_2 \xi_3 \xi_4 \xi_5 - \\
& - (T_1 T_2 T_3 T_4 T_5 \xi_1 \xi_2 \xi_3 \xi_4 \xi_5) / (T_2 T_3 T_4 T_5 \xi_1 + T_1 T_3 T_4 T_5 \xi_2 + T_1 T_2 T_4 T_5 \xi_3 + \\
& + T_1 T_2 T_3 T_5 \xi_4 + T_1 T_2 T_3 T_4 \xi_5))) / (T_3 T_4 T_5 \xi_1 \xi_2 + T_2 T_4 T_5 \xi_1 \xi_3 + T_1 T_4 T_5 \xi_2 \xi_3 + \\
& + T_2 T_3 T_5 \xi_1 \xi_4 + T_1 T_3 T_5 \xi_2 \xi_4 + T_1 T_2 T_5 \xi_3 \xi_4 + T_2 T_3 T_4 \xi_1 \xi_5 + T_1 T_3 T_4 \xi_2 \xi_5 + \\
& + T_1 T_2 T_4 \xi_3 \xi_5 + T_1 T_2 T_3 \xi_4 \xi_5 - (T_1 T_2 T_3 T_4 T_5 (T_4 T_5 \xi_1 \xi_2 \xi_3 + T_3 T_5 \xi_1 \xi_2 \xi_4 + \\
& + T_2 T_5 \xi_1 \xi_3 \xi_4 + T_1 T_5 \xi_2 \xi_3 \xi_4 + T_3 T_4 \xi_1 \xi_2 \xi_5 + T_2 T_4 \xi_1 \xi_3 \xi_5 + T_1 T_4 \xi_2 \xi_3 \xi_5 + \\
& + T_2 T_3 \xi_1 \xi_4 \xi_5 + T_1 T_3 \xi_2 \xi_4 \xi_5 + T_1 T_2 \xi_3 \xi_4 \xi_5)) / (T_2 T_3 T_4 T_5 \xi_1 + T_1 T_3 T_4 T_5 \xi_2 + \\
& + T_1 T_2 T_4 T_5 \xi_3 + T_1 T_2 T_3 T_5 \xi_4 + T_1 T_2 T_3 T_4 \xi_5)))
\end{aligned}$$

(Eq. 48)

$$c_{1,6} = \xi_1 \xi_2 \xi_3 \xi_4 \xi_5 \quad (\text{Eq. 49})$$

$$\begin{aligned}
c_{2,1} = & \xi_4 \xi_5 T_1 T_2 T_3 + \xi_3 \xi_5 T_1 T_2 T_4 + \xi_2 \xi_5 T_1 T_3 T_4 + \xi_1 \xi_5 T_2 T_3 T_4 + \xi_3 \xi_4 T_1 T_2 T_5 + \\
& + \xi_2 \xi_4 T_1 T_3 T_5 + \xi_1 \xi_4 T_2 T_3 T_5 + \xi_2 \xi_3 T_1 T_4 T_5 + \xi_1 \xi_3 T_2 T_4 T_5 + \xi_1 \xi_2 T_3 T_4 T_5
\end{aligned}$$

(Eq. 50)

$$\begin{aligned}
c_{2,2} = & \xi_3 \xi_4 \xi_5 T_1 T_2 + \xi_2 \xi_4 \xi_5 T_1 T_3 + \xi_1 \xi_4 \xi_5 T_2 T_3 + \xi_2 \xi_3 \xi_5 T_1 T_4 + \xi_1 \xi_3 \xi_5 T_2 T_4 + \\
& + \xi_1 \xi_2 \xi_5 T_3 T_4 + \xi_2 \xi_3 \xi_4 T_1 T_5 + \xi_1 \xi_3 \xi_4 T_2 T_5 + \xi_1 \xi_2 \xi_4 T_3 T_5
\end{aligned}$$

