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A. Ashyralyev[1,2,3,*], B. Haso[4,5]

[1] *Bahcesehir University, Istanbul, Turkey;*
[2] *Peoples Friendship University of Russia, Moscow, Russian Federation;*
[3] *Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan;*
[4] *Near East University, Nicosia, Turkey;*
[5] *Department of Mathematics, High School, Duhok, Iraq*
(E-mail: aallaberen@gmail.com, bashar.chato1988@gmail.com)

Stability of the time-dependent identification problem for delay hyperbolic equations

Time-dependent and space-dependent source identification problems for partial differential and difference equations take an important place in applied sciences and engineering, and have been studied by several authors. Moreover, the delay appears in complicated systems with logical and computing devices, where certain time for information processing is needed. In the present paper, the time-dependent identification problem for delay hyperbolic equation is investigated. The theorems on the stability estimates for the solution of the time-dependent identification problem for the one dimensional delay hyperbolic differential equation are established. The proofs of these theorems are based on the Dalambert's formula for the hyperbolic differential equation and integral inequality.

Keywords: hyperbolic equation, time delay, Hilbert space, source identification, stability.

Introduction

There is always a major interest for the theory of source identification problems for partial differential equations since they have widespread applications in modern physics and technology. Subsequently, the stability of various source identification problems for partial differential and difference equations have been studied extensively by many researchers (see, e.g., [1–25] and the references given therein). In many fields of the contemporary science and technology, systems with delaying terms appear. The dynamical processes are described by systems of delay ordinary and partial differential and difference equations. The stability of the delay differential and difference equations have also been studied in many papers (see, e.g., [26–35] and the references given therein). In the present paper, the time-dependent identification problem

$$\begin{cases} \frac{\partial^2 u(t,x)}{\partial t^2} - \frac{\partial^2 u(t,x)}{\partial x^2} = b\frac{\partial^2 u(t-\omega,x)}{\partial x^2} + p(t)q(x) + f(t,x), \\ 0 < t < \infty, x \in (-\infty,\infty), \\ u(t,x) = g(t,x), -\omega \le t \le 0, x \in (-\infty,\infty), \\ \int\limits_{-\infty}^{\infty} \alpha(x)u(t,x)dx = \zeta(t), t \ge 0 \end{cases} \tag{1}$$

for one-sdimensional delay hyperbolic equation is considered. Here $u(t,x)$ and $p(t)$ are unknown functions. Under compatibility conditions, problem (1) has a unique solution $(u(t,x),p(t))$ for the smooth functions $f(t,x)((t,x) \in (0,\infty)\times(-\infty,\infty))$, $g(t,x)((t,x) \in [-\omega,0]\times(-\infty,\infty))$, $\zeta(t)(t \ge 0), q(x)$, and $\alpha(x)$,
$x \in (-\infty,\infty)$. Here b is a constant.

*Corresponding author.
E-mail: aallaberen@gmail.com

The theorems on stability

We have the following theorems on the stability of problem (1).

Theorem 1. Assume that $\int\limits_{-\infty}^{\infty} \alpha(x)q(x)dx \neq 0$ and $\int\limits_{-\infty}^{\infty} |\alpha(x)|\, dx \leq \alpha < \infty$. Then for the solution of problem (1) the following stability estimates holds:

$$\max_{0\leq t\leq\omega} |p(t)|\,,\ \max_{0\leq t\leq\omega} \|u_{tt}\|_{C(-\infty,\infty)}\,,\ \max_{0\leq t\leq\omega} \|u_t\|_{C^{(1)}(-\infty,\infty)}\,,\ \max_{0\leq t\leq\omega} \|u\|_{C^{(2)}(-\infty,\infty)}$$

$$\leq M\,(q,\alpha)\left[a_0 + \max_{0\leq t\leq\omega} \left\|f'(t)\right\|_{C(-\infty,\infty)} + \|f\,(0)\|_{C(-\infty,\infty)} + \max_{0\leq t\leq\omega} \left|\zeta''\right|\right],$$

