Theorem 1. Let A_1, A_2be existentially closed models of some perfect Jonsson existentially complete theory T of given S-act which is closed under Cartesian products of their models. Let T_1, T_2be existentially complete Jonsson fragments of A_1 and A_2. With the condition that T_1 is perfect. Then T_2 will be perfect iff T_1 is model consistent with T_2.

Theorem 2. Let A_1, A_2 be existentially closed models of some perfect Jonsson existentially complete theory T of given S-act which is closed under Cartesian products of their models. Let T_1, T_2be existentially complete Jonsson fragments of A_1 and A_2. With the condition that T_1 is perfect. The hybrid of T_1 and T_2 will be perfect iff it is model consistent with T.

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THE PROPERTY OF NON-MULTIDIMENSIONALITY FOR J-BEAUTIFUL PAIRS IN ADMISSIBLE ENRICHMENTS

Yeshkeyev A.R., Kassymetova M.T., Zhumabekova G.E.
Karaganda Buketov University, Karaganda, Kazakhstan
E-mail: *aibat.kz@gmail.com, mairusha@mail.ru,galkatai@mail.ru*

We need the following definitions in order to formulate the main result.

Definition 1. [1] Jonsson theory T is called a perfect theory, if each a semanticmodel of theory T is saturated model of T^*.

Definition 2.[2] An enrichment of the Jonsson theory T is said to be permissible if any $\exists$-type in this enrichment is definable in the framework of considered stability.

Definition 3.[2] The Jonsson theory is said to be hereditary, if in any of its permissible enrichment any extension of it in this enrichment will be Jonsson theory.

Definition 4.[3]A set X is called a Jonsson set in the theory T, if itsatisfies the following properties:

1) X is a definable subset of C_T , where C_T is a semantic model of the theory T;

2) $dcl(X)$is a universe of existentially closed submodel C_T , where $dcl(X)$ isdefinable closure of X.

Definition 5. Let T be an $\exists$-complete Jonsson theory Jonsson theory of a countable language L, $N, M \in E_T$ and $M \preccurlyeq_{\exists_1} N$. We will call a pair$(N, M)$ is called a J-beautiful pair if it satisfies the following conditions:

1. M is $|T|^+ \exists_1$-saturated;

2. for each $\bar{b} \in N$ each $\exists$-type over $M \cup \{\bar{b}\}$ is realized in N.

Let T be a perfect $\exists$-complete Jonssontheory of a countable language L, C is its semantic model.

Let class $K = \{(C, M) \mid M \preccurlyeq_{\exists_1} C,\ (C, M) - J - \text{beautiful pairs}\}$. Consider the Jonsson spectrum of the class K:

$$JSp(K) = \{\Delta \mid \Delta\ is\ Jonsson\ theory\ and\ \Delta = Th_{\forall\exists}(C, M), where\ (C, M) \in K\}$$

We say that T_1 is cosemanticto $T_2 (T_1 \bowtie T_2)$if $C_{T_1} = C_{T_2}$,where C_{T_i} is semantic model of T_i , $i = 1; 2$. [3]

It is easy to notice that $JSp(K) /_{\bowtie}$ is the factor set of the Jonssonspectrum of the class $K$by $\bowtie$.

Consider some enrichment of the signatureσ and consider the central type of this enrichment for all Jonsson hereditarytheories$T \in [\Delta]$,$[\Delta] \in JSp(K) /_{\bowtie}$.

Further all considered theory will be hereditary, let C be the semantic model of the theory T, $A \subseteq C$. Let $\sigma_\Gamma(A) = \sigma \cup \{c_a | a \in A\} \cup \Gamma, \Gamma = \{P\} \cup \{c\}$. Let $\bar{T} = T \cup Th_{\forall\exists}(C, a)_{a\in A} \cup \{P(c)\} \cup \{"P \subseteq"\}$, where $\{"P \subseteq"\}$ is an infinite set of sentences expressing the this fact that the interpretation of the symbol P is an existentially closed submodel in the language of the signature $\sigma_\Gamma(A)$. I.e. the interpretation of the symbol P is the solution of the following equation $P(C) = M \in E_T$ in the language $\sigma_\Gamma(A)$. By virtue of the hereditary of the theory T the theory$\bar{T}$ will be a Jonsson theory. Consider all the completions of the theory $\bar{T}$ in the signature language $\sigma_\Gamma(A)$. Since $\bar{T}$ is a Jonsson theory, it has its center, and we denote it by $\bar{T}^*$ and this center is one of the above completions of the theory $\bar{T}$. This enrichment is denoted by $\odot$.

Further we will consider the notion "type p does not fork over A" in the meaning of the theorem8 from [3].

Definition 6.Let p be complete $\exists$-type over A, A is a Jonsson subset of CThen p is J-stationary over A if

1) p does not fork over A;

2) p has a unique consistent extension that does not fork over A.

Definition 7.

1) If $p(\bar{x}_1)$, $q(\bar{x}_2)$ are complete $\exists$-types over A, A is a Jonsson subset of C. $p$is said to be J-weakly orthogonal to q if and only if $p(\bar{x}_1) \cup q(\bar{x}_2)$ is an $\exists$-complete type (over A).