(Eq. 51)

$$\begin{aligned}
c_{2,3} = & T_5 \xi_1 \xi_2 \xi_3 \xi_4 + T_4 \xi_1 \xi_2 \xi_3 \xi_5 + T_3 \xi_1 \xi_2 \xi_4 \xi_5 + T_2 \xi_1 \xi_3 \xi_4 \xi_5 + T_1 \xi_2 \xi_3 \xi_4 \xi_5 - \\
& - (T_1 T_2 T_3 T_4 T_5 \xi_1 \xi_2 \xi_3 \xi_4 \xi_5) / (T_2 T_3 T_4 T_5 \xi_1 + T_1 T_3 T_4 T_5 \xi_2 + T_1 T_2 T_4 T_5 \xi_3 + \\
& + T_1 T_2 T_3 T_5 \xi_4 + T_1 T_2 T_3 T_4 \xi_5)
\end{aligned}$$

(Eq. 52)

$$c_{2,4} = \xi_1 \xi_2 \xi_3 \xi_4 \xi_5$$

$$c_{2,5} = 0$$

$$c_{2,6} = 0$$

(Eq. 53)

$$c_{3,1} = \xi_2 \xi_3 \xi_4 \xi_5 T_1 + \xi_1 \xi_3 \xi_4 \xi_5 T_2 + \xi_1 \xi_2 \xi_4 \xi_5 T_3 + \xi_1 \xi_2 \xi_3 \xi_5 T_4 + \xi_1 \xi_2 \xi_3 \xi_4 T$$
(Eq. 54)

$$c_{3,2} = \xi_1 \xi_2 \xi_3 \xi_4 \xi_5$$

$$c_{3,3} = 0$$

$$c_{3,4} = 0$$

$$c_{3,5} = 0$$

$$c_{3,6} = 0;$$

(Eq. 55)

$$c_{1,6} = \xi_1 \xi_2 \xi_3 \xi_4 \xi_5$$

(Eq. 56)

$$\xi_1 \langle 0; \xi_2 \rangle 0$$

$$\xi_3 > 0$$

$$\xi_4 > 0$$

$$\xi_5 > 0$$

$$\xi_1 > 0$$

$$\xi_2 \langle 0; \xi_3 \rangle 0$$

$$\xi_4 > 0$$

$$\xi_5 > 0$$

$$\xi_1 > 0$$

$$\xi_2 > 0$$

$$\xi_3 \langle 0; \xi_4 \rangle 0$$

$$\xi_5 > 0$$

$$\xi_1 > 0$$

$$\xi_2 > 0$$

$$\xi_3 > 0$$

$$\xi_4 \langle 0; \xi_5 \rangle 0$$

$$\xi_1 > 0$$

$$\xi_2 > 0$$

$$\xi_3 > 0$$

$$\xi_4 > 0$$

$$\xi_5 < 0$$

(Eq. 57)

(Eq. 58)

(Eq. 59)

(Eq. 60)

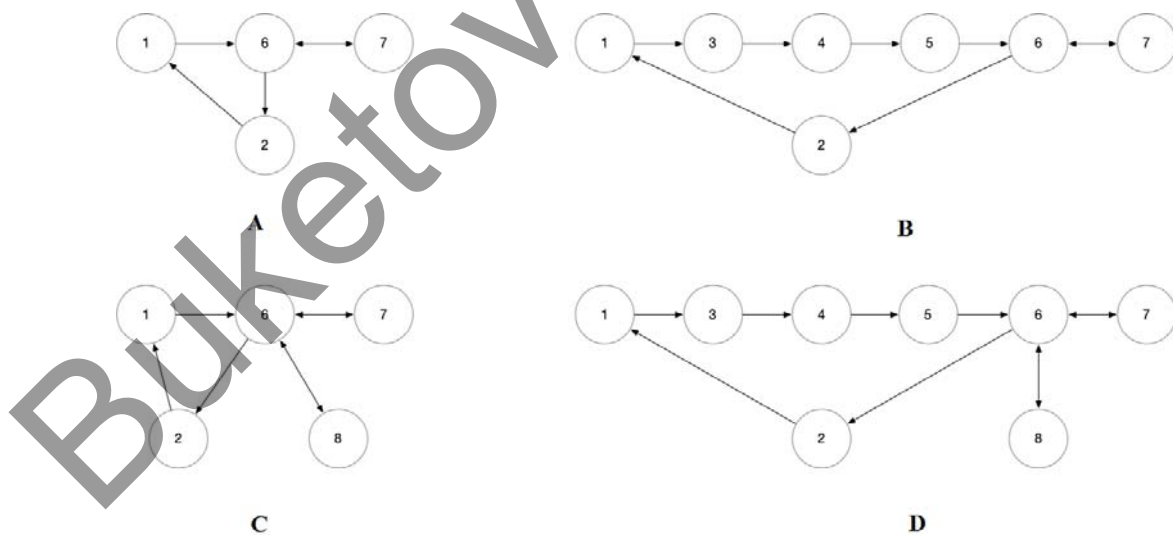
(Eq. 61)