$$a_0 = \max\left\{\max_{-\omega\leq t\leq 0} \|g_{tt}(t)\|_{C(-\infty,\infty)}\,,\ \max_{-\omega\leq t\leq 0} \ \|g_t(t)\|_{C^{(1)}(-\infty,\infty)}\,,\right.$$

$$\left.\max_{-\omega\leq t\leq 0} \|g(t)\|_{C^{(2)}(-\infty,\infty)}\right\},$$

and

$$\max_{n\omega\leq t\leq(n+1)\omega} |p(t)|\,,\quad \max_{n\omega\leq t\leq(n+1)\omega} \|u_{tt}\|_{C(-\infty,\infty)}\,,\ \max_{n\omega\leq t\leq(n+1)\omega} \|u_t\|_{C^{(1)}(-\infty,\infty)}\,,$$

$$\max_{n\omega\leq t\leq(n+1)\omega} \|u\|_{C^{(2)}(-\infty,\infty)} \leq M\,(q,\alpha)\left[a_n + \max_{(n-1)\omega\leq t\leq n\omega} |p(t)|\right.$$

$$\left.+ \max_{n\omega\leq t\leq(n+1)\omega} \left\|f'(t)\right\|_{C(-\infty,\infty)} + \|f\,(n\omega)\|_{C(-\infty,\infty)} + \max_{n\omega\leq t\leq(n+1)\omega} \left|\zeta''\right|\right],$$

$$a_n = \max\ \left\{\max_{(n-1)\omega\leq t\leq n\omega} \|u_{tt}(t)\|_{C(-\infty,\infty)}\,,\max_{(n-1)\omega\leq t\leq n\omega} \|u_t(t)\|_{C^{(1)}(-\infty,\infty)}\,,\right.$$

$$\left.\max_{(n-1)\omega\leq t\leq n\omega} \|u(t)\|_{C^{(2)}(-\infty,\infty)}\right\}, n = 1, 2, \cdots .$$

Here $C(-\infty,\infty)$ refers to the vector space of continuous functions $w(x)$ from the entire real line to $R = (-\infty,\infty)$ with norm

$$\|w\|_{C(-\infty,\infty)} = \sup_{x\in(-\infty,\infty)} |w\,(x)|\,.$$

Proof. We will seek $u(t,x)$, using the substitution

$$u(t,\ x) = w(t,\ x) + \eta(t)q(x), \tag{2}$$

where $\eta(t)$ is the function defined by the formula

$$\eta(t) = \int_{(n-1)\omega}^{t} (t-s)p(s)ds,\ \ \eta((n-1)\,\omega) = \eta'((n-1)\,\omega) = 0, n = 1, 2, \cdots .$$

It is easy to see that $w(t,\,x)$ is the solution of the problems

$$\begin{cases} \frac{\partial^2 w(t,x)}{\partial t^2} - \frac{\partial^2 w(t,x)}{\partial x^2} = \eta(t)q''(x) + bg_{xx}(t-\omega,x) + f(t,x), \\ 0 < t < \omega, x \in (-\infty,\infty)\,, \\ w(0,\ x) = g(0,x),\ \ w_t(0,\ x) = g_t(0,x),\ \ x \in (-\infty,\infty)\,, \end{cases} \tag{3}$$

and

$$\begin{cases} \frac{\partial^2 w(t,x)}{\partial t^2} - \frac{\partial^2 w(t,x)}{\partial x^2} = b\frac{\partial^2 w(t-\omega,x)}{\partial x^2} \\ + \left(\eta(t) + b\eta(t-\omega)\right) q''(x) + f(t,x), \\ (n-1)\,\omega < t < n\omega, x \in (-\infty,\infty)\,, \quad n = 2,3,\cdots, \\ w((n-1)\,\omega+,\ x) = w((n-1)\,\omega-,\ x), \\ w_t((n-1)\,\omega+,\ x) = w_t((n-1)\,\omega-,\ x), \\ x \in (-\infty,\infty)\,, n = 2,3,\cdots. \end{cases} \tag{4}$$

