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THE INVERSE PROBLEM FOR A FOURTH-ORDER PARABOLIC EQUATION WITH A COMPLEX-VALUED COEFFICIENT

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This article presents the results of studies of inverse problems for a fourth-order parabolic equation with a variable complex-valued coefficient. The existence and uniqueness of the solution of inverse problems for a one-dimensional fourth-order equation is established

$$u_t(x,t) + \frac{\partial^4}{\partial x^4} u(x,t) + q(x)\, u(x,t) = f(x) \tag{1}$$

with Dirichlet boundary conditions

$$u(-1,t) = 0,\ u(1,t) = 0,\ u_{xx}(-1,t) = 0,\ u_{xx}(1,t) = 0,\ \ t \in [0,T]. \tag{2}$$

We have to find a pair of functions $u(x,t)$ and $f(x)$ satisfying equation (1) in the domain Ω and conditions

$$u(x,0) = \varphi(x),\ u(x,T) = \psi(x),\ x \in [-1,1], \tag{3}$$

where $\varphi(x)$ and $\psi(x)$ are given sufficiently smooth functions and $q(x) = q_1(x) + iq_2(x)$. We will use $\Omega = \{-1 < x < 1,\ 0 < t < T\}$ to denote an open domain, and $\bar{\Omega} = \{-1 \le x \le 1,\ 0 \le t \le T\}$ to denote a closed domain.

Let us introduce a non-self-conjugate fourth-order differential operator $L_q : D(L_q) \subset L_2(-1,1) \to L_2(-1,1)$ by the formula

$$L_q y = y^{IV}(x) + q(x)\, y(x),\ -1 \le x \le 1,$$

with the domain of definition

$$D(L_q) = \left\{ y(x) \in C^3[-1,1] : y^{IV}(x) \in L_2(-1,1) \right\}.$$

Theorem. Let $q(x) \in C^4[-1,1]$, and functions $\varphi,\ \psi$ are such that $\varphi,\ \psi,\ L_q\varphi,\ L_q\psi \in D(L_q)$. Then inverse problem (1), (2), (3) has a unique solution, which can be represented as Fourier series

$$u(x,t) = \varphi(x) + \sum_{k=0}^{\infty} \frac{\varphi_k - \psi_k}{1 - e^{-\lambda_k T}} \left(e^{-\lambda_k t} - 1\right) X_k(x),$$

and

$$f(x) = L_q\varphi(x) - \sum_{k=0}^{\infty} \frac{\varphi_k - \psi_k}{1 - e^{-\lambda_k T}} \lambda_k \cdot X_k(x).$$

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FEATURES OF THE JOINT SOLUTION OF DEGENERATE HYPERGEOMETRIC SYSTEMS AND BESSEL-TYPE SYSTEMS

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Introduction. In the theory of degenerate hypergeometric functions of two variables, an important place is occupied by the study of the properties of 20 degenerate hypergeometric functions of two variables [1]. These were obtained from four Appell $F_1 - F_4$ functions [2] by limiting transitions.

The Italian mathematician Lauricella (1890) constructed systems of n equations: $(F_A)\,, (F_B)\,, (F_C)$ and (F_D) and constructed their solutions F_A, F_B, F_C and F_D in the form of generalized power series in n variables [3].

In this paper, the subject of our study is degenerate hypergeometric systems

$$z_i\left(1-z_i\right)\frac{\partial^2 W}{\partial z_i^2}+\sum_{j=1, j\neq i}^{n} z_j\frac{\partial^2 W}{\partial z_i\partial z_j}+\left[\gamma-\left(\alpha_i+\beta_i+1\right)z_i\right]\frac{\partial W}{\partial z_i}-\alpha_i\beta_i W=0, \left(i=\overline{1,k}\right) \quad (1)$$

$$\sum_{j=1}^{n} z_j\frac{\partial^2 W}{\partial z_j\partial z_i}+\left(\gamma-z_i\right)\frac{\partial W}{\partial z_i}-\alpha'_{i-k}W=0, \quad (i=k+1,\ldots,k+l) \quad (2)$$

$$\sum_{j=1}^{n} z_j\frac{\partial^2 W}{\partial z_j\partial z_i}+\gamma\frac{\partial W}{\partial z_i}-W=0, \left(i=\overline{k+l+1,n}\right) \quad (3)$$

obtained by passing to the limit from the Lauricella system (F_B).

Studying the degenerate hypergeometric system (1)-(3), V.I. Khudozhnikov introduced a new function:

$$\Phi_{B,n}^{k,l}\left(\begin{matrix}(\alpha_k)\,, & (\alpha'_l)\,, & (\beta_k)\\ & \gamma & \end{matrix}\middle|\,(z_n)\right)=\sum_{i_1,\ldots,i_n}\frac{\prod_{j=1}^{k}\left(\alpha_j\right)_{i_j}\left(\beta_j\right)_{i_j}}{(\gamma)_{\sum_{j=1}^{n} i_j}}\prod_{j=k+1}^{k+l}\left(\alpha'_{j-k}\right)_{i_j}\prod_{j=1}^{n}\frac{(z_j)^{i_j}}{i_j!} \quad (4)$$

where the following abbreviations and notations are used [4]:

$$(a)_n=(a_1,a_2,\ldots,a_n)\,, \quad \prod(\alpha_k)_{i_n}=\prod_{k=1}^{n}(\alpha_k)_{i_k}\,, \quad \sum_{i_1,\ldots,i_n}=\sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\cdots\sum_{i_n=0}^{\infty} \quad (5)$$