$$\begin{aligned}
 \xi_1 &> 0 \\
 \xi_2 &> 0 \\
 \xi_3 &> 0 \\
 \xi_4 &> 0 \\
 \xi_5 &> 0
 \end{aligned}
 \tag{Eq. 62}$$

$$\xi_{1,2} + \xi_{1,2}\xi_{3,4} + \xi_{1,2}\xi_5 + \xi_{3,4}\xi_5 - \frac{(\xi_{1,2}^2(\xi_{3,4} + \xi_5) + \xi_{3,4}^2(\xi_{1,2} + \xi_5) + \xi_{1,2} * \xi_{3,4} * \xi_5)}{\xi_{1,2} + \xi_{3,4} + \xi_5}
 \tag{Eq. 63}$$

$$\begin{aligned}
 \xi_1 &> 0 \\
 \xi_2 &> 0 \\
 \xi_3 &> 0 \\
 \xi_4 &> 0 \\
 \xi_5 &> 0
 \end{aligned}
 \tag{Eq. 64}$$

$$\xi_{1,2} + \xi_{1,2}\xi_{3,4} + \xi_{1,2}\xi_5 + \xi_{3,4}\xi_5 > \frac{(\xi_{1,2}^2(\xi_{3,4} + \xi_5) + \xi_{3,4}^2(\xi_{1,2} + \xi_5) + \xi_{1,2}\xi_{3,4}\xi_5)}{\xi_{1,2} + \xi_{3,4} + \xi_5}
 \tag{Eq. 65}$$



**Figure 1.** Block Diagrams of Autonomous Power Supply System based on WDPP: 1 – WDPP; 2 – capacitor bank; 3 – controller; 4 – battery pack; 5 – inverter; 6 – synchronizer; 7 – consumer; 8 – diesel generator