Now we will take an estimate for $|p(t)|$. Applying the integral overdetermined condition

$$\int\limits_{-\infty}^{\infty} \alpha(x)u(t,x)dx = \zeta(t)$$

and substitution (2), we get

$$\eta(t) = \frac{\zeta(t) - \int\limits_{-\infty}^{\infty} \alpha(x)w(t,x)dx}{\int\limits_{-\infty}^{\infty} \alpha(x)q(x)dx}.$$

From that and $p(t) = \eta''(t)$, it follows that

$$p(t) = \frac{\zeta''(t) - \int\limits_{-\infty}^{\infty} \alpha(x)\frac{\partial^2}{\partial t^2}w(t,x)dx}{\int\limits_{-\infty}^{\infty} \alpha(x)q(x)dx}.$$

Then, using the triangle inequality, we obtain

$$|p(t)| \leq \frac{|\zeta''(t)| + \int\limits_{-\infty}^{\infty} \left|\alpha(x)\frac{\partial^2}{\partial t^2}w(t,x)\right| dx}{\left|\int\limits_{-\infty}^{\infty} \alpha(x)q(x)dx\right|} \tag{5}$$

$$\leq k(q,\alpha)\left[\left|\zeta''(t)\right| + \left\|\frac{\partial^2}{\partial t^2}w(t,.)\right\|_{C(-\infty,\infty)}\right]$$

for all $t \in (0,\infty)$. Now, using substitution (2), we get

$$\frac{\partial^2 u\,(t,x)}{\partial t^2} = \frac{\partial^2 w\,(t,x)}{\partial t^2} + p(t)q(x).$$

Applying the triangle inequality, we obtain

$$\left\|\frac{\partial^2 u\,(t,\cdot)}{\partial t^2}\right\|_{C(-\infty,\infty)} \leq \left\|\frac{\partial^2 w\,(t,\cdot)}{\partial t^2}\right\|_{C(-\infty,\infty)} + |p(t)|\,\|q\|_{C(-\infty,\infty)}$$

for all $t \in (0, \infty)$. Therefore, the proof of Theorem 1 is based on the following theorem.

Theorem 2. Under assumptions of Theorem 1, for the solution of problems (3) and (4) the following stability estimates holds:

$$\max_{0\le t\le\omega}\|w_{tt}\|_{C(-\infty,\infty)},\ \max_{0\le t\le\omega}\|w_t\|_{C^{(1)}(-\infty,\infty)},\ \max_{0\le t\le\omega}\|w\|_{C^{(2)}(-\infty,\infty)} \tag{6}$$

$$\le M(q,\alpha)\left[a_0+\max_{0\le t\le\omega}\left\|f'(t)\right\|_{C(-\infty,\infty)}+\|f(0)\|_{C(-\infty,\infty)}+\max_{0\le t\le\omega}\left|\zeta''\right|\right],$$

$$a_0=\max\left\{\max_{-\omega\le t\le 0}\|g_{tt}(t)\|_{C(-\infty,\infty)},\ \max_{-\omega\le t\le 0}\|g_t(t)\|_{C^{(1)}(-\infty,\infty)},\ \max_{-\omega\le t\le 0}\|g(t)\|_{C^{(2)}(-\infty,\infty)}\right\},$$

$$\max_{n\omega\le t\le(n+1)\omega}\|w_{tt}\|_{C(-\infty,\infty)},\ \max_{n\omega\le t\le(n+1)\omega}\|w_t\|_{C^{(1)}(-\infty,\infty)},\ \max_{n\omega\le t\le(n+1)\omega}\|w\|_{C^{(2)}(-\infty,\infty)} \tag{7}$$

$$\le M(q,\alpha)\left[a_n+\max_{n\omega\le t\le(n+1)\omega}\left\|f'(t)\right\|_{C(-\infty,\infty)}+\|f(n\omega)\|_{C(-\infty,\infty)}+\max_{n\omega\le t\le(n+1)\omega}\left|\zeta''\right|\right],$$

$$a_n=\max\left\{\max_{(n-1)\omega\le t\le n\omega}\|w_{tt}(t)\|_{C(-\infty,\infty)},\max_{(n-1)\omega\le t\le n\omega}\|w_t(t)\|_{C^{(1)}(-\infty,\infty)},\right.$$

$$\left.\max_{(n-1)\omega\le t\le n\omega}\|w(t)\|_{C^{(2)}(-\infty,\infty)}\right\},n=1,2,\cdots.$$

