DOI 10.31489/2023M4/185-193
UDC 510.67

G.E. Zhumabekova*

Karaganda Buketov University, Karaganda, Kazakhstan;
Karaganda Industrial University, Karaganda, Kazakhstan
(E-mail: galkatai@mail.ru)

Model-theoretic properties of semantic pairs and e.f.c.p. in Jonsson spectrum

The article is committed to the study of model-theoretic properties of stable hereditary Jonsson theories, wherein we consider Jonsson theories that retain jonssonnes for any permissible enrichment. The paper proves a generalization of stability that relates stability and classical stability for Jonsson spectrum. This paper introduces new concepts such as "existentially finite cover property" and "semantic pair". The basic properties of e.f.c.p. and semantic pairs in the class of stable perfect Jonsson spectrum are studied.

Keywords: Jonsson theory, semantic model, permissible enrichment, central type, hereditary theory, stable theory, perfect theory, fundamental order, saturated model, e.f.c.p., existentially closed pair, semantic pair.

Introduction

The concept of language enrichment plays a significant role in description the model-theoretic characteristics of both theories itself and models. Language enrichment options are limited by first-order language rules. In this article we are dealing with language enrichment using a one-place predicate symbol and some constant symbol. The next important point of novelty and relevance of this work is the fact that all new concepts and corresponding statements were concerned within the system of the study within the framework of the study generally speaking incomplete theories. Namely, in the class of Jonsson theories. This class is quite broad and its application covers many areas of modern mathematics. The remark about the incompleteness of the theories under consideration is relevant in the sense that the modern apparatus of model theories is developing within the system of the study of complete theories. This article presents results that clarify previously obtained theorems related to the classical concept of stability within the framework of complete theories and its generalizations.

In this work we are going to highlight the fact that we consider many classical concepts associated with the concept of stability for Jonsson theories and their types within the framework of such a new concept as the Jonsson spectrum of cosemanticness a model or class models. This concept allows us to classify Jonssons theories regarding the relation of cosemantic. Also, to find analogues of basic theorems from stability theory, such as with theorems associated with the concept f.c.p. [1], in our case, for these purposes, the idea of using the concept of the central type of Jonsson theories is used.

And here we present results related to the concept of stability of perfect Jonsson theories, and also obtain results regarding the Jonsson spectrum for semantic pairs. Semantic pairs are a generalization of beautiful pairs, which started to be explored deliberately in the work [1] of B. Poizat. In this work, B. Poizat investigated structures of a common form in which elementary substructures are distinguished. He formed the question of finding for conditions under which the theory of elementary pairs is complete. Subsequently, the works of [2–8] and others were devoted to the study of this issue. Commonly, reflection of the work of [2–8] played a significant role in the study of the issue of incomplete theory, that is, Jonsson theories. In the works of A.R. Yeshkeyev we can find a complete description of Jonsson theories regarding this issue [9–14].

*Corresponding author.
E-mail: galkatai@mail.ru

Buketov University

1 Local properties of the Jonsson spectrum in stable theory

The main result in the article is developed within the framework of the Jonsson theory. Since this work is not the first work in the study of the Jonsson theory, I did not want to rewrite the definition and related original concepts and theorems. A detailed description of the Jonsson theory and the initial concepts and theorems related to the theory can be found in work [15–19].

Further we will prove the main results for some fixed Jonsson spectrum. Before that, a number of results related to Jonsson spectrum were obtained. In particular, the generalization of the classical theorem on elementary equivalence of abelian groups and modules, which is one of the important concepts in algebra, is given in works [20–22].

Definition 1. [21] Let σ be an arbitrary signature, L be the set of all formulas of signature σ. Let $\mathcal{B}$ be an arbitrary model of some fixed signature σ, $\mathcal{B} \in Mod\sigma$. Let us call the Jonsson spectrum of the model $\mathcal{B}$ the set:

$$JSp(\mathcal{B}) = \{T/\mathcal{B} \in ModT, T \text{ is Jonsson theory of the signature } \sigma\}.$$

Next, we obtain the following factor set by the cosemantic relation

$$JSp(\mathcal{B})/_{\bowtie} = \{[T]|T \in JSp(\mathcal{B})\}.$$

Let $[T] \in JSp(\mathcal{B})/_{\bowtie}$. Since each theory $\Delta \in [T]$ has $\mathcal{C}_\Delta = \mathcal{C}_T$, then the semantic model of the $[T]$ class will be called the semantic model of the T theory: $\mathcal{C}_{[T]} = \mathcal{C}_T$. The center of the Jonsson class $[T]$ will be called the elementary theory $[T]^*$, its semantic model $\mathcal{C}_{[T]}$, i.e. $[T]^* = Th(C_{[T]})$ and $[T]^* = Th(\mathcal{C}_\Delta)$ for any $\Delta \in [T]$. Denote by $E_{[T]} = \bigcup_{\Delta \in [T]} E_\Delta$ the class of all existentially closed models of the class $[T] \in JSp(\mathcal{B})/_{\bowtie}$. Note that $\bigcap_{\Delta \in [T]} \neq \emptyset$, since at least for each $\Delta \in [T]$ we have $\mathcal{C}_{[T]} \in E_\Delta$.