2) Let p_1 be a $\exists$-complete or J-stationary type and p_2 be a $\exists$-complete or J-stationary type. Then p_1 is J-orthogonal to p_2 if for any A, $Dom\, p_1 \cup Dom\, p_2 \subseteq A$, $A$is the universe of an $\exists_1$-saturated model and q_1 is weakly J-orthogonal to q_2, where q_1,q_2 are any J-non-forking extensions of p_1 and p_{21}over A respectively.

Definition 8. Let A be a Jonsson subset of the $C_T$semantic model, where T is some Jonsson theory. An $\exists$-complete type p is said to be J-multidimensional if p is orthogonal to any complete $\exists$-type over A. If T has a J -multidimensional type, then T is called a J-multidimensional theory. Otherwise, the theory T is called$J$ -non-multidimensional, or the theory of J -restricted dimension.

Finally, the main result is the following theorem for above enrichment $\odot$.

Theorem 1. Let T be a perfect, J-λ-stable $\exists$-complete Jonsson theory,K be the class of J-beautiful pairs of T. Let $[\Delta] \in JSp(K) /_{\bowtie}$be a complete for $\exists$-sentences class. Then the following conditions are equivalent:

1) the theory $\overline{[\Delta]}^*$ is non-multidimensional (in classical meaning [4]);

2) the theory $\overline{[\Delta]}$ is J-non-multidimensional.

Theorem 2.If the theory $\overline{[\Delta]}$is$J$-P- λ - stable, then it is J -non-multidimensional.

All concepts that are not defined in this abstract can be extracted from [1, 2,3, 4].

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FORCING COMPANIONS OF MUTUALLY CONSISTENT THEORIES IN PERMISSIBLE ENRICHMENTS

Yeshkeyev A.R.[1], Tungushbayeva I.O.[1], Omarova M.T.[1,2]
[1]*Karaganda Buketov University, Karaganda, Kazakhstan*
[2]*Karaganda University of Kazpotrebsoyuz, Karaganda, Kazakhstan*
E-mail: aibat.kz@gmail.com; intng@mail.ru; omarovamt_963@mail.ru

This study is devoted to the study of the forcing companions of the Jonsson AP-theories in the enriched signature. It is proved that the forcing companion of the theory does not change when expanding the theories under consideration, which have some properties, by adding new predicate and constant symbols to the language. The model-theoretic results obtained in the general form are supported by examples from differential algebra.

Definition 1 [3]. Let T be a theory of the language L. A forcing companion of the theory T is a theory T^f that satisfies the following equation:

$$T^f = \{\phi \mid T \Vdash \neg\neg\phi\}.$$

The following results were proved by J. Barwise and A. Robinson:

Theorem 1 [3]. Let T_1 and T_2 be the theories of the language L. Then T_1 and T_2 are mutually model consistent if and only if $T_1^f = T_2^f$.

Theorem 2 [3]. Let T be mutually model consistent with some inductive theory T'. Then $T' \subseteq E^a$. Therefore, if T is an inductive theory then $T \subseteq E^a$.

We are working within the framework of the following definition of Jonsson theory published in the Russian edition of [1].

Definition 2 [1, p. 80]. A theory T is called Jonsson if the theory T has at least one infinite model; T is an inductive theory;T has the amalgam property (AP) and the joint embedding property (JEP).

Definition 3 [2]. A theory T is called an AP-theory if, from the fact that it has the amalgam property, it follows that T also has the joint embedding property, i.e. $AP \rightarrow JEP$.

We consider the theories Δ_1, Δ_2, $\Delta_3$to satisfy the following conditions:

1) Δ_1 is an inductive theory that is not a Jonsson theory, but has a model companion which is the theory Δ_3,

2) Δ_2 is a hereditary Jonsson AP-theory that has a model companion, which is also Δ_3.

All three theories are mutually model consistent because Δ_3 is mutually model consistent with both Δ_1 and Δ_2, for which Δ_3 is the model companion, which means that Δ_1 and Δ_2 are mutually model consistent with each other. At the same time, according to Theorem 1, the forcing companions of mutually model consistent theories must coincide, which means that $\Delta_1^f = \Delta_2^f$. Δ_2 is a perfect Jonsson theory, while $\Delta_2^* = Th(C) = \Delta_3$, C is a semantic model of Δ_2. In addition, Δ_3 is also a forcing companion of Δ_2, i.e. $\Delta_3 = \Delta_2^f$. So we get $\Delta_1^f = \Delta_2^f = \Delta_3$.

We consider the following extensions of the theories Δ_1, Δ_2, Δ_3 in various language enrichment L by adding new constant and predicate symbols c and P. Let $\overline{\Delta_1}$ be a theory extending Δ_1 by enriching the language L with the predicate symbol P as follows:

$$\overline{\Delta_1} = \Delta_1 \cup \Delta_1^f \cup \{P, \subseteq\},$$

where $\{P, \subseteq\}$ is an infinite list of $\exists$-sentences and interpretation of P is an existentially closed submodel in amodel of Δ_1.