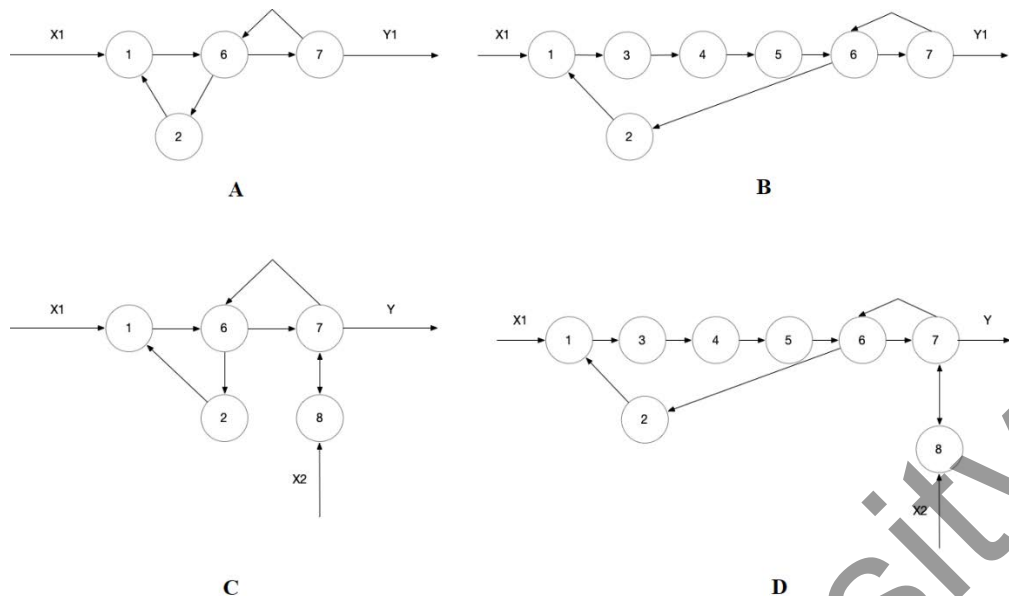


Figure 2. Graphs of Autonomous Power Supply System based on WDPP