Definition 2. [21] The class $JSp(B)/_{\bowtie}$ is called perfect (further, $PJSp(A)/_{\bowtie}$) if each class $[T] \in JSp(B)/_{\bowtie}$ is perfect, $[T]$ is called perfect if $C_{[T]}$ is a saturated model.

$$PJSp(\mathcal{B}) = \{T|T \text{ is perfect Jonsson theory in language } \sigma \text{ and } \mathcal{B} \in ModT\}.$$

It is clear that $PJSp(\mathcal{B}) \subsetneq JSp(B)$.

Theorem 1. [17]. Let T be a perfect Jonsson theory. Then the following conditions are equivalent:
1) T^* is a model companion of the T theory;
2) $ModT^* = E_T$;
3) $T^* = T^f$, where E_T is the class of T-existentially closed models T, $T^f = Th(F_T)$, where F_T is the class of generic T models (in the sense of Robinson's finite forcing).

Let T be a Jonsson theory, $S^J(X)$ the set of all existential n-complete types over X consistent with T for every finite n.

Definition 3. [17] We say that a Jonsson theory T is J-λ-stable if for any T-existentially closed model A, for any subset X of the set A, $|X| \leq \lambda \Rightarrow |S^J(X)| \leq \lambda$.

At one time, the author in [23] proved a theorem that connects the concepts of J-stability and classical stability for perfect Jonsson theories. And this result generalizes the concepts of stability. Now we want to define the concepts for the Jonsson spectrum.

Theorem 2. Let $[T]$ be a perfect Jonsson $\exists$-complete class, $\lambda \geq \omega$. Let $\mathcal{C}_{[T]}$ be its semantic model, $\mathcal{A} \preceq_{\exists_1} C_{[T]}$ and $\mathcal{A}$ is the existentially closed model of $[T]$, $[T] \in JSp(\mathcal{A})/_{\bowtie}$. The Jonsson class $[T]$ is J-λ-stable if and only if the center of the Jonsson class $[T]^*$ is λ-stable (in the classical sense).

Buketov University

Proof. We will work only with perfect Jonsson theories $PJSp(A)/_{\bowtie}$. Let $[T] \in PJSp(T)/_{\bowtie}$ and $E_n([T])$ be the distributive lattice of equivalence classes

$$\varphi^{[T]} = \{\psi \in E_n(L) \mid [T]^* \models \varphi \leftrightarrow \psi, \varphi \in E_n(L)\}.$$

We will call the Jonsson class $[T]$ stable if every theory $\Delta \in [T]$ is a stable theory by the Definition 3.

If $[T] \subset [T]^*$, then $E_n([T]) \subset E_n(Th(C_{[T]}))$, where $E_n([T])$, $E_n(Th(C_{[T]}))$ are the corresponding lattices of existential formulas. The class $[T]$ is complete for existential propositions, which means that if every theory in $[T]$ is a complete theory, therefore $E_n([T]) = E_n(Th(C_{[T]}))$. $[T] \in JSp(\mathcal{A})/_{\bowtie}$ is perfect, then the semantic model $\mathcal{C}_{[T]}$ is saturated, every Jonsson theory $\Delta \in [T]$ is perfect. Then, by Theorem 1, each $\Delta \in [T]$ has a model companion. Since $[T] \in JSp(\mathcal{A})/_{\bowtie}$ is perfect $[T]^*$ is model complete by Theorem 2.9.15 [17] , $[T]^* = Th(C_{[T]})$ if and only if $\forall n < \omega, \forall \varphi \in F_n(\Delta^*)\ \exists \theta \in E_n(\Delta^*) : \Delta^* \vdash \varphi \leftrightarrow \theta$.

Let the Jonsson class $[T]$ be J-λ-stable, this means that if in the class there is a theory from $[T]$ that is stable, then by Definition 3 for each model $\mathcal{A} \in E_\Delta$ we have that for each subset $X \subset A$, if $|X| \leq \lambda$ then $|S^J(X)| \leq \lambda$.

Note that if the class is perfect, then all E_Δ for $\Delta \in [T]$ are equal to each other.