Proof. First, we will prove that

$$\max_{0\le t\le\omega}\|w_{tt}\|_{C(-\infty,\infty)}\le M(q,\alpha)\left[a_0+\max_{0\le t\le\omega}\left\|f'(t)\right\|_{C(-\infty,\infty)}+\|f(0)\|_{C(-\infty,\infty)}+\max_{0\le t\le\omega}\left|\zeta''\right|\right]. \tag{8}$$

Applying the Dalambert's formula, we get the following formula

$$w(t,x)=\frac{g(0,x+t)+g(0,x-t)}{2}+\frac{1}{2}\int_{x-t}^{x+t}g_t(0,\xi)d\xi$$

$$+\int_0^t\frac{1}{2}\int_{x-(t-\tau)}^{x+(t-\tau)}\left[\eta(\tau)q''(\xi)+bg_{\xi\xi}(\tau-\omega,\xi)+f(\tau,\xi)\right]d\xi d\tau$$

for any $t\in[0,\omega]$, $x\in(-\infty,\infty)$. From that it follows that

$$w(t,x)=\frac{g(0,x+t)+g(0,x-t)}{2}+\frac{1}{2}\int_{x-t}^{x+t}g_t(0,\xi)d\xi$$

$$+\int_0^t\frac{\eta(\tau)}{2}\left[q_{x+(t-\tau)}(x+(t-\tau))-q_{x-(t-\tau)}(x-(t-\tau))\right]d\tau$$

$$+\int_0^t\frac{b}{2}\left[g_{x+(t-\tau)}(\tau-\omega,x+(t-\tau))-g_{x-(t-\tau)}(\tau-\omega,x-(t-\tau))\right]d\tau$$

$$+\int_0^t \frac{1}{2} \int_{x-(t-\tau)}^{x+(t-\tau)} f(\tau,\xi)d\xi d\tau.$$

Taking the derivatives, we get

$$w_t(t,x) = \frac{g_t(0,x+t)+g_t(0,x-t)}{2} + \frac{1}{2}\left[g_t(0,x+t)-g_t(0,x-t)\right]$$

$$+\int_0^t \frac{\eta(\tau)}{2}\left[q_{x+(t-\tau),t}(x+(t-\tau)) - q_{x-(t-\tau),t}(x-(t-\tau))\right]d\tau$$

$$+\int_0^t \frac{b}{2}\left[g_{x+(t-\tau),t}(\tau-\omega,x+(t-\tau)) - g_{x-(t-\tau),t}(\tau-\omega,x-(t-\tau))\right]d\tau$$

$$+\int_0^t \frac{1}{2}\left[f(\tau,x+(t-\tau)) - f(\tau,x-(t-\tau))\right]d\tau,$$

$$w_{tt}(t,x) = \frac{g_{tt}(0,x+t)+g_{tt}(0,x-t)}{2} + \frac{1}{2}\left[g_{tt}(0,x+t)-g_{tt}(0,x-t)\right]$$

$$+\int_0^t \frac{\eta(\tau)}{2}\left[q_{x+(t-\tau),tt}(x+(t-\tau)) - q_{x-(t-\tau),tt}(x-(t-\tau))\right]d\tau$$

$$+\int_0^t \frac{b}{2}\left[g_{tt}(-\omega,x+t) - g_{tt}(-\omega,x-t)\right]d\tau$$

$$+\int_0^t \frac{1}{2}\left[f_t(\tau,x+(t-\tau)) - f_t(\tau,x-(t-\tau))\right]d\tau.$$

Applying this formula and the triangle inequality and estimate (5), we get

$$\|w_{tt}(t,\cdot)\| \le M(q,\alpha)\left[a_0 + \max_{0\le t\le\omega}\left\|f'(t)\right\|_{C(-\infty,\infty)} + \|f(0)\|_{C(-\infty,\infty)} + \left|\zeta''(t)\right|\right]$$
$$+M(q)\int_0^t \|w_{\tau\tau}(\tau,\cdot)\|\,d\tau$$

for any $t\in[0,\omega]$. By the integral inequality, we get the estimate (8). Applying equation (3) and triangle inequality and estimate (8), we get estimate (6).

