(ii) The restriction R_1 of R to the subgroup U_1 generated by the minimal M-submodules of U has no nilpotent element.

(iii) By ABC 2008 p. 44, R_1 is a finite product of fields; therefore U_1 is a product of vector spaces over these fields.

(iv) If N is the normalizer of M, N° preserves this decomposition of U_1, and acts linearly on each of the components; since the action is free, it has no unipotents.

(v) In characteristic p, by POIZAT 2001 no simple group is involved, and N° is commutative.

(vi) In characteristic 0, the intersection of all the Z(M) is infinite, and we finally obtain a vector space over a field K of characteristic 0 on which T° acts linearly and without unipotents; then use Lie-Kolchin-Mal'cev.

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FAMILIES OF ELEMENTARY THEORIES AND THEIR BASIC CHARACTERISTICS

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We present a survey of results on families of elementary theories and their basic characteristics including the following main items:

1. Topological, spectral and syntactic characterization of total transcendence for families of theories and their closures both in general for complete and incomplete theories, and for families of theories of abelian groups [1, 2, 3, 4, 5].

2. Description of rank values and their dynamics for various families of theories and their subfamilies [1, 2, 6, 7].

3. Description of the approximability and approximations of theories by various families [8, 9].

4. Characterization and description of generating sets, P-closures $\mathrm{Cl}_P(T)$ and E-closures $\mathrm{Cl}_E(T)$ for families T of theories and their combinations [5, 10, 11].

5. Characterization and description of formulas for families of theories, as well as of their characteristics [3, 12, 13].

6. Description of arities of theories, their dynamics and characteristics under transition to closures [14, 15, 16].

7. Description of countable spectra and Hasse diagrams for various families of theories [17, 18] including linearly, circularly and spherically ordered ones [19, 20, 21].

8. Minimality conditions for ordered structures [22, 23, 24].

As for linear and circular orders we divide the class of spherically ordered structures into dense, discrete and mixed ones. For these cases we introduce minimality conditions similar to ones in [22, 23, 24]. In particular, dense spherical orders produce analogues of (weak) o-minimality, and discrete minimal ones assume possibilities to define finite or cofinite sets only. Moreover, these spherical orders are connected with correspondent linear ones, as well as their automorphism groups are naturally linked.

Modifying a result in [21] the following theorem on values of countable spectrum for spherically ordered theories is based both on combinations of minimal dense and discrete spherical orders, and it is spread for more complicated spherically ordered structures of finite convexity ranks.

Theorem. Let T be a countable expansion by disjoint convex unary predicates of a disjoint combination of dense n-spherical theories T_n, for some fixed $n > 1$. Then either T has 2^{ω} countable models or T has exactly $\prod\limits_{k\in n\setminus\{1\}}\left(2^k+2\right)^{r_r}$ countable models, where r_k are natural numbers. Moreover, for any $r_0,\ldots,r_{n-1}\in\omega$ there is an aforesaid theory T with exactly $\prod\limits_{k\in n\setminus\{1\}}\left(2^k+2\right)^{r_r}$ countable models.

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ON NON-ESSENTIALITY OF AN O-STABLE EXPANSION OF $(\mathbb{Z},<,+)$

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Model theory of ordered groups is a sufficiently important application of mathematical logic to algebra. Here we consider such a quite classical object as the ordered group of integers and its possible expansions.O. Belegradek, Y. Peterzil and F. Wagner proved in [1] that there is no quasi-o-minimal expansion of $(\mathbb{Z},<,+)$. Later in this direction ordered groups were investigated in [2–4]. Quite recently E. Walsberg proved some results inthis context [5], in particular showing that a dp-minimal expansion $G$of a discretely ordered group is interdefinable with a model of the theory of$(\mathbb{Z},<,+)$ if and only if G does not admit a nontrivial definable convex subgroup. Here we consider o-stable expansions of $(\mathbb{Z},<,+)$.

Recall, that notion of o-stability was introduced in [6], where it was proved that quasi-o-minimal theories are o-superstable.

Definition.An ordered structure$M$ is o-λ-stable if for every $A \subseteq M$ of size at most λ and every cut (C,D) in M, at most λcomplete 1-types over A are consistent with (C,D). A theory T is o-stable if there is someinfinite λ such that every model of T is o-λ-stable, and it is o-superstable if there is some μsuch that for every $\lambda \geq \mu$, T is o-λ-stable.

Model theory of o-stable ordered groups has been developed in [7–9], where in particular it was proved that an o-stable ordered group is abelian and an o-stable ordered field without definable non-trivial convex subgroups of the additive group is weakly o-minimal and by Marker–Macpherson–Steinhorn theorem is real closed.More general theory of relative stability has been investigated in [10–11]. In [12] V. Verbovskiy proved that a dp-minimal theory with a definable linear order is o-stable, so the class of o-stable theories includes each of the following classes: o-minimal, weakly o-minimal, quasi-o-minimal [6], dp-minimal with a definable linear order.

Theorem. Any o-stable expansion of $(\mathbb{Z},<,+)$is not essential, that is, if G is an o-stable expansion of $(\mathbb{Z},<,+)$then any definable with parameters subset of G is also definable with parameters in $(\mathbb{Z},<,+)$.

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