Suppose that $[T]^*$ is not λ-stable. Then there exists $\mathcal{A} \in E_\Delta = Mod\Delta^*$, by Theorem 1, so there is $X \subset A$ such that $|X| < \lambda, \exists n < \omega \Rightarrow |S^J(X)| > \lambda$. For each formula $\varphi \in p$, where $p \in S_n(X)$, we replace φ with θ satisfying the properties $\Delta^* \vdash \varphi \leftrightarrow \theta$ and $\theta \in E_n([T]^*)$. Let p' be p after replacement. Then $p' \in S^J(X)$ and $|S^J(X)| > \lambda$. This contradicts the J-λ-stability of the class $[T]$.

2 The central type of a semantic pair

Since our main goal in this article is to consider the special properties of central types, we will work with some signature enrichments in which some fixed Jonsson theory is given, other questions regarding this can be found [18, 19, 24, 25].

In the future, the entire theory under consideration will be hereditary. We gave a detailed description of the hereditary theory in paper [19]. Now let's talk about the hereditary class. A class is hereditary if every theory in that class is hereditary.

Let us consider some extension of signature σ and consider the central type of this extension for all Jonsson theories $[T] \in PJSp(T)/_{\bowtie}$. And the central types here are taken from enrichment $\odot$ in the previous work [19].

Next, we consider the concept of "finite cover property" which arises from the work of Shelah [26]. In his works Shelag shows the following: an unstable theory has f.c.p., but this is not of great importance for us, since we will only consider stable theories.

Δ denotes a set of formulas of the form $\varphi(\bar{x}, \bar{y})$. An m-formula, or φ-m-formula is a formula of the form $\varphi(\bar{x}, \bar{y})$ or $\varphi(\bar{x}, \bar{a})$ where $l(\bar{x}) = m$ and we consider $\bar{y}$ as a sequence of parameters for which we will usually substitute some $\bar{a}$ and get $\varphi(\bar{x}, \bar{a})$.

Definition 4. [26; 62] Let $\varphi(\bar{x}, \bar{y}) \in L$, $\bar{a}^0, \ldots, \bar{a}^{n-1} \in A$.

(1) $\varphi(\bar{x}, \bar{y})$ has the finite cover property (f.c.p.) if for arbitrarily large natural numbers n there $\bar{a}^0, \ldots, \bar{a}^{n-1}$ such that $\models \neg(\exists \bar{x}) \bigwedge_{k<n} \varphi(\bar{x}, \bar{a}^k)$ but for every $l < n$, $\models (\exists \bar{x}) \bigwedge_{k<n, k \neq l} \varphi(\bar{x}, \bar{a}^k)$.

(2) T has the f.c.p. if there exists a formula $\varphi(x, \bar{y})$ which has the f.c.p.

And so, for a special case, we form matrices where rows consist of $\varphi - 1$-type. And this matrix will be the central type, and all partitions of the central type from the enrichment $\odot$ will be f.c.p.

$$p^c = \begin{pmatrix} \varphi_1^s(x, b_1) & \varphi_1^s(x, b_2) & \ldots & \varphi_1^s(x, b_n) & \ldots \\ \varphi_2^s(x, b_1) & \varphi_2^s(x, b_2) & \ldots & \varphi_2^s(x, b_n) & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \varphi_n^s(x, b_1) & \varphi_n^s(x, b_2) & \ldots & \varphi_n^s(x, b_n) & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{pmatrix} s = \{0, 1\}.$$

Buketov University

For example, $n = 3$, $k = 1$, $l = 2$, then f.c.p:

$$\begin{pmatrix} \varphi_1^0(x,a_1) & \varphi_1^1(x,a_2) & \varphi_1^1(x,a_3) \\ \varphi_2^1(x,a_1) & \varphi_2^0(x,a_2) & \varphi_2^1(x,a_3) \\ \varphi_3^1(x,a_1) & \varphi_3^1(x,a_2) & \varphi_3^0(x,a_3) \end{pmatrix}.$$

From this it can be seen that f.c.p. can be extended to the central type, while p^c must preserve the hereditary property. When central type = f.c.p., then the theory will be unstable.

In this work, B. Poizat's results on beautiful pairs are generalized on the case of $\exists$-complete J-λ-stable hereditary Jonsson theory. Instead of f.c.p. and a type, we consider existentially finite cover property (e.f.c.p.) and a central type, correspondingly, in a specific expansion of the signature. Professor A. Yeshkeyev first made a report on this at the conference Logic Colloquium-2023 [27].