Second, we will prove that

$$\max_{n\omega\le t\le(n+1)\omega}\left\|\frac{\partial^2 w(t,\cdot)}{\partial t^2}\right\|_{C(-\infty,\infty)} \le M(q,\alpha)\left[a_n\right.$$

$$\left.+\max_{n\omega\le t\le(n+1)\omega}\left\|f'(t)\right\|_{C(-\infty,\infty)} + \|f(n\omega)\|_{C(-\infty,\infty)} + \max_{n\omega\le t\le(n+1)\omega}\left|\zeta''\right|\right], n=1,2,\cdots$$

Applying the Dalambert's formula, we get the following formula

$$w\left(t,x\right)=\frac{w(n\omega,x+t)+w(n\omega,x-t)}{2}+\frac{1}{2}\int\limits_{x-t}^{x+t}w_t(n\omega,\xi)d\xi$$

$$+\int\limits_{n\omega}^{t}\frac{1}{2}\int\limits_{x-(t-\tau)}^{x+(t-\tau)}\left[\left(\eta(\tau)+b\eta(\tau-\omega)\right)q''(\xi)+bw_{\xi\xi}(\tau-\omega,\xi)+f(\tau,\xi)\right]d\xi d\tau.$$

for any $t\in\left[n\omega,\left(n+1\right)\omega\right],x\in\left(-\infty,\infty\right).$ From that it follows that

$$w\left(t,x\right)=\frac{w(n\omega,x+t)+w(n\omega,x-t)}{2}+\frac{1}{2}\int\limits_{x-t}^{x+t}w_t(n\omega,\xi)d\xi$$

$$+\int\limits_{n\omega}^{t}\frac{\left(\eta(\tau)+b\eta(\tau-\omega)\right)}{2}\left[q_{x+(t-\tau)}(x+(t-\tau))-q_{x-(t-\tau)}(x-(t-\tau))\right]d\tau$$

$$+\int\limits_{n\omega}^{t}\frac{b}{2}\left[w_{x+(t-\tau)}(\tau-\omega,x+(t-\tau))-w_{x-(t-\tau)}(\tau-\omega,x-(t-\tau))\right]d\tau$$

$$+\int\limits_{n\omega}^{t}\frac{1}{2}\int\limits_{x-(t-\tau)}^{x+(t-\tau)}f(\tau,\xi)d\xi d\tau.$$

Taking the derivatives, we get

$$w_t\left(t,x\right)=\frac{w_t(n\omega,x+t)+w_t(n\omega,x-t)}{2}$$

$$+\frac{1}{2}\left[w_t(n\omega,x+t)-w_t(n\omega,x-t)\right]$$

$$+\int\limits_{n\omega}^{t}\frac{\left(\eta(\tau)+b\eta(\tau-\omega)\right)}{2}\left[q_{x+(t-\tau),t}(x+(t-\tau))-q_{x-(t-\tau),t}(x-(t-\tau))\right]d\tau$$

$$+\int\limits_{n\omega}^{t}\frac{b}{2}\left[w_{x+(t-\tau),t}(\tau-\omega,x+(t-\tau))-w_{x-(t-\tau),t}(\tau-\omega,x-(t-\tau))\right]d\tau$$

$$+\int\limits_{n\omega}^{t}\frac{1}{2}\left[f(\tau,x+(t-\tau))-f(\tau,x-(t-\tau))\right]d\tau,$$

