

$h_n(\bar{x}) = \prod_{i=1}^k h_{n_i}(x_i)$, $w_n(\bar{x}) = \prod_{i=1}^k w_{n_i}(x_i)$. Через $E_n^h(f)_X$, $(E_n^w(f)_X)$ соответственно будем обозначать

наилучшее приближение функции $f \in X[0,1]^N$ полиномами по системе Хаара (Уолша) порядка не выше $n_1 \times \dots \times n_N$ ($n_j \in N$) в метрике $X[0,1]^N$, где $X[0,1]^N = BC_p[0,1]^N$, $1 < p < \infty$ или

$X[0,1]^N = BV_p[0,1]^N$, $1 \leq p < \infty$, $E_n^-(f)_X = \inf_{c_i} \left\| f(x_1, \dots, x_n) - \sum_{i=1}^{s_i} c_i \chi_i(\bar{x}) \right\|_X$. Через $S_n^h(f)$, $(S_n^w(f))$

обозначим частичную сумму ряда Фурье по системе Хаара (Уолша) функции f . Через $K_{\alpha, \beta, \dots, \gamma}$ - обозначим положительные постоянные, зависящие от параметров $\alpha, \beta, \dots, \gamma$, вообще говоря, различные в разных формулах. Основной целью данной работы является доказательство следующей теоремы, являющейся аналогом прямой теоремы теории приближения функций полиномами по системе Уолша или Хаара.

Теорема 1. Пусть $f \in C_p[0,1]^N$, $1 < p < \infty$. Тогда верны неравенства

$$E_n^h(f)_{BV_p} \leq K_p \omega_{1-\frac{1}{p}}\left(f, \frac{1}{n_1}, \dots, \frac{1}{n_N}\right), \quad E_n^w(f)_{BV_p} \leq K_p \omega_{1-\frac{1}{p}}\left(f, \frac{1}{n_1}, \dots, \frac{1}{n_N}\right).$$

Теорема 2. Пусть $f \in C_p[0,1]^N$, $1 < p < \infty$. Тогда верны неравенства

$$\omega_{1-\frac{1}{p}}\left(f, \frac{1}{n_1}, \dots, \frac{1}{n_N}\right) \leq K_p E_n^h(f)_{BV_p}, \quad \omega_{1-\frac{1}{p}}\left(f, \frac{1}{n_1}, \dots, \frac{1}{n_N}\right) \leq K_p E_n^w(f)_{BV_p}.$$

Для случай функций одной переменной подобные теоремы были доказаны в работе [4].

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ON THE ESTIMATE OF THE NORM OF THE VECTOR-VALUED FOURIER MULTIPLIERS ON GENERALIZED PERIODIC MORREY SPACES

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In this paper we present the estimate of the norm of the Fourier multipliers on generalized periodic Morrey space by the norm on Bessel potential space.

Let $0 \leq p < \infty$ and let $\varphi : (0, \infty) \rightarrow (0, \infty)$. Then the generalized periodic Morrey space $M_p^\varphi(T^d)$ is the collection of all functions $f : R^d \rightarrow C$, 2π -periodic in each component, such that $f \in L_p(B(x, r))$ for all $x \in R^d$ and all $r > 0$ and

$$\|f\|_{M_p^\varphi(T^d)} := \sup_{x \in [-\pi, \pi]^d} \sup_{0 < r \leq 2\sqrt{d}\pi} \varphi(r) \left(\frac{1}{|B(x, r)|} \int_{B(x, r)} |f(y)|^p dy \right)^{\frac{1}{p}} < \infty.$$

In the definition of generalized Morrey space we assume, that $\varphi \in G_p$.

Let $0 < p < \infty$. Then $\varphi: (0, \infty) \rightarrow (0, \infty)$ belongs to the class G_p , if φ is essentially nondecreasing and there exist positive constants C, C' such that the inequalities $\varphi(t_1) \leq C\varphi(t_2)$ and $t_2^{\frac{d}{p}}\varphi(t_2) \leq C't_1^{\frac{d}{p}}\varphi(t_1)$ hold for all $0 < t_1 \leq t_2 < \infty$.

Let $M = \{M_j(x)\}_{j=0}^{\infty} \subset L_{\infty}(R)$ be a sequence of functions which can be represented as $M_j(x) = \int_{-\infty}^{x_1} d\mu_j(y)$; $j = 0, 1, 2, \dots$, where the μ_j 's; $j = 0, 1, 2, \dots$; are finite measures with uniformly bounded variation, i.e. $\sup_{j \in \mathbb{N}} \int_R |d\mu_j| < \infty$.

Let $k \geq 0$. Then Bessel potential space $H_2^k(R)$ is the collection of all $L_2(R)$ such that

$$\|f|H_2^k(R)\| := \|F^{-1}[(1+|\xi|^2)^{k/2}Ff(\xi)]|L_2(R)\| < \infty$$

Theorem. Let $1 \leq p < \infty$, $1 \leq q \leq \infty$ and a function $\varphi: (0, \infty) \rightarrow (0, \infty)$, $\varphi \in G_p$. Let $(\Lambda_j)_j$ be a given family of finite and nontrivial intervals. With d_j we denote the length of Λ_j . Let

$k > \frac{1}{2} + \frac{1}{\min(p, q)}$. Then there exists a constant C such that the inequality

$$\left\| \left(\sum_{j=0}^{\infty} \left| \sum_{k=-\infty}^{\infty} M_j(k) c_k(f_j) e^{ikx} \right|^q \right)^{1/q} |M_p^{\varphi}(T, l_q) \right\| \leq C \sup_{j=0, 1, \dots} \|M_j(d_j \cdot)|H_2^k(R)\| \| (f_j)_j |M_p^{\varphi}(T, l_q)\|$$

holds for all sequences of functions $M_j \in H_2^k(R)$ and all sequences $(f_j)_j$ of trigonometric polynomials such that $c_k(f_j) = 0$ if $k \notin \Lambda_j$, $j \in \mathbb{N}_0$. Theorem is a generalization of the corresponding result for $\varphi = |B(0, r)|^{\frac{1}{p}}$, for we refer to [1].

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SEMIFINITE TRACIAL SUBALGEBRAS

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Noncommutative Hardy space theory has received considerable progress since the seminal paper by Arveson [1] in 1967. He introduced the notion of finite, maximal, subdiagonal algebras \mathbf{A} of \mathbf{M} , as noncommutative analogues of weak* Dirichlet algebras. Many classical results of Hardy space have been successfully transferred to the noncommutative setting. The first author [2] obtained that if a tracial subalgebra has L_p -factorization ($0 < p < \infty$), then it is a subdiagonal algebra. In [3], the first author and Ospanov proved that if a tracial subalgebra \mathbf{A} has L_E -factorization, then \mathbf{A} is a subdiagonal algebra, where E is a symmetric quasi Banach space on $[0, 1]$.

We continue this line of investigation. The aim of this talk is to prove some characterizations of subdiagonal algebras of semifinite von Neumann algebras.

Theorem. *Let \mathbf{A} be a tracial subalgebra of \mathbf{M} with respect to \mathbf{D} . Then the following conditions are equivalent:*

1. \mathbf{A} is a subdiagonal algebra of \mathbf{M} .
2. \mathbf{A} is a τ -maximal tracial subalgebra of \mathbf{M} satisfying the unique normal state extension property.
3. \mathbf{A} satisfies L_2 -density and the unique normal state extension property.