Definition 5. [27] Let T be the Jonsson L-theory and $f(\bar{x},\bar{y})$ be an $\exists$ formula of L language. If for any arbitrary large n exists $\bar{a}^0,\ldots,\bar{a}^{n-1}$ in some existentially closed model of T and $\bar{a}^0,\ldots,\bar{a}^{n-1}$ satisfies $\neg(\exists\bar{x})\bigwedge_{k<n} f(\bar{x},\bar{a}^k)$ and for any $l<n$ $\neg(\exists\bar{x})\bigwedge_{k<n} f(\bar{x},\bar{a}^k)$, then $f(\bar{x},\bar{y})$ is said to have e.f.c.p. (existentially finite cover property).

In [1], the connection between fundamental order and definability was defined.

The fundamental order is a tool of comparing types over models of a complete theory: it measures the degree of complexity of a type in the realization. This order is especially effective in the case of a stable theory. Since the center of the T^* Jonsson theory is a complete theory, and we can consider the fundamental order for central types. If the Jonsson theory is a perfect theory, then T^* will be a Jonsson theory. And also, due to the perfection of the theory of T, any formula is existential in T^*.

Definition 6. [20] Let $A \subseteq M$, $\exists$-formula $\varphi(\bar{x},\bar{y}) \in L(A)$ be called representable in $p \in S^J(M)$ if there exists a tuple $\bar{m} \in M$ such that $p \vdash \varphi(\bar{x},\bar{m})$.

Definition 7. [20] M, N are existentially closed submodels of the semantic model C_T of the theory of T. If $p \in S_1^J(M)$, and $q \in S_1^J(N)$, $p \geq q$ in the sense of fundamental order, if any formula represented by p is also represented by q.

Definition 8. [1] If p and q represent the same formulas, we say that they are equivalent, and they even have a class in fundamental order.

Theorem 3. [1] Let T be a stable theory, M, N are $|T|^+$ be saturated models of T, $p \in S_1(M)$, $q \in S_1(N)$. Let $A \subset M$ be $|A| \leq |T|$, such that p is defined for all formulas $f(x,\bar{y})$: $g(\bar{y},\bar{a})$ can be taken with parameters $\bar{a}$ in A; then p and q are equivalent in fundamental order T if there is an $A' \subset N$ of the same type as A such that q is the definable type of a formula of the form $g(\bar{y},\bar{a}')$, where a' corresponds to a.

A theory T is stable if and only if for any model M of T and all p from $S_1(M)$, p is definable.

In the framework of the study of Jonsson theories, which are generally incomplete, and in some expanded language with new unary predicate and constant symbols, we refine in such generalization the earlier result obtained on beautiful pairs for complete theories from [1] (Theorem 4).

Definition 9. [27] Let C_T be a semantic model of T and N, M be existentially closed submodels of C_T. A pair (N, M) is called existentially closed pair, if M is an existentially closed submodel of N.

Lemma 1. If theory T is a perfect Jonsson theory, then theory $Th_{\forall\exists}(\mathcal{C},\mathcal{M})$ is a perfect Jonsson theory.

Definition 10. [27] An existentially closed pair (C_T, M) is a semantic pair, if the following conditions hold:

1) M is $|T|^+$-$\exists$-saturated (it means that it is $|T|^+$-saturated restricted up to existential types);

2) for any tuple $\bar{a} \in C$ each its $\exists$-type in sense of T over $M \cup \{\bar{a}\}$ is satisfiable in C.

By definition we see that it generalizes the excellent pair in [1], but weaker, because in the definition the number of tuples is finite and by Definition 2.4.4 [17] the power of the semantic model is ω^+ and it does not reach 2^ω: $2^\omega > \omega^+, \omega^+ + \omega = \omega^+$. Using the following Theorem 4 we can show the elementary equivalence of semantic pairs.

Let class K be $\{(\mathcal{C}, \mathcal{M}) | \mathcal{M} \preceq_{\exists_1} \mathcal{C}, (\mathcal{C}, \mathcal{M})$ is semantic pair$\}$.

Consider the Jonsson spectrum of class K:

$$JSp(K) = \{\nabla | \nabla \text{ is Jonsson theory}, \nabla = Th_{\forall\exists}(\mathcal{C}, \mathcal{M}), \text{where } (\mathcal{C}, \mathcal{M}) \in K\}.$$

It is easy to see that $JSp(K)/_{\bowtie}$ is the factor set of the Jonsson spectrum of class K by $\bowtie$, $[\nabla] \in JSp(K)/_{\bowtie}$.