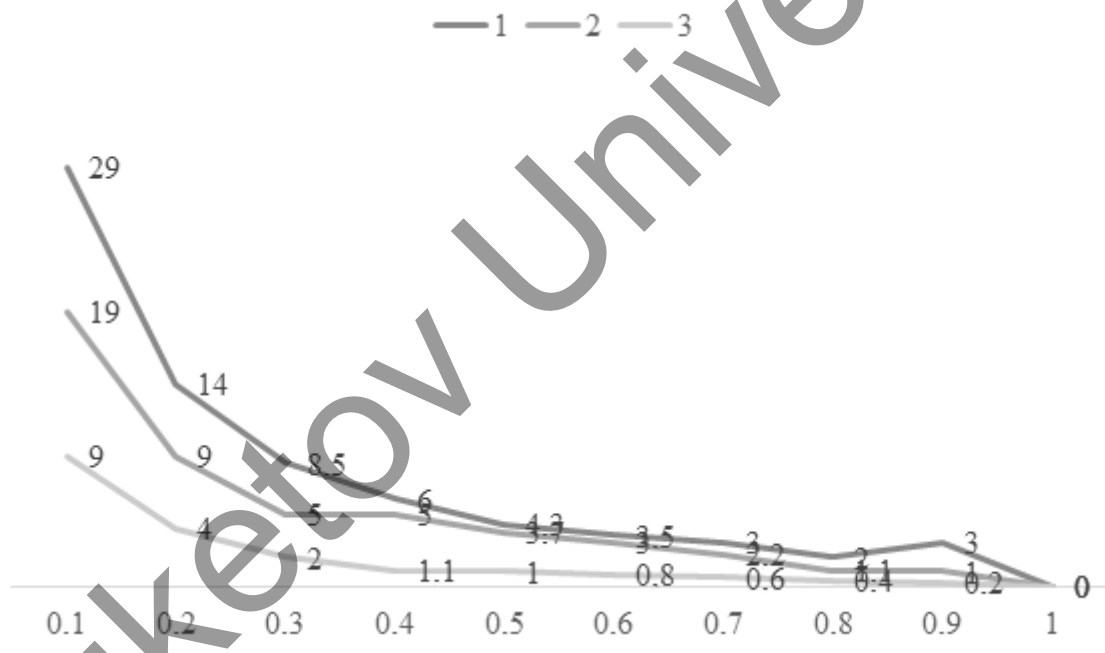
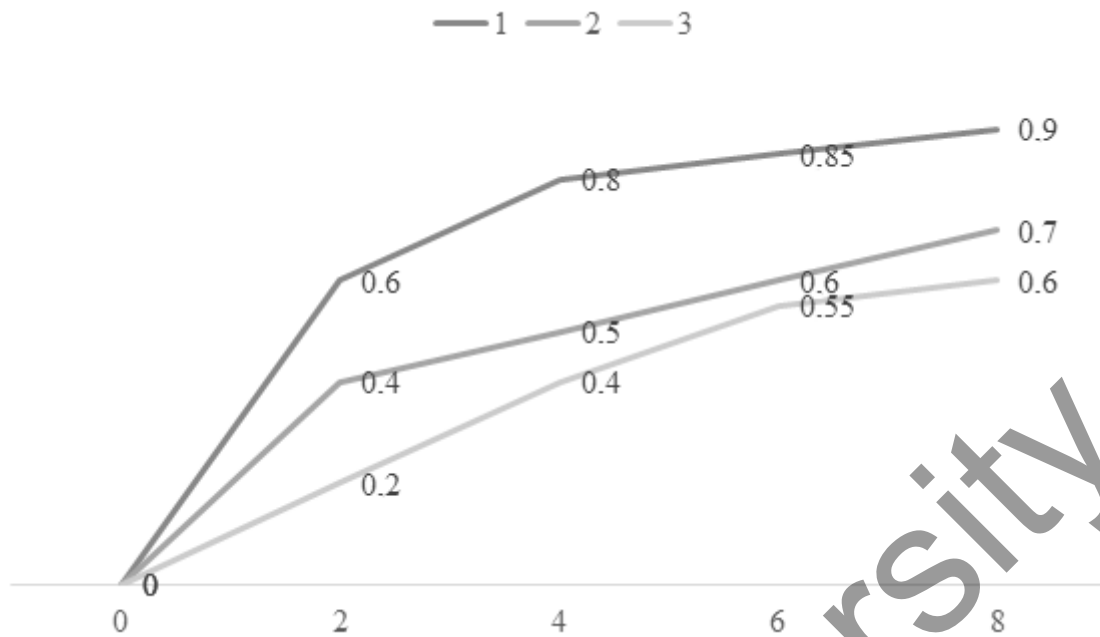
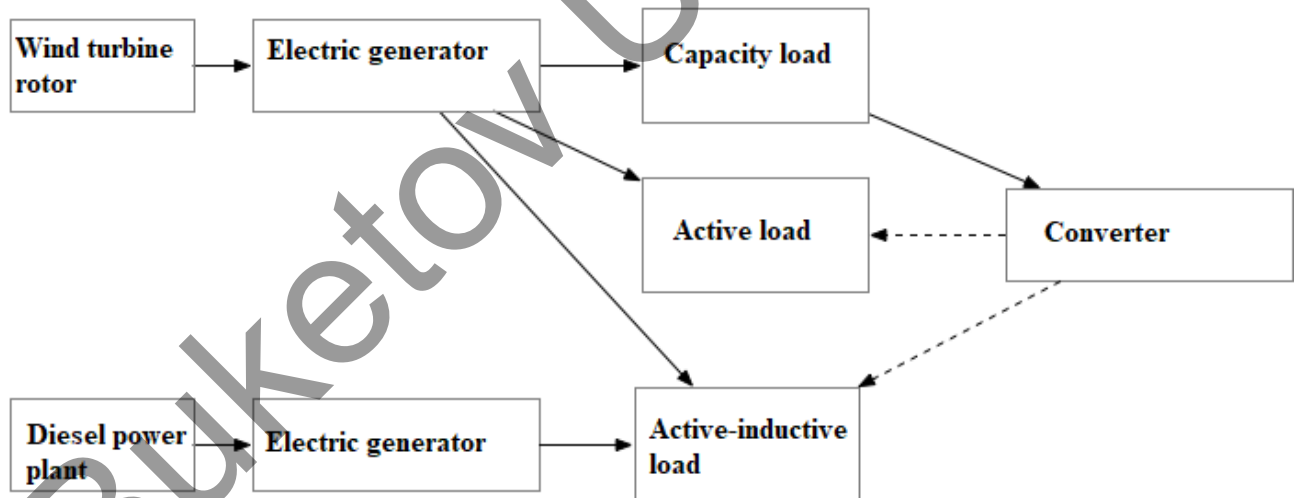