$$w_{tt}\left(t,x\right)=\frac{w_{tt}(n\omega,x+t)+w_{tt}(n\omega,x-t)}{2}$$

$$+\frac{1}{2}\left[w_{tt}(n\omega,x+t)-w_{tt}(n\omega,x-t)\right]$$

$$+\int\limits_{n\omega}^{t}\frac{(\eta(\tau)+b\eta(\tau-\omega))}{2}\left[q_{x+(t-\tau),tt}(x+(t-\tau))-q_{x-(t-\tau),tt}(x-(t-\tau))\right]d\tau$$

$$+\int\limits_{n\omega}^{t}\frac{b}{2}\left[w_{tt}(-\omega,x+t)-w_{tt}(-\omega,x-t)\right]d\tau$$

$$+\int\limits_{n\omega}^{t}\frac{1}{2}\left[f_t(\tau,x+(t-\tau))-f_t(\tau,x-(t-\tau))\right]d\tau.$$

Applying this formula and the triangle inequality and estimate (5), we get

$$\|w_{tt}(t,\cdot)\|\leq M\left(q,\alpha\right)[a_n$$

$$+\max_{n\omega\leq t\leq(n+1)\omega}\left\|f'(t)\right\|_{C(-\infty,\infty)}+\left\|f\left(n\omega\right)\right\|_{C(-\infty,\infty)}+\max_{n\omega\leq t\leq(n+1)\omega}\left|\zeta''\right|\Big]$$

$$+M\left(q\right)\int\limits_{n\omega}^{t}\|w_{\tau\tau}(\tau,\cdot)\|\,d\tau$$

for any $t\in[n\omega,(n+1)\,\omega]$. By the integral inequality, we get the estimate (6). Applying equation (4) and triangle inequality and estimate (6), we get estimate (7). This completes the proof of Theorem 2.

Moreover, we have that

Theorem 3. Assume that $\int\limits_{-\infty}^{\infty}\alpha(x)q(x)dx\neq 0$ and $\int\limits_{-\infty}^{\infty}|\alpha(x)|^q\,dx\leq\alpha<\infty, 1\leq q<\infty, \frac{1}{q}+\frac{1}{p}=1.$

Then for the solution of problem (1) the following stability estimates holds:

$$\max_{0\leq t\leq\omega}|p(t)|\,,\ \max_{0\leq t\leq\omega}\|u_{tt}\|_{L_p(-\infty,\infty)}\,,\ \max_{0\leq t\leq\omega}\|u_t\|_{W_p^1(-\infty,\infty)}\,,\ \max_{0\leq t\leq\omega}\|u\|_{W_p^2(-\infty,\infty)}$$

$$\leq M\left(q,\alpha\right)\left[a_0+\max_{0\leq t\leq\omega}\left\|f'(t)\right\|_{L_p(-\infty,\infty)}+\left\|f\left(0\right)\right\|_{L_p(-\infty,\infty)}+\max_{0\leq t\leq\omega}\left|\zeta''\right|\right],$$

$$a_0=\max\left\{\max_{-\omega\leq t\leq 0}\|g_{tt}(t)\|_{L_p(-\infty,\infty)}\,,\ \max_{-\omega\leq t\leq 0}\ \|g_t(t)\|_{W_p^1(-\infty,\infty)}\,,\right.$$

$$\left.\max_{-\omega\leq t\leq 0}\|g(t)\|_{W_p^2(-\infty,\infty)}\right\},$$

$$\max_{n\omega\leq t\leq(n+1)\omega}|p(t)|\,,\quad\max_{n\omega\leq t\leq(n+1)\omega}\|u_{tt}\|_{L_p(-\infty,\infty)}\,,\ \max_{n\omega\leq t\leq(n+1)\omega}\|u_t\|_{W_p^1(-\infty,\infty)}\,,$$

$$\max_{n\omega\leq t\leq(n+1)\omega}\|u\|_{W_p^2(-\infty,\infty)}\leq M\left(q,\alpha\right)\left[a_n+\max_{(n-1)\omega\leq t\leq n\omega}|p(t)|\right.$$

$$\left.+\max_{n\omega\leq t\leq(n+1)\omega}\left\|f'(t)\right\|_{L_p(-\infty,\infty)}+\left\|f\left(n\omega\right)\right\|_{L_p(-\infty,\infty)}+\max_{n\omega\leq t\leq(n+1)\omega}\left|\zeta''\right|\right],$$