Let $[\nabla]$ be $\exists$-complete and J-λ-stable Jonsson class, $\mathcal{C}_{[\nabla]}$ be a semantic model of the theory $[\nabla]$, $\overline{[\nabla]} = [\nabla]$ in the enrichment of $\odot$, $\overline{[\nabla]}^*$ is the center of the $\overline{[\nabla]}$, $p, q \in S(\overline{[\nabla]}^*)$, $\nabla' = Th_{\forall\exists}(\mathcal{C}, \mathcal{M})$.

Theorem 4. $(\mathcal{C}_{[\nabla]}, M_1)$ and $(\mathcal{C}_{[\nabla]}, M_2)$ are two semantic pairs, $\bar{a}$ and $\bar{b}$ tuples taken from each of them, M_1, $M_2 \in E_{[\nabla]}$. Then $(\mathcal{C}_{[\nabla]}, M_1) \equiv_{\forall\exists} (\mathcal{C}_{[\nabla]}, M_2)$, if their central types are equivalent by the fundamental order $\overline{\nabla}^*$.

Proof. Follows from Theorem 6 in [1] and from Theorem 3.

Theorem 5. Let $[\nabla]$ be a hereditary, $\exists$-complete perfect, and J-λ-stable Jonsson class. Then the following conditions are equivalent:

1) $\overline{[\nabla]}^*$ does not have e.f.c.p.;

2) Any $|T|^+$-saturated model from ∇' is a semantic pair;

3) Two tuples $\bar{a}$ and $\bar{b}$ from the models of $\overline{[\nabla]}^*$ have the same type if and only if their central types in sense of $\overline{[\nabla]}^*$ over $\mathcal{M}$ are equivalent by fundamental order $\overline{[\nabla]}^*$;

4) Two tuples $\bar{a}$ and $\bar{b}$ from models of ∇' and that are in $\mathcal{C}_{[\nabla]} \setminus M$ have the same central types in the sense of $\overline{[\nabla]}$ if and only if they have the same central types in the sense of $\overline{[\nabla]}^*$.

Proof. 1) $\Rightarrow$ 2). $\overline{[\nabla]}^*$ the center of $\overline{[\nabla]}$ theory in the permissible enrichment $\odot$. And by Theorem 2. it is λ-stable theory. In [26], Shelah showed that stable theories do not have f.c.p. If (N, M) is a $|T|^+$-$\exists$-saturated model from ∇', M is $|T|^+$-$\exists$-saturated.

Let us assume that $\overline{[\nabla]}^*$ is not λ-stable. Then there exists $\mathcal{M} \in E_\Delta = Mod\overline{[\nabla]}^*$, by Theorem 1, so there is $X \subset M$ such that $|X| < \lambda, \exists n < \omega \Rightarrow |S^J(X)| > \lambda$. For each formula $\varphi \in p$, where $p \in S_n(X)$, we replace φ with θ satisfying the properties $\Delta^* \vdash \varphi \leftrightarrow \theta$ and $\theta \in E_n([T]^*)$. Let p' be p after replacement. Then $p' \in S^J(X)$ and $|S^J(X)| > \lambda$. This contradicts the J-λ-stability of the class $\overline{[\nabla]}^*$. Hence $\overline{[\nabla]}^*$ is stable and has a saturated model.

2) $\Rightarrow$ 3). By Definition 10, since any sufficiently saturated model is a semantic pair.

3) $\Rightarrow$ 4). Because if $\bar{a}$ and $\bar{b}$ are in $\mathcal{C}_{[\Delta]} \setminus M$ let's say that their types are over $\mathcal{C}_{[\Delta]} \setminus M$ are fundamentally equivalent, that is, they implement a type over $\emptyset$.

4) $\Rightarrow$ 1). If $\overline{\Delta}^*$ does not have e.f.c.p., then by Definition 5 there would not exist an arbitrarily large number n. In the semantic pair $(\mathcal{C}_{[\Delta]}, M)$ for arbitrarily large n we find $\bar{a}_n$ in M [1]. Moreover, any $\bar{b}$ of a semantic pair, $\bar{b} \in M$ is of the same type as $\bar{a}$ over $\emptyset$ in the sense of $\overline{\Delta}$ would satisfy the opposite. Therefore, $\bar{a}$ and $\bar{b}$ will not implement the same type in the sense of Δ', which contradicts (4).

Theorem 6. Let $[\nabla]$ be a hereditary, $\exists$-complete perfect, and J-λ-stable Jonsson class. If $\overline{[\nabla]}^*$ does not have e.f.c.p. and λ-stable class, then the class If $[\nabla]'$ is J-λ-stable and does not have e.f.c.p.

Proof. The proof follows from Theorems 4 and 5.