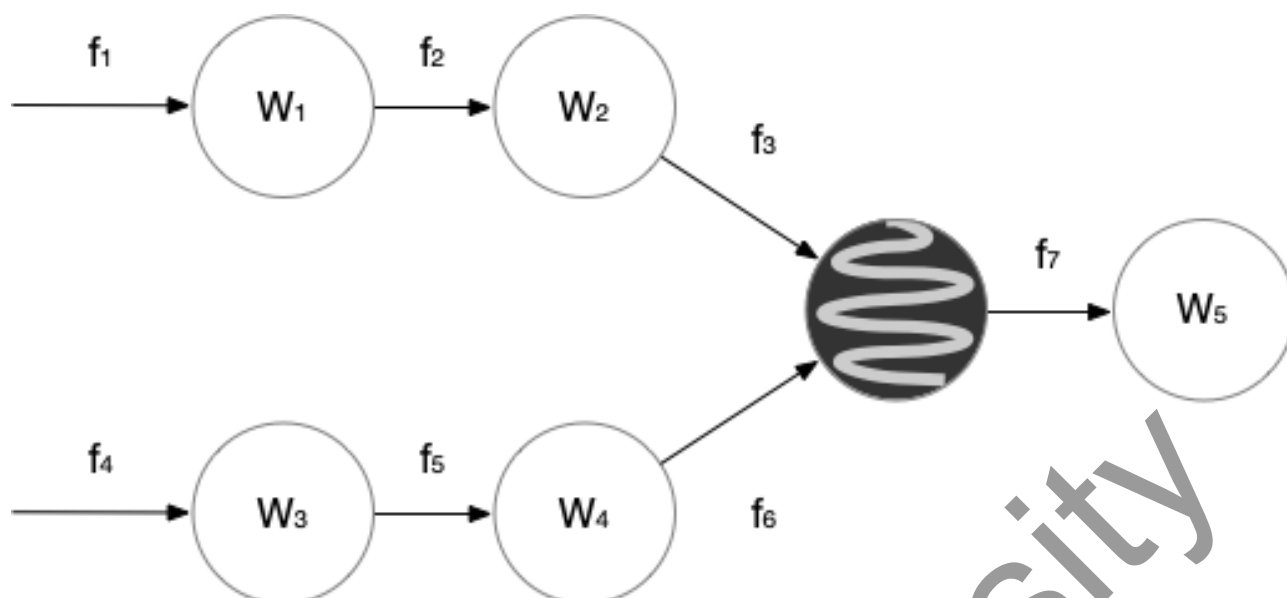
Figure 3. The Dependence of the Load Intensity on the Capacity of a Single-Channel (3), Two-Channel (2), and Three-Channel (1) System



**Figure 4.** Probability of Power Supply of the Consumer: 1-Probability of Power Supply from One Power Source; 2-Probability of Power Supply of the Consumer from Two Power Sources at the Same Time; 3 – from Three Power Sources



**Figure 5.** Block Diagram of an Autonomous Wind-Diesel Power System



**Figure 6.** Functional diagram of an autonomous wind-diesel power plant:  $W_1$  – rotor link;  $W_2$  – electric generator link;  $W_3$  – diesel engine link;  $W_4$  – electric generator link;  $W_5$  – load link.

**Table 1.** Raus Table

$r_i$	$i / k$	1	2	3
–	1	$c_{1,1} = \alpha_0$	$c_{2,1} = \alpha_2$	$c_{3,1} = \alpha_4$
–	2	$c_{1,2} = \alpha_1$	$c_{2,2} = \alpha_3$	$c_{3,2} = \alpha_5$
$r_3 = \frac{c_{1,1}}{c_{1,2}}$	3	$c_{1,3} = c_{2,1} - r_3 c_{2,2}$	$c_{2,3} = c_{3,1} - r_3 c_{3,2}$	$c_{3,3} = 0$
$r_3 = \frac{c_{1,2}}{c_{1,3}}$	4	$c_{1,4} = c_{2,2} - r_4 c_{2,3}$	$c_{2,4} = c_{3,2} - r_4 c_{3,3}$	$c_{3,4} = 0$
$r_3 = \frac{c_{1,3}}{c_{1,4}}$	5	$c_{1,5} = c_{2,3} - r_5 c_{2,4}$	$c_{2,5} = 0$	$c_{3,5} = 0$
$r_3 = \frac{c_{1,4}}{c_{1,5}}$	6	$c_{1,6} = c_{2,4} - r_6 c_{2,5}$	$c_{2,6} = 0$	$c_{3,5} = 0$