$$a_n=\max\left\{\max_{(n-1)\omega\leq t\leq n\omega}\|u_{tt}(t)\|_{L_p(-\infty,\infty)}\,,\ \max_{(n-1)\omega\leq t\leq n\omega}\|u_t(t)\|_{W_p^1(-\infty,\infty)}\,,\right.$$

$$\left.\max_{(n-1)\omega\leq t\leq n\omega}\|u(t)\|_{W_p^2(-\infty,\infty)}\right\},n=1,2,\cdots.$$

Here $L_p(-\infty,\infty)$ refers to the vector space of functions $w(x)$ from the entire real line to $R=(-\infty,\infty)$ satisfy the condition

$$\int_{-\infty}^{\infty} |w(x)|^p \, dx < \infty.$$

Conclusion

This paper is devoted to the time-dependent identification problems for delay hyperbolic partial differential equations with unknown parameter p(t). The theorems on stability estimates for the solution of this problem are established.

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А. Ашыралыев[1,2,3], Б. Хасо[4,5]

[1] *Бахчешехир университеті, Стамбул, Түркия;*
[2] *Ресей халықтар достығы университеті, Мәскеу, Ресей Федерациясы;*
[3] *Математика және математикалық модельдеу институты, Алматы, Қазақстан;*
[4] *Таяу Шығыс университеті, Никосия, Түркия;*
[5] *Математика факультеті, орта мектеп, Духок, Ирак*

Гиперболалық кешігу теңдеулері үшін стационарлы емес сәйкестендіру есебінің тұрақтылығы

Дербес туындылы дифференциалдық және айырымдық теңдеулер үшін уақытқа және кеңістікке тәуелді көзді анықтау есептері қолданбалы ғылымдар мен техникада маңызды орын алады және бірнеше авторлармен зерттелген. Сонымен қатар, кешігу логикалық және есептеуіш құрылғылары бар күрделі жүйелерде туындайды, мұнда ақпаратты өңдеу үшін белгілі бір уақыт қажет. Мақалада кешігуі бар гиперболалық теңдеу үшін стационарлы емес сәйкестендіру есебі зерттелген. Кешігуі бар бірөлшемді гиперболалық дифференциалдық теңдеу үшін стационарлы емес сәйкестендіру есебін шешу үшін орнықтылықты бағалау туралы теоремалар анықталған. Бұл теоремаларды дәлелдеу гиперболалық дифференциалдық теңдеу мен интегралдық теңсіздік үшін Даламбер формуласына негізделген.

Кілт сөздер: гиперболалық теңдеу, кешігу, Гильберт кеңістігі, көзді анықтау, тұрақтылық.

А. Ашыралыев[1,2,3], Б. Хасо[4,5]

[1] *Университет Бахчешехир, Стамбул, Турция;*
[2] *Университет дружбы народов России, Москва, Россия;*
[3] *Институт математики и математического моделирования, Алматы, Казахстан;*
[4] *Ближневосточный университет, Никосия, Турция;*
[5] *Средняя школа, Духок, Ирак*

Устойчивость нестационарной задачи идентификации для гиперболических уравнений с запаздыванием

Зависящие от времени и пространства задачи идентификации источника для дифференциальных и разностных уравнений в частных производных занимают важное место в прикладных науках и технике и изучались несколькими авторами. Кроме того, задержка возникает в сложных системах с логическими и вычислительными устройствами, где требуется определенное время для обработки информации. В настоящей работе исследована нестационарная задача идентификации для гиперболического уравнения с запаздыванием. Установлены теоремы об оценках устойчивости решения нестационарной задачи идентификации для одномерного гиперболического дифференциального уравнения с запаздыванием. Доказательства этих теорем основаны на формуле Даламбера для гиперболического дифференциального уравнения и интегрального неравенства.

Ключевые слова: гиперболическое уравнение, запаздывание, гильбертово пространство, идентификация источника, устойчивость.