Buketov University

References

1 Poizat B. Paires de structure stables / B. Poizat // J. Symb. Logic. — 1983. — 48 — P. 239–249.

2 Нуртазин А.Т. Об элементарных парах в несчётно-категоричной теории / А.Т. Нуртазин // Тр. Сов.-фр. коллокв. по теории моделей. — Караганда, 1990. — С. 126–146.

3 Bouscaren E. Dimensional order property and pairs of models / E. Bouscaren // Annals of Pure and Appl. Logic. — 1989. — 41. — P. 205–231.

4 Bouscaren E. Elementary pairs of models / E. Bouscaren // Annals of Pure and Applled Logic. — 1989. — 45. — P. 129–137.

5 Мустафин Т.Г. Новые понятия стабильности теорий / Т.Г. Мустафин // Тр. сов.-фр. коллокв. по теории моделей. — Караганда, 1990. — С. 112–125.

6 Мустафин Т.Г. О p-стабильности полных теорий / Т.Г. Мустафин, Т.А. Нурмагамбетов // Структурные свойства алгебраических систем. — Караганда, 1990. — С. 88–100.

7 Палютин Е.А. E^*-стабильные теории / Е.А. Палютин // Алгебра и логика. — 2003. — № 42(2). – С. 194–210.

8 Нурмагамбетов Т.А. О числе элементарных пар над множествами / Т. Нурмагамбетов, Б. Пуаза // Исследования в теории алгебраических систем. — Караганда, 1995. — С. 73–82.

9 Yeshkeyev A.R. Small models of hybrids for special subclasses of Jonsson theories / A.R. Yeshkeyev, N.M. Mussina // Bulletin of the Karaganda University. Mathematics Series. — 2019. — No. 3(95). — P. 68–73.

10 Yeshkeyev A.R. Strongly minimal Jonsson sets and their properties / A.R. Yeshkeyev // Bulletin of the Karaganda University. Mathematics Series. — 2015. — No. 4(80). — P. 47–51.

11 Yeshkeyev A.R. Properties of lattices of the existential formulas of Jonsson fragments / A.R. Yeshkeyev, M.T. Kasymetova // Bulletin of the Karaganda University. Mathematics Series. — 2015. — No. 3(79). — P. 25–32.

12 Yeshkeyev A.R. On Jonsson varieties and quasivarieties / A.R. Yeshkeyev // Bulletin of the Karaganda University. Mathematics Series. — 2021. — No. 4(104). — P. 151–157.

13 Yeshkeyev A.R.Connection between the amalgam and joint embedding properties / A.R. Yeshkeyev, I.O. Tungushbayeva, M.T. Kassymetova // Bulletin of the Karaganda University. Mathematics Series. — 2022. — No. 1(105). — P. 127–135.

14 Yeshkeyev A.R. Companions of (n(1), n(2))-Jonsson theory / A.R. Yeshkeyev, M.T. Omarova // Bulletin of the Karaganda University. Mathematics Series. — 2019. — No. 4(96). — P. 75–80.

15 Барвайс Дж. Теория моделей: справ. кн. по мат. логике. — Ч. 1 / Дж. Барвайс. — М.: Наука, 1982. — 392 с.

16 Mustafin Y.T. Quelques proprietes des theories de Jonsson / Y.T. Mustafin // J. Symb. Log. — 2002. — 67. — No. 2. — P. 528–536.

17 Ешкеев А.Р. Йонсоновские теории и их классы моделей / А.Р. Ешкеев, М.Т. Касыметова. — Караганда: Изд-во Караганд. гос. ун-та, 2016. — 370 с.

18 Yeshkeyev A.R. Companions of fragments in admissible enrichments / A.R. Yeshkeyev, G.E. Zhumabekova // Bulletin of the Karaganda University. Mathematics Series. — 2018. — No. 4(92). — P. 105–111.

19 Yeshkeyev A.R. The J-minimal sets in the hereditary theories / A.R. Yeshkeyev, M.T. Omarova, G.E. Zhumabekova // Bulletin of the Karaganda University. Mathematics Series. — 2019. — No. 2(94). — P. 92–98.

20 Yeshkeyev A.R. Independence and simplicity in Jonsson theories with abstract geometry / A.R. Yeshkeyev, M.T. Kassymetova, O.I. Ulbrikht // Siberian Electronic Mathematical Reports.

Buketov University

— 2021. — 18. — No. 1. — P. 433–455.

21 Yeshkeyev A.R., Ulbrikht O.I. JSp-cosemanticness and JSB property of Abelian groups // Siberian Electronic Mathematical Reports. — 2016. — 13. — P. 861–874.

22 Yeshkeyev A.R. Model-theoretical questions of the Jonsson spectrum / A.R. Yeshkeyev // Bulletin of the Karaganda University. Mathematics Series. — 2020. — No. 2(98). — P. 165–173.

23 Yeshkeyev A.R. On Jonsson stability and some of its generalizations / A.R. Yeshkeyev // Journal of Mathematical Sciences. — 2010. — 166. — No. 5. — P. 646–654.

24 Yeshkeyev A.R. An essential base of the central types of the convex theory / A.R. Yeshkeyev, M.T. Omarova // Bulletin of the Karaganda University. Mathematics Series. — 2021. — No. 1(101). — P. 119–126.

25 Yeshkeyev A.R. An algebra of the central types of the mutually model-consistent fragments / A.R. Yeshkeyev, N.M. Mussina // Bulletin of the Karaganda University. Mathematics Series. — 2021. — No. 1(101). — P. 111–118.

26 Shelah S. Classification theory and the number of nonisomorphic models / S. Shelah. — Amsterdam: North-Holland, 1978.

27 Yeshkeyev A.R. The central type of a semantic pair / A.R. Yeshkeyev, I.O. Tungushbayeva, G.E. Zhumabekova // Book of Abstracts – Logic Colloquium. — 2023. — P. 184–185.

Г.Е. Жумабекова

Академик Е.А. Бөкетов атындағы Қарағанды университеті, Қарағанды, Қазақстан;
Қарағанды индустриялық университеті, Қарағанды, Қазақстан

Йонсондық спектрлердегі йонсондық семантикалық қосар мен шекті жабудың экзистенциалды қасиетінің моделді-теоретикалық қасиеттері

Мақала кез келген рұқсаттылығы бар байытуда йонсондылықты сақтайтын стабилді әрі мұралы йонсондық теориялардың моделді-теоретикалық касиеттерін зерттеуге арналған. Жұмыста стабилділік пен классикалық стабилділікті байланыстыратын стабилділіктің жалпыламасы йонсондық спектрлер үшін дәлелденген. Ұсынылып отырған жұмыста "шекті жабудың экзистенциалды қасиеті" мен "семантикалық қосар" секілді жаңа ұғымдар енгізілген. Және осы семантикалық қосар мен шекті жабудың экзистенциалды қасиетінің стабилді кемел йонсондық спектрлер үшін негізгі қасиеттері зерттелген.

Кілт сөздер: йонсондық теория, семантикалық модель, рұқсатылығы бар байыту, централды тип, мұралы теория, стабилді теория, кемел теория, фундаменталды рет, қаныққан модель, шекті жабудың экзистенциалды қасиеті, экзистенциалды-тұйық қосар, семантикалық қосар.

Buketov University

Г.Е. Жумабекова

Карагандинский университет имени академика Е.А. Букетова, Караганда, Казахстан;
Карагандинский индустриальный университет, Караганда, Казахстан

Теоретико-модельные свойства сематических пар и e.f.c.p. в йонсоновских спектрах

Статья посвящена изучению теоретико-модельных свойств стабильных наследственных йонсоновских теорий, при этом мы рассматриваем йонсоновские теории, которые сохраняют йонсоновость при любом допустимом обогащении. Авторами доказано обобщение стабильности, связывающее стабильность и классическую стабильность для йонсоновских спектров. Введены новые понятия, такие как «экзистенциальное свойство конечного покрытия» и «семантическая пара». Изучены основные свойства e.f.c.p. и семантических пар в классе стабильных совершенных йонсоновских спектров.

Ключевые слова: йонсоновская теория, семантическая модель, допустимое обогащение, центральный тип, наследственная теория, стабильная теория, совершенная теория, фундаментальный порядок, насыщенная модель, e.f.c.p., экзистенциально-замкнутая пара, семантическая пара.

Buketov University

References

1 Poizat, B. (1983). Paires de structure stables. *J. Symb. Logic, 48*, 239–249.

2 Nurtazin, A.T. (1990). Ob elementarnykh parakh v neschetno-kategorichnoi teorii [On elementary pairs in uncountably categorical theory]. *Trudy Sovetsko-frantsuzskogo kollokviuma po teorii modelei — Proceedings of the Soviet-French colloquium on model theory*, 126–146 [in Russian].

3 Bouscaren, E. (1989). Dimensional order property and pairs of models. *Annals of Pure and Appl.Logic, 41*, 205–231.

4 Bouscaren, E. (1989). Elementary pairs of models. *Annals of Pure and Appl.Logic, 45*, 129–137.

5 Mustafin, T.G. (1990). Novye poniatiia stabilnosti teorii [New concepts of theory stability]. *Trudy sovetsko-frantsuzskogo kollokviuma po teorii modelei — Proceedings of the Soviet-French colloquium on model theory*, 112–125 [in Russian].

6 Mustafin, T.G. & Nurmagambetov, T.A. (1990). O P-stabilnosti polnykh teorii [On p-stability of complete theories]. *Strukturnye svoistva algebraicheskikh sistem — Structural properties of algebraic systems*, 88–100 [in Russian].

7 Palyutin, E.A. (2003). E^*-stabilnye teorii [E^*-stable theories]. *Algebra i logika — Algebra and logic, 42*(2), 194–210 [in Russian].

8 Nurmagambetov,T.A. & Poizat, B. (1995). O chisle elementarnykh par nad mnozhestvami [On the number of elementary pairs over sets]. *Issledovaniia v teorii algebraicheskikh sistem — Research in the theory of algebraic systems*, 73–82 [in Russian].

9 Yeshkeyev, A.R., & Mussina, N.M. (2019). Small models of hybrids for special subclasses of Jonsson theories. *Bulletin of the Karaganda University. Mathematics Series,* 3(95), 68–73.

10 Yeshkeyev, A.R. (2015). Strongly minimal Jonsson sets and their properties. *Bulletin of the Karaganda University. Mathematics Series,* 4(80), 47–51.

11 Yeshkeyev, A.R. (2015). Properties of lattices of the existential formulas of Jonsson fragments. *Bulletin of the Karaganda University. Mathematics Series,* 3(79), 25–32.

12 Yeshkeyev, A.R. (2021). On Jonsson varieties and quasivarieties. *Bulletin of the Karaganda University. Mathematics Series,* 4(104), 151–157.

13 Yeshkeyev A.R.Connection between the amalgam and joint embedding properties / A.R. Yeshkeyev, I.O. Tungushbayeva, M.T. Kassymetova // Bulletin of the Karaganda University. Mathematics Series. — 2022. — No. 1(105). — P. 127–135.

14 Yeshkeyev, A.R., & Omarova, M.T. (2019). Companions of (n1,n2)-Jonsson theory. *Bulletin of the Karaganda University. Mathematics Series,* 4(96), 75–80.

15 Barwise, J. (1982). Teoriia modelei: spravochnaia kniga po matematicheskoi logike. Chast 1 [Model theory: Handbook of mathematical logic. Part 1]. Moscow: Nauka [in Russian].

16 Mustafin, Y.T. (2002). Quelques proprietes des theories de Jonsson. *J. Symb. Log, 67*(2), 528–536.

17 Yeshkeyev, A.R., & Kassymetova, M.T. (2016). Ionsonovskie teorii i ikh klassy modelei [Jonsson Theories and their Classes of Models]. Karaganda: Izdatelstvo Karagandinskogo gosudarstvennogo universiteta [in Russian].

18 Yeshkeyev, A.R, & Zhumabekova, G.E. (2018). Companions of fragments in admissible enrichments. *Bulletin of the Karaganda University. Mathematics Series,* 4(92), 105–111.

19 Yeshkeyev, A.R., Omarova, M.T., & Zhumabekova, G.E. (2019). The J-minimal sets in the hereditary theories. *Bulletin of the Karaganda University. Mathematics Series,* 2(94), 92–98.

20 Yeshkeyev, A.R., Kassymetova, M.T., & Ulbrikht, O.I. (2021). Independence and simplicity in Jonsson theories with abstract geometry. *Siberian Electronic Mathematical Reports, 18*(1), 433–455.

21 Yeshkeyev, A.R. & Ulbrikht, O.I. (2016). JSp-cosemanticness and JSB property of Abelian groups. *Siberian Electronic Mathematical Reports, 13,* 861–874.

22 Yeshkeyev, A.R. (2020). Model-theoretical questions of the Jonsson spectrum. *Bulletin of the Karaganda University. Mathematics Series,* 2(98), 165–173.

23 Yeshkeyev, A.R. (2010). On Jonsson stability and some of its generalizations. *Journal of Mathematical Sciences, 166*(5), 646–654.

24 Yeshkeyev, A.R., & Omarova, M.T. (2021). An essential base of the central types of the convex theory. *Bulletin of the Karaganda University. Mathematics Series,* 1(101), 119–126.

25 Yeshkeyev, A.R., & Mussina, M.M. (2021). An algebra of the central types of the mutually model-consistent fragments. *Bulletin of the Karaganda University. Mathematics Series,* 1(101), 111–118.

26 Shelah, S. (1978). *Classification theory and the number of nonisomorphic models.* Amsterdam: North-Holland.

27 Yeshkeyev, A.R., Tungushbayeva, I.O. & Zhumabekova, G.E. (2023). The central type of a semantic pair. *Book of Abstracts – Logic Colloquium*, 184–185.